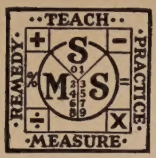
STANDARD SERVICE ARITHMETICS

GRADE SEVEN

KNIGHT
STUDEBAKER
RUCH

Standard Mathematical Service



Standard Mathematical Service

Edited by George W. Myers

Standard Service Arithmetics

GRADE SEVEN

by

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PREFACE

The *Standard Service Arithmetics,* of which the present book is for the seventh grade, is a main unit of an organized undertaking known as the *Standard Mathematical Service.* Fuller detail both as to the larger program and the series of arithmetics may be seen in the prefaces to the texts for Grades Three, Four, Five, and Six.

For the reader's convenience the salient features of the Series in general and of this volume in particular are briefly listed here:

(1) To secure a sound start for each half-year's work, the text provides carefully built diagnostic pre-tests and inventories of basic essentials (pp. 1-17 and 191-213) with a body of correlated re-teaching and remedial units (The Self-Help Study and Practice Lessons, pp. 387-469). Thus the text provides material which makes it practical for the busy teacher to conduct remedial work on the basis of individual needs.

(2) To guarantee genuine mastery from the start, new topics are developed gradually through multiple and varied models, always with due regard to vocabulary, illustrations, and explanations (see, for example, pp. 34-38, 61-67, 79-87, 115-149). The teacher is requested to make an unusually critical study of the instructional sections of the book. Such a study will reveal not perfect teaching but substantially successful teaching. Difficulties for the young learner are not lightly passed over. Questions to facilitate the "active attack" on the part of the pupil are liberally used. The text claims to be a **learning instrument** as well as a drill and problem book.

(3) To review and to organize items of knowledge and skill previously learned as isolated units, scrupulously calculated drills and reviews are provided for both skill-building (e. g. pages 34-40, 43-46, 95) and skill-holding (e. g. pages 41, 51-53, 58, 128-129).

(4) As means of arousing pupil interest, the following motivating devices are systematically used: (*a*) Competition with nation-wide groups of pupils of similar grade through standards objectively obtained (the Self-Testing Drills and Problem Scales, pp. 24-29 and 32); (*b*) competition with one's own past record (pp. 24-29); (*c*) prevention of initial discouragement through leisurely attack and mastery of new things (pp. 34-38, 61-67, 79-87, 144-147, 242-247); (*d*) persistent use of the principle of "awareness of success or failure at the time of learning" through Self-Testing Drills, Problem Scales, and other self-measuring devices; (*e*) variety in forms and types of drills and problems (pp. 28, 41, 52-53, 71, 91-93, 141-142); (*f*) socialized approaches to new topics presented (pp. 79, 90, 101, 119, 132, 245-246, 323).

(5) To arouse a personal sense of responsibility for learning, provision is made for an active, critical, self-reliant attitude toward arithmetic (pp. 2-3, 17-19, 23, 24-25, 32, 49, 56-57, 171-174, 213, 387, 467).

(6) To aid in mastering problem-solving there are definitely designed and consistently related lessons on problem study, problem analysis, and estimating and judging results (pp. 29-31, 171-174, 217-218, 369-370).

(7) To stimulate and facilitate self-remedial efforts of pupils, there is a persistent and consistent measuring and recording of individual achievement, with plainly keyed remedial material (See Chapters I and V, pp. 387-469, and the *Self-Help Index*). This material is available throughout the year at any time an individual weakness is revealed.

(8) To train the pupil into habits of using his book as means of discovering his own needs and of doing for himself the things that will meet those needs, a definite technique is provided (pp. 17-19, 23, 35, 187-189, 191-192, and the *Self-Help Index*).

(9) To prevent so far as possible the need for corrective arithmetic, special guidance is given to guard the pupil against the

known causes of common errors and bad habits (pp. 56-57, 133 171-173, 235-241, 281, 337-338).

The itemized major reasons for devoting a generous volume to each grade, which may strike some as a rather radical departure from traditional book-making for school arithmetic, are cited fully in the earlier books of this series. For convenience, these reasons are briefly stated here:

I. Specific learning demands for expansive treatment. Compression of subject-matter in a text that is actually to be used as a learning tool by children is fatal to the end desired. Specific difficulties cannot be ignored and explanations cannot be presented briefly if the child mind is to grasp the ideas.

(1) Step by step, development of the real difficulties of each new topic and process, so copiously exemplified, illustrated, and practiced as to be continually within the child's easy grasp.

(2) Chatty and informal silent reading exercises. This material clearly makes for bulk, but classroom use of the *Standard Service Arithmetics* has demonstrated the value of these exercises. They have been shown to be a marked factor in pupil appeal, in making the pupil feel that he is an actual partner in the learning process instead of a passive object of the teacher's efforts.

(3) Specially designed lessons to assist the child in acquiring skill in problem-thinking, problem-solving, and problem-checking.

(4) Every effort has been made in the problems of this text to present a complete problem situation with its full social setting, rather than to offer mere verbalized examples. The individual problems throughout the book therefore bulk greater than is usually the case.

(5) A series of drills for rapid oral work on fundamental skills and important ideas are set in "boxes." Experience has demonstrated the value of this scheme because of its attractiveness and convenience in handling.

(6) There are chapter summaries furnishing quick test-surveys of pupil-mastery of the main ideas of arithmetic. The thought,

judgment, and fact basis of arithmetic, often slighted, is given an organic part in this text.

II. Demands of individualizing instruction and provision for optional work. A rightly constructed text may greatly aid the teacher in the problem of dealing with individual differences. This text seeks to render such assistance in two main ways, viz.:

(1) By careful gradation of content to meet by easy stages the diverse abilities present in typical classrooms. This undertaking provides also for the superior levels of ability.

(2) By scientifically built diagnostic exercises continually reviving the essentials of previous years, the items of which are keyed into the units of Self-Help Study and Practice Lessons. A Self-Help Index, designed especially for pupil use, directs the pupil to crucial study and practice articles which he may use when his work on the Self-Testing Drills and other measuring devices shows him his "weak spots."

III. Mechanical demands of a working text. Any attempt to cut down the number of pages of a book through undue compression of the subject-matter must result in the sacrifice of appearance and of convenience in the actual use of the material. Attractiveness and workability have always been the first considerations in the building of these texts.

(1) To make plain to both teacher and pupil the plan and the "flow of the organization" there are (*a*) chapter headings of large units; (*b*) center headings of large sub-units, and (*c*) numbered article titles for smaller sub-units, with parenthetical phrases suggesting how to handle these articles in the class-rooms.

(2) Thirty standardized Self-Testing Drills for maintaining acquired skills. The typographical arrangement essential to the proper presentation of this material with its standards and explanations necessitates space.

(3) The explanations and standards necessitated by fourteen standardized Problem Scales demand space.

(4) Much care and space are given to pictorial, diagrammatic, and graphic aids to clearness and insight.

The foregoing list of space demands reveals the inherent need for more generous texts of arithmetic than are commonly available. Through years of patient, intensive, experimental study the authors proved to themselves that a genuinely modern treatment of arithmetic, based on the scientific findings of the last decade, demands leisurely and expansive development. They were then faced with a critical question: Should they attempt to compress their treatment into books of the conventional size, or should they break with tradition and present the work in the manner dictated by their findings?

The decision made is evident from an examination of the *Standard Service Arithmetics*. In the minds of the authors there could be no thought of trimming the final product to the dimensions and character of the customary reference-book type of text, thereby discarding the results of careful investigations made over a period of years.

Experimental use of the general scheme and the various devices of the *Standard Service Arithmetics* in a large number of classes widely distributed over the country proves that expansive treatment takes not more but less time than condensed textual material. The authors therefore lay heavy stress on the fact that expansive treatment **increases the rate of progress** while **decreasing** both the **difficulties** and the **failures.**

The *Standard Service Arithmetics* were made for, and largely in and by the nation-wide American public school. A large number of courses of study representing as many well-distributed sections of the country were canvassed for the basic, staple topics of seventh-grade arithmetic. If topics desired by certain seventh-grade teachers are not treated here, it is because they are not among the basic topics of nation-wide use in arithmetic.

There is at present wide disagreement as to what may be regarded as the best content of seventh-grade arithmetic. It is

agreed however by all courses of study and by all competent experts in this field that **the main functions of seventh-grade arithmetic must be the following,** which this text has made its prime objectives:

I. A genuine insight into and a real skill in the four fundamental processes with whole, fractional, decimal, and denominate numbers and with percentage and simple interest relations.

II. A real power of application of the foregoing processes to social and simple industrial and commercial situations.

III. A genuine though informal start toward a working control of the formula, the equation, and of the rudiments of intuitional and practical geometry.

Specific attention is given in this text to the seventh-grade function of weaving together into an organized general mastery the numerous items which the pupil has learned separately as distinct units. The technique of this solidifying function is found in the distributive rapid oral drills, the Self-Testing drills, the Problem Scales, and the numerous topic, chapter, half-yearly, and yearly summaries.

For average pupils this book will be found relatively self-sufficient, needing only in rare instances, well outside the province of a single text, support from independent drill and test services.

The authors invite especial attention to the technique employed to guarantee that the year's work be started right and that it be properly closed. Every effort has been put forth to make the pupil realize that everywhere and always it is **his learning, his arithmetic, his power to do** that are the objects of his concern. The treatment builds on the tenet that **learning is personal.**

The authors' acknowledgments are herewith made to the teachers and administrators, unnamed here only because their number runs to hundreds, who generously aided in passing this material through the refining fires of classroom experimentation.

THE AUTHORS

CONTENTS

PART ONE

CHAPTER I

TESTS OF ABILITIES BASIC TO SEVENTH GRADE. LEARNING TO USE YOUR BOOK

PAGE

Testing Arithmetic Ability 2
Test in Integers 3
Test in Fractions and Mixed Numbers 5
Test in Decimal Fractions 7
Test on Tables of Measure 8
Test on Denominate Numbers 10
Tests on Percentage 11
Test in Mensuration 14
Test in Checking 17
Learning to Use Your Book 17
Cumulative Review of Ideas of Arithmetic 19

CHAPTER II

THE SELF-TESTING DRILLS AND PROBLEM SCALES. THIRD USE OF PERCENTAGE. CHECKS FOR ACCURACY. ELECTRIC AND GAS METERS

The Self-Testing Drills 24
Self-Testing Drill No. 1 28
Practice in Problem Solving 29
Measurement of Problem-Solving Ability 32
Problem Scale No. I 32
The Third Use of Percentage 34
Self-Testing Drill No. 2 42
Continuing with Percentage 43
Checks with Integers 47
Self-Testing Drill No. 3 50
Percentage (Practice Work) 51
Problem Scale No. II 54

PAGE

Checks for Fractions and Mixed Numbers 56
Checks for Percentage 59
Self-Testing Drill No. 4 60
Electric and Gas Meters 61
Self-Testing Drill No. 5 67
Problem Scale No. III 69
Summary 71

CHAPTER III

FACTORS. CHECKING AND SAVINGS ACCOUNTS. MAKING CHANGE. BUDGET MAKING

Factors 73
Self-Testing Drill No. 6 78
The Essentials of Checking Accounts 79
Self-Testing Drill No. 7 88
Balancing Accounts 90
Problem Scale No. IV 96
Self-Testing Drill No. 8 99
The Savings Bank 100
Computing Interest on Savings 103
Accuracy in Making Change 110
Self-Testing Drill No. 9 113
Review of Percentage 115
Introduction to Budget Making 119
Problem Scale No. V 123
Self-Testing Drill No. 10 126
Summary 128

CHAPTER IV

MEASUREMENTS. THE FORMULA. THE ARITHMETIC OF COMMISSION

Some Fundamental Facts of Measurement. 130
Linear Measurements 132
Accurate Measurement of Short Distances 135
Drawing to Scale 138
Self-Testing Drill No. 11 143
The Formula 144
Problem Scale No. VI 150

PAGE

Measurement of Area 152
Self-Testing Drill No. 12 160
The Circle. 161
Area of the Circle 166
Self-Testing Drill No. 13 170
Problem Solving 171
Problem Scale No. VII 174
Self-Testing Drill No. 14 178
The Use of Commission 179
Self-Testing Drill No. 15 186
Summary 187

PART TWO

CHAPTER V

TESTING ARITHMETIC ABILITY

Test in Integers 191
Test in Fractions and Mixed Numbers 193
Test in Decimal Numbers 195
Test in Percentage 197
Test on Tables of Measure 203
Test on Denominate Numbers. 205
Test of Ability to Find References 206
Critical Thinking in Mensuration 208
Test on Checking 211
Test on Estimating 212

CHAPTER VI

THE MEASUREMENT OF VOLUMES

Kinds of Solids 214
Self-Testing Drill No. 16 216
Problem Scale No. VIII 218
The Measurement of Volumes 220
Self-Testing Drill No. 17 226
Review of Fundamentals 227
Problem Scale No. IX 231
Self-Testing Drill No. 18 234
Summary 235

CHAPTER VII

FINDING INTEREST BY THE FORMULA AND BY USE OF THE INTEREST TABLE. PROMISSORY NOTES. BANK DISCOUNTS

PAGE

Main Ideas of Interest 242
Self-Testing Drill No. 19 247
Mixed Practice with Interest 249
Computing Interest (The Interest Formula) 251
Amount 254
Self-Testing Drill No. 20 259
Problem Scale No. X 260
The Interest Table 262
Self-Testing Drill No. 21 270
Promissory Notes 271
Bank Discounts 277
Practice in Estimating 280
Mixed Practice 281
Self-Testing Drill No. 22 282
Problem Scale No. XI 284
Summary 286

CHAPTER VIII

GRAPHS. FINDING THE INTEREST RATE. PROFIT AND LOSS

Graphs as Pictures of Facts 290
Kinds of Graphs 291
Self-Testing Drill No. 23 305
Interest in Business 307
How to Find the Interest Rate 311
Self-Testing Drill No. 24 318
Problem Scale No. XII 319
Essentials of Percentage (Review Work) 320
Profit and Loss 323
How Profit Is Computed 327
Self-Testing Drill No. 25 333
Review of Mensuration 334
Problem Scale No. XIII 335
Finding and Correcting Errors 337
Self-Testing Drill No. 26 338
Summary 339

CHAPTER IX

ANGLES. THE MEASUREMENT OF DIRECTION. THE PROTRACTOR AND THE COMPASS. GETTING READY FOR EIGHTH GRADE

PAGE

The Use of the Angle in the Measurement of Direction . . . 343
Angular Measurement 345
The Protractor 349
Vocabulary of Angles 353
Self-Testing Drill No. 27 358
The Magnetic Compass 359
Review of Angular Measurement 362
Getting Ready for Eighth Grade 366
Self-Testing Drill No. 28 368
Problem Solving 369
Problem Scale No. XIV. 371
Mixed Review 373
Self-Testing Drill No. 29 377
Mixed Review 378
Self-Testing Drill No. 30 382
Inventory of Arithmetic Ideas 383

SELF-HELP STUDY AND PRACTICE LESSONS

How to Use the Self-Help Study and Practice Lessons . . . 387
Addition of Whole Numbers 388
Subtraction of Whole Numbers 392
Multiplication of Whole Numbers 396
Division of Whole Numbers 401
Checking for Accuracy 410
Estimating 412
Words Used in Arithmetic 414
Addition of Fractions and Mixed Numbers 415
Subtraction of Fractions and Mixed Numbers 420
Multiplication of Fractions and Small Mixed Numbers . . . 424
Division of Fractions and Small Mixed Numbers 431
Addition and Subtraction of Decimals 436
Multiplication of Decimals 437
Division of Decimals 438
Changing the Form of Denominate Numbers 439
Addition of Denominate Numbers 441

PAGE

Subtraction of Denominate Numbers 443
Multiplication and Division of Denominate Numbers . . . 444
Mensuration 445
Percentage 449
Bank Checking Accounts 462
Answer Key for Self-Help Lessons 463

Tables of Measure 470

Self-Help Index 471

General Index 479

Blank Progress Charts 485

STANDARD SERVICE ARITHMETICS

TELLER

SEVENTH-GRADE ARITHMETIC

PART ONE

CHAPTER I

TESTS OF ABILITIES BASIC TO SEVENTH GRADE. LEARNING TO USE YOUR BOOK

1. Uses of Arithmetic. (CLASS WORK.) Why should seventh-grade pupils be expected to study arithmetic? Do you think the only reason for teaching this subject in the seventh grade is that many pupils find it interesting and like to work problems and examples? Is arithmetic used very much by business men and women, or is it a subject that is never needed after a pupil leaves school?

How successful do you think a person could be if he could not even add and subtract accurately? Look at the pictures on the opposite page. Do they suggest some uses of arithmetic in life outside of school?

Most of you in a few years will ask some business man to employ you so that you can earn your own living. It is well known that business men prefer to employ boys and girls who can do arithmetic well. That is, the mastery of arithmetic helps you in securing and holding a worth-while position, and skill in arithmetic often helps to make promotions in the business world more rapid for the person who has such skill. In managing our own money matters, as

well as in earning a living, we need to be able to work arithmetic accurately and easily.

TESTING YOUR ARITHMETIC ABILITY

2. Getting a Good Start in Seventh-Grade Arithmetic. (CLASS WORK.) You can see that there is little use in learning new ways of doing work and new ideas in arithmetic until you are sure that you understand thoroughly the important things you have already studied before reaching the seventh grade. Therefore, the first thing to do is to judge your own ability in arithmetic. This you can do by taking the three *Location Tests* given in Arts. 3, 4, and 5, and the tests on certain other important things in arithmetic (Arts. 6 to 12). After each test there are "standards" by which you can decide whether you need to do some reviewing or whether you are ready to go on to new work.

MY TEST RECORD
John Emerson

Whole Numbers	Score	Need Review
Addition	4	No
Subtraction	3	Yes
Multiplication	3	No
Division	1	Yes
Fractions		
Addition	3	No
Subtraction	1	Yes
Multiplication	3	No
Division	3	No
Decimals		
Addition	2	No
Subtraction	3	No
Multiplication	2	Yes
Division	1	Yes

You may find it interesting and worthwhile to keep a complete record of your scores on the tests and of your needed review work as shown by your scores. Such a record will help you to know at any time exactly the "spots" on which you need more study and practice in order to do good seventh-grade work. The time given in class for review may not be all that you need. Your Test Record will guide you in doing extra review work. Even after all the tests have been completed, some of these things you have been tested on may still trouble

you as you go on into the new work of the seventh grade. Your Test Record will always show you just where you were weak in arithmetic when you began seventh-grade work.

There are a number of kinds of records you might make. The picture shows how one boy kept his complete record of review tests. If you need extra study on certain "spots" that trouble you, start as soon as you can to do all of the extra studying and practicing that are necessary to put those "trouble spots" out of your way.

On pages 387-469 you will find **Self-Help Study and Practice Lessons** that will give you just the review work you may need to overcome your "trouble spots." Turn now to page 387 and read Article 212.

You are not asked to spend any time on unnecessary review work. If you are up to standard, you do not need restudy. While others are spending time on needed review, spend your time in useful ways. Here are some suggestions: (*a*) Work on some other school subject on which you need review. (*b*) Get started on the reading of some good book. (*c*) Look through your arithmetic text to "survey" the year's work.

3. A Location Test in Integers. (WRITTEN WORK.) Your standing on the test on the next page will "locate" your ability in the addition, subtraction, multiplication, and division of integers, or whole numbers.

If you reach the standards given in the table which follows the test, you need not spend time in reviewing integers. If you fall below the standard given for any one of these processes (addition, subtraction, multiplication, and division), you should study all of the articles suggested for that process in the right-hand column of the table on page 5. These articles are on pages 388-409.

Notice that you find your standing on each of the four processes **separately.** Thus, your answers to Examples 1, 6, 8, and 13 will give you your score in addition. If you do not work at least three of the examples correctly, you will need to review addition of integers.

Directions for the test: You will be allowed **20 minutes** to work the 16 examples. Number your papers correctly and work the examples in order. Copy examples when necessary. Wait for your teacher to say "Go!" before beginning to work.

After your teacher says "Stop," she will read the answers. As your teacher reads the correct answers, mark each correct answer with a "C." You can then count up your score and see what review work you need, if any. Every example you work correctly counts one.

LOCATION TEST NO. I

1.
6454
9789
647
1065
4156

2.
Multiply:
6749
86

3.
Subtract:
13725
8267

4.
$49\overline{)44443}$

5.
Subtract:
62725
34459

6.
572
899
8735
3156
3858

7.
Multiply:
853
709

8.
996
7076
124
2097
4428

9.
$58\overline{)470308}$

10.
Subtract:
95370
26704

11.
Multiply:
7216
379

12.
Subtract:
126919
48529

13.	14.	15.	16.
346	$607\overline{)3157007}$	*Subtract:*	*Multiply:*
917		159757	5398
389		86978	453
645			
959			

STANDARDS FOR LOCATION TEST NO. I
GUIDE TO REVIEW WORK

PROCESS	EXAMPLES	POSSIBLE SCORE	STANDARDS (Number you should get right.)	ARTICLES FOR STUDY (You should do the work in these articles if you fall below the standard for any process. See pages 388-409.)
ADDITION	1, 6, 8, 13	4	3 or 4	Arts. 213-217
SUBTRACTION	3, 5, 10, 12, 15	5	4 or 5	Arts. 218-222
MULTIPLICATION	2, 7, 11, 16	4	3 or 4	Arts. 223-228
DIVISION	4, 9, 14	3	2 or 3	Arts. 229-235

4. A Location Test in Fractions and Mixed Numbers. (WRITTEN WORK.) The test on the next page is like the one in Art. 3 except that this test will "locate" your ability in fractions and mixed numbers.

Directions for the test: You will be allowed **20 minutes** to work the 16 examples. Wait for your teacher to say "Go!" before you begin work. **Copy** and work the examples in order. Number your examples correctly.

After your teacher says "Stop," she will read the answers. Count up your score; then look in the table of standards and see what processes, if any, you need to review.

LOCATION TEST NO. II

1.
$6\frac{1}{2}$
9
$3\frac{3}{4}$
$11\frac{5}{8}$

2. *Subtract:*
$14\frac{1}{4}$
$8\frac{3}{4}$

3. $\frac{2}{3} \times \frac{6}{7}$

4. $\frac{4}{5} \div \frac{1}{2}$

5. $9 \div 3\frac{3}{4}$

6. $\frac{4}{5}$ of $3\frac{3}{4}$

7. $3\frac{3}{4} + 9\frac{1}{6} + \frac{2}{3} + 11 + 4\frac{1}{2}$

8. $18 - 11\frac{1}{4}$

9. *Subtract:*
$17\frac{1}{2}$
$9\frac{2}{3}$

10.
$16\frac{1}{6}$
$11\frac{3}{8}$
$8\frac{1}{4}$
7

11. $6\frac{2}{3} \div 2\frac{3}{8}$

12. $5\frac{1}{12} \times \frac{4}{9}$

13.
$12\frac{3}{5}$
$6\frac{2}{3}$
21
$7\frac{1}{2}$
14

14. *Subtract:*
$11\frac{1}{6}$
$8\frac{3}{8}$

15. $4\frac{1}{2} \times 3\frac{1}{6}$

16. $2\frac{1}{12} \div 2\frac{1}{2}$

Standards for Location Test No. II
Guide to Review Work

Process	Examples	Possible Score	Standards (Number you should get right.)	Articles for Study (If you need review, see pages 415-435.)
Addition	1, 7, 10, 13	4	3 or 4	Arts. 239-247
Subtraction	2, 8, 9, 14	4	3 or 4	Arts. 248-252
Multiplication	3, 6, 12, 15	4	3 or 4	Arts. 253-262
Division	4, 5, 11, 16	4	3 or 4	Arts. 263-267

5. A Location Test in Decimal Fractions. (WRITTEN WORK.) Below is a test on decimals similar to the Location Tests you have already taken on integers and fractions. Use the table of standards on page 8 to "locate" your ability in decimals and to see whether you need further review of any of the four processes with decimals. The table also shows you which articles you should use if you fail to reach the standards.

Directions for the test: You will be allowed **20 minutes** to work the 15 examples. Number your paper correctly, make your figures easy to read, and work the examples in order. Wait for the teacher to give the signal, "Go!"

LOCATION TEST NO. III

1. *Copy and find the sum:* 8.23, 3.24, 8.38, 3.26
2. *Copy and add:* .87, .84, .21, .63, .26
3. Find the difference of 12.514 and 7.198.
4. *Subtract:* 1647.76 − 695.97

5. *Find the product:*

$$\begin{array}{r} 67.03 \\ \times\ 4.8 \\ \hline \end{array}$$

6. $24\overline{)120.96}$

7. $2.39\overline{)62.14}$

8. *Multiply:*

$$\begin{array}{r} 492 \\ \times\ 9.8 \\ \hline \end{array}$$

9. $12.624 \div 4.8$

10. *Multiply:*

$$\begin{array}{r} 79.4 \\ \times\ 5.07 \\ \hline \end{array}$$

11. $.57\overline{)28.272}$

12. *Subtract:*

$$\begin{array}{r} 82.943 \\ -\ 38.769 \\ \hline \end{array}$$

13. *Multiply:*

$$\begin{array}{r} 38.65 \\ \times\ 8.17 \\ \hline \end{array}$$

14. *Subtract:*

$$\begin{array}{r} 1238.33 \\ -\ 940.95 \\ \hline \end{array}$$

15. $.81\overline{)4811.4}$

STANDARDS FOR LOCATION TEST NO. III
GUIDE TO REVIEW WORK

PROCESS	EXAMPLES	POSSIBLE SCORE	STANDARDS (Number you should get right.)	ARTICLES FOR STUDY (If review is needed, see pages 436-438.)
ADDITION	1, 2	2	2	Art. 270
SUBTRACTION	3, 4, 12, 14	4	3 or 4	Art. 270
MULTIPLICATION	5, 8, 10, 13	4	3 or 4	Art. 271
DIVISION	6, 7, 9, 11, 15	5	4 or 5	Art. 272

TEST ON TABLES OF MEASURE

6. Test on Knowledge of the Tables of Measure. (WRITTEN WORK.) In each of the first 10 examples select the one of the five possible answers which makes a true statement. Number your papers correctly and write after each number the answer you select for that statement.

Time allowed: 8 minutes.

1. 30 oz. are about: $1\frac{1}{4}$ lb. $1\frac{1}{2}$ lb. 2 lb. 3 lb. $3\frac{1}{2}$ lb.

2. 18 pk. are: 144 qt. 108 qt. 90 qt. 72 qt. 36 qt.

3. 20 T. are: 400 lb. 2000 lb. 4000 lb. 20,000 lb. 40,000 lb.

4. 15 gal. are: 30 qt. 45 qt. 60 qt. 75 qt. 90 qt.

5. 1800 yd. are about: $\frac{1}{5}$ mi. $\frac{1}{3}$ mi. $\frac{2}{3}$ mi. 1 mi. 2 mi.

6. 5 sq. ft. are: 60 sq. in. 600 sq. in. 620 sq. in. 720 sq. in. 7200 sq. in.

7. 2 bu. are: 8 pk. 75 pt. 65 qt. 8 qt. 16 qt.

8. 1 cu. ft. equals: 27 cu. yd. 144 cu. in. $\frac{1}{3}$ cu. yd. 1728 cu. in. 12 cu. in.

9. $2\frac{1}{2}$ ft. are: 30 sq. in. 25 in. 30 in. 42 in. $\frac{1}{2}$ yd.

10. 640 A. equal: 1 sq. mi. 2 sq. mi. 3 sq. mi. 4 sq. mi. 5 sq. mi.

Write as your answers for Examples 11–20 the numbers which should stand on the dotted lines. Be sure to number the examples correctly.

11. 7 pt. = ... qt. ..... pt.

12. 142 in. = ... ft. ..... in.

13. 80 pk. = ... bu. ..... pk.

14. 2 ft. 6 in. = ... in.

15. 2 T. 1000 lb. = ... lb.

16. 18 ft. = ... yd. ..... ft.

17. 195 min. = ... hr. ..... min.

18. 21 qt. = ... gal. ..... qt.

19. 4 lb. 22 oz. = 5 lb. ..... oz.

20. 4 yd. 7 ft. = 6 yd. ..... ft.

If you did not get at least 12 of these examples correct, study the Tables of Measure on page 470 and use the Self-Help Lessons in Arts. 274-275.

TEST ON DENOMINATE NUMBERS

7. Test of Your Ability to Use Denominate Numbers. (WRITTEN WORK.) This test will show you your "trouble spots" in denominate numbers, if you have any.

After you have finished the examples, look over your answers a second time to make sure that they are in proper form. **Time allowed: 20 minutes.**

Change the numbers in 1 to 4 to the best form for answers.

1. 4 ft. 17 in. **2.** 3 lb. 20 oz. **3.** 3 hr. 72 min.

4. 3 hr. 65 min. 80 sec.

Work Examples 5-16. Answers must be in reduced form.

5.
4 ft. 6 in.
2 ft. 8 in.
1 ft. 11 in.
3 ft. 2 in.

6.
Subtract:
4 yd. 1 ft.
2 yd. 2 ft.

7.
Multiply:
4 lb. 9 oz.
4

8.
6)14 min. 12 sec.

9.
Subtract:
5 gal.
2 gal. 2 qt.

10.
3 pk. 4 qt.
6 qt.
4 pk.
2 pk. 3 qt.

11.
4)2 yd. 1 ft.

12.
Multiply:
3 ft. 9 in.
4

13.
3 gal. 3 qt.
2 gal. 1 qt. 1 pt.
4 gal. 2 qt. 1 pt.

14.
6)11 qt. 1 pt.

15.
Multiply:
4 bu. 3 pk.
7

16.
Subtract:
8 lb.
2 lb. 10 oz.

If you missed more than one example in any process, use the following as a guide for study:

Process	*Examples*	*Articles for Study*
Correct Form for Answers	1 to 4	275
Addition	5, 10, and 13	276, 277
Subtraction	6, 9, and 16	278
Multiplication	7, 12, and 15	279
Division	8, 11, and 14	280

TESTS ON PERCENTAGE

8. Test on Changing Forms. (WRITTEN WORK.) Write as your answer to each of the 16 examples below, the number which ought to be on the dotted line. Number the examples carefully.

Time allowed: 5 minutes.

1. $\frac{1}{4}$ = ... %
2. 1.30 = ... %
3. 50% = ... (fraction)
4. .195 = ... %
5. $16\frac{2}{3}$% = ... (fraction)
6. $\frac{9}{100}$ = ... %
7. 125% = ... (fraction)
8. .01 = ... %
9. $\frac{2}{3}$ = ... %
10. .175 = ... %
11. $\frac{2}{5}$ = ... %
12. $33\frac{1}{3}$% = ... (fraction)
13. $1\frac{1}{2}$ = ... %
14. $6\frac{1}{2}$% = ... (decimal)
15. $12\frac{1}{2}$% = ... (fraction)
16. $4\frac{1}{4}$% = ... (decimal)

If you had fewer than 12 examples correct in 5 minutes, use the Self-Help Lesson in Art. 287, pages 450–452.

9. Judgment Test in Percentage. (WRITTEN WORK.) Five ways are shown for working each example. Only one of these ways is correct. Write as your answer the **letter** of the correct way. Do **not** work the examples.

Time allowed: 7 minutes.

1. What is 6% of 897?

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
897 ×.6	.06)897	897 ×.06	897 ×.006	$\frac{.06}{897}$ or 897).06

2. 89 is what per cent of 766?

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
766 ×.89	$\frac{766}{89}$ or 89)766	766 ×89	766−89	$\frac{89}{766}$ or 766)89.00

3. Find $5\frac{1}{4}$% of 435.

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
435 ×.525	435 ×.0525	$435 \div 5\frac{1}{4}$	435 $\times 5\frac{1}{4}$	.0525)435.

4. What per cent of 386 is 655?

(*a*) $\frac{655}{386}$ or 386)655.00 (*b*) 386 ×.655 (*c*) 386 ×6.55

(*d*) $\frac{386}{655}$ or 655)386.00 (*e*) 655−386

5. What number equals 140% of 845?

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
845 ×.140	845 ×140	$\frac{140}{845}$ or 845)140.	845 ×1.40	$\frac{845}{140}$ or 140)845

The Self-Help Lesson in Art. 291, pages 457-460, gives more practice on examples like those on page 12 for pupils who had difficulty with the judgment test.

10. Calculating Percentage. (WRITTEN WORK.) Work the following examples. Take time to look over your answers after you have finished to make sure that they are correct and in good form.

Time allowed: 15 minutes.

1. Find 8% of \$5000.
2. 34 is what per cent of 85?
3. Find $5\frac{1}{2}\%$ of \$3600.
4. What per cent of \$3650 is \$226.30?
5. What number is 120% of 460?
6. 420 is what per cent of 350?
7. Find $4\frac{1}{4}\%$ of \$2500.
8. 2.6 is what per cent of 260?
9. If $700 = 100\%$ of N, what number equals 84% of N?
10. If 2460 equals 100% of a certain number, what per cent of the number does 320 equal?

If you did not get at least six examples correct in fifteen minutes, study the explanations and work the examples in the Self-Help Lessons, Arts. 288, 289, 290, 292.

MENSURATION

11. Informal Test in Mensuration. (Class Work.) Several answers are suggested for each example below. Select the best one as your answer.

1. Find the area of Figure I below.

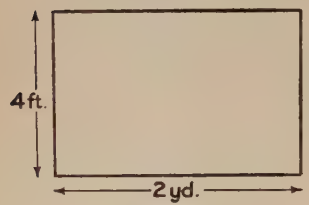


Fig. I

Answer (*a*). The answer is 8 yd. 8 yd. is correct, because to find the area of a rectangle we multiply the base by the altitude. Answers should be named; so the area is 8 yd. instead of 8 ft., because yards are bigger than feet.

Answer (*b*). The area is 8 sq. yd. Area is measured by square units; so the answer is 8 sq. yd.

Answer (*c*). The area is 24 ft. Dimensions cannot be multiplied until they are expressed in the same unit; so 2 yd. should be changed to 6 ft. When this is done, we get 24 ft.

Answer (*d*). The area is 24 sq. ft. Express both dimensions in the same unit (ft.). Multiply. Area is expressed in square units.

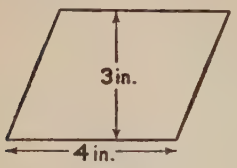


Fig. II

2. Find the area of Figure II below.

Answer (*a*). This is a "trick" example which cannot be worked because the length of only one side is given.

Answer (*b*). The area is 6 sq. in. To find the area of a parallelogram we take $\frac{1}{2}$ of the base times the altitude. $\frac{1}{2}$ of 3×4 is 6. The name of the answer is "sq. in.," because the dimensions are given in inches.

Answer (*c*). The area of the figure is 12 sq. in. To find the area of a parallelogram multiply the base by the altitude when both of these dimensions are expressed in the same unit. The answer will be expressed in square inches, because areas are always measured in square units.

3. Find the area of Figure III.

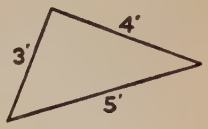


FIG. III

Answer (*a*). This is another example which cannot be worked, because we are not told which side is the base.

Answer (*b*). Area is 20 sq. ft. We find the area of a right triangle by finding $\frac{1}{2}$ of the longest side times the next longest side. 5 and 4 are the longest and next longest sides.

Answer (*c*). The area is 12 sq. in. Find the area of a right triangle by taking $\frac{1}{2}$ of the base times the altitude. The answer is expressed in sq. in., because the (′) is the short way of writing inch or inches.

Answer (*d*). The area is 6 sq. ft. (′) means foot or feet. The way to get the area of a right triangle is to find one half the product of the base times the altitude.

4. Find the area of Figure IV.

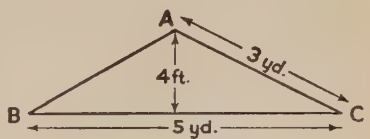


FIG. IV

Answer (*a*). This example cannot be worked, because we do not know the length of the line *AB*.

Answer (*b*). The example cannot be worked, but the reason given in (*a*) is wrong. The real reason why it cannot be worked is that the altitude and the base are not expressed in the same unit.

Answer (*c*). The area is $3\frac{1}{3}$ sq. yd. We find the area of any triangle by taking $\frac{1}{2}$ of the base times its altitude when these are expressed in the same unit. Thus: $\frac{1}{2}$ of 5 yd. $\times \frac{4}{3}$ yd. equals $3\frac{1}{3}$ sq. yd.

Answer (*d*). When base *BC* is expressed in feet, the area in sq. ft. equals $\frac{1}{2}$ of 15×4. The answer is 30 sq. ft. (Can answers (*c*) and (*d*) both be correct? Notice that one is expressed in sq. yd. and the other in sq. ft.)

5. Find the volume of Figure V below.

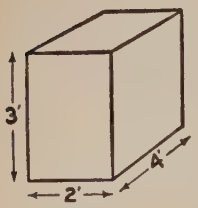


FIG. V

Answer (*a*). We have never been taught to find the volume of solids like Figure V; so of course we cannot work the example.

Answer (*b*). The volume is 24 sq. ft. To find the volume of a rectangular solid we multiply length times width times height. Volume is expressed in square units just as area is; so the answer to the problem must be in square feet.

Answer (*c*). We cannot find the volume, because we do not know what the figure represents.

Answer (*d*). The volume is 24 cu. ft. In finding the volume of a rectangular solid, use the rule volume = length $\times$ width $\times$ height. Remember that in finding volumes all dimensions used must be expressed in the same unit, and volume is measured in cubic units.

A careful study of Self-Help Lessons, Arts. 281 to 285, pages 446 to 450 will help to keep you from making such foolish answers as many of those given above.

CHECKING

12. Informal Test in Checking. (WRITTEN WORK.) Some of the answers to the examples below are wrong. If by checking you find any printed answers which are wrong, give the correct answers on your papers.

Time allowed: 10 minutes.

1.

786
347
21
831
72
2057

2.

Subtract:
43218
7142
47076

3.

807
208
6465
16140
167865

4.

7.32
2.4)17.581

5.

Subtract:
4 3 2.9 8
2 1.4
4 3 0 8.4

6.

314
400
72
632
89
1517

7.

61.03
42.6
36608
12206
24412
2599.868

8.

64
39)23556

If you have forgotten how to check, turn to the Self-Help Lesson, Art. 236, pages 410-411.

LEARNING TO USE YOUR BOOK

13. A Word to Seventh-Grade Pupils. (SILENT READING.) In studying any school subject it is important for you to know how your textbook is to be used. The facts on the next two pages will help you to use your arithmetic book in such way that it will be most helpful to you.

First: This book is divided into chapters. You are now studying Chapter I. Each chapter has a title which tells you what main topics are to be discussed. Look through Chapter II and see what work is taken up in that chapter. On what page does Chapter III begin? Chapter IV? Chapter VII? What does Chapter V cover?

Second: Each chapter is divided into several "articles," numbered in black-faced type at the left of the page. These numbers are called article numbers.

You are now reading Article 13. Each article has a title. Article 13 is called "A Word to Seventh-Grade Pupils." Find Article 50. On what page is it? What is its title? What are the page and title of Article 93?

Notice that after the title of each article there are some words in parentheses. Thus, the full title of Article 13 is **A Word to Seventh-Grade Pupils** (SILENT READING). The words in parentheses (SILENT READING) tell you how the article is to be studied. Find an article that is marked "WRITTEN WORK."

"Written Work" means that you will work alone with pencil and paper. "Class Work" means that your teacher will probably have a class discussion of the article. "Silent Reading" means that you will study the article by yourself and be ready to answer questions about it or to ask your teacher to explain anything that you do not understand.

There are several other kinds of titles in parentheses in this book. They will tell you in each case how the article is to be used.

Third: At times you will have to look back and review an article that you have already studied. Look now at Article 24. Do you see that it tells you to turn back to

Article 22 which you will already have studied when you get to Article 24?

Fourth: Spend a few minutes looking through this book. Do you find some of the **New-Work** signs like the one at the beginning of this article (page 17) and Article 21 (page 34)? Why do you suppose these signs are used?

Fifth: The **Self-Help Index,** beginning on page 471, is intended to help you to find quickly articles or sections of work to review whenever some sort of example or problem causes you trouble. Turn now to page 471 and read Article 295 carefully.

GENERAL REVIEW

14. Cumulative Review of the Ideas of Arithmetic. (Class or Written Work.) In this review work are a number of things about arithmetic that were not included in the tests you took in this chapter. It is important for you to know them. Supply the right word for each blank.

Use the Self-Help Lessons (pages 387-469) when you need to. Thus, for Examples 1 to 8, use Article 238. The article numbers following the other examples give the Self-Help Lessons that will help you in this review.

1. The answer in addition is called the.....
2. The number subtracted from is called the.....
3. The number which is subtracted is called the.....
4. Minuend less subtrahend is called the .....
5. The result in multiplication is called the .....
6. The number which is multiplied by another is known as the.....

7. In the example $675 \div 25$, the 675 is the.....and the 25 is the.....

8. The result in division is called the.....

9. Remainders in division can be expressed as..... or..... (231)

10. A rectangular solid has.....dimensions. (285)

11. To find the average of 5 numbers, first find their .....and then divide it by..... (238)

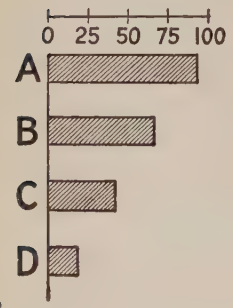
12. The dimensions of a rectangular solid are....., ..., and ..... (285)

13. A figure which has four square corners and four equal sides is called a..... (282)

14. In the fraction $\frac{3}{4}$, 3 is the.....and 4 is the ..... (238)

15. Fractions must have the same.....before they can be added or..... (249)

16. The.....of a fraction shows into how many equal parts a whole is divided, and the..... shows how many of these equal parts are being taken or used. (238)



17. To divide by a fraction, multiply by its..... (264)

18. A drawing used to picture facts, like the one at left, is called a.....

19. To change the form of a fraction without changing its....., multiply or.....both.....by the same number. (240)

20. The sum of the sides of a rectangle is called the .....of the rectangle. (282)

21. Area is always expressed in.....units. (283)

22. "Of" with fractions means..... (254)

23. Dimensions must be in the same.....before multiplying to find.....or..... (283, 285)

24. The statement 1″:10 mi., means that a distance of 50 miles would be represented in a drawing by a line..... inches long.

25. When two or more fractions have the same denominator, they are said to have a.....denominator. (238)

26. (*a*) 4%+8% =.....%.

(*b*) What is the average of 6%, 8% and 10%?

27. In the following numbers: (238)

(*a*) $\frac{1}{4}$ is a..... .....

(*b*) 5 is an integer or a ....

(*c*) 65.7 is a..... .....

(*d*) $3\frac{1}{2}$ is a..... .....

(*e*) $\frac{5}{3}$ is an..... .....

28. Fractions in answers should be in.....terms. (242)

29. Cancellation is possible in.....and.....of fractions, but not in.....or.....

30. To change a mixed number to an improper fraction, ...the whole number by the....., add this product to the....., and write the sum over the..... (256)

31. When a whole number or a fraction is inverted, the resulting fraction is called the.....of the number or fraction. (238)

32. $\frac{7}{10}$ is written in.....form, seven-tenths is written in word form, and .7 is written in.....form. (287)

33. Volume is expressed in.....units. (285)

34. 94 bu. is a.....number. (238)

35. There are as many decimal places in a product as there are in the.....and the.....together. (271)

36. To divide by 100, move the decimal point..... places to the....., and insert.....where necessary. (272)

37. To divide decimals, move the decimal point in the divisor as many places to the right as are needed to make the divisor a.....number. Next, move the decimal point in the dividend as many places to the right as you did in the..... Place decimal point in the..... exactly above the new decimal point in the..... Divide as you do in..... ..... (272)

38. Before a bank check can be "cashed," it must be .....on the back. (294)

39. At the end of each month a bank gives you a record of deposits and withdrawals called a..... ..... (294)

40. Savings banks pay.....on deposits. (293)

41. Percentage is a business way of saying..... (287)

42. An amount by which a bill is reduced for cash, prompt payment, or quantity buying is called a..... (293)

43. An article marked at "$1.00 less 40%" actually sells for..... (293)

44. To find what per cent 6 is of 11,.....6 by 11 and change the decimal answer to its.....form. (290)

45. Interest is commonly reckoned by the..... (293)

46. To find what per cent a certain number is of another, think what.....the one number is of the other. (290)

TO THE PUPIL

If you have taken these tests honestly and have done the study and practice that you need, you have made a good start in your arithmetic work for grade seven. You have done much toward learning to **help yourself,** and that is one of the valuable things arithmetic can help teach us.

Throughout the seventh and eighth grades you will be studying about many uses of arithmetic in life. Many new ideas, such as interest, commission, formulas, banking, etc., will be presented to you. But you should remember that first of all you must be able to work accurately with numbers. In all these new things that you are learning, you will have to add, subtract, multiply, and divide with whole numbers, decimals, and fractions. It will do you little good to know about interest and areas if you cannot accurately work the examples and problems that you will have to do.

The Self-Help Lessons are always ready to explain to you the hard spots in working with numbers, and they give you practice so that you can become an **accurate** worker. Don't neglect to help yourself when your work on the Self-Testing Drills shows that you are "falling down" regularly on certain kinds of examples.

CHAPTER II

THE SELF-TESTING DRILLS AND PROBLEM SCALES. THIRD USE OF PERCENTAGE. CHECKS FOR ACCURACY. ELECTRIC AND GAS METERS

THE SELF-TESTING DRILLS

15. Measuring Your Progress in Arithmetic. (SILENT READING.) Your teacher, your principal, and your superintendent of schools spend many hours during the year making and keeping records of the work you are doing. Why do they do this? They do it because it is the only way they have of telling whether the schools in your town or city are steadily becoming better or whether they are "going down hill." Those in charge of your schools want to send you out with the best kind of training, so that you will be able to do your work in the world to the best of your ability. They can't tell how well they are succeeding unless they keep records of your work and watch them closely.

On a modern dairy farm, the owner keeps records of the daily milk production of his cows. Farmers try different ways of improving crops and keep records to show which methods bring greatest results. Factory managers from month to month and year to year consult their charts and other forms of records to see whether their sales are increasing, whether their costs of manufacturing are being cut down, etc.

You know why people in all these different kinds of work keep records. They want to improve and to succeed, and

they can't tell whether they are improving unless they know their past records. Success in arithmetic is much like success in any other line of work. **You must know your past arithmetic records and try to beat them.** Your teacher wants you to make a splendid record, and she is ready to help you, but, after all, the biggest thing each of us can learn is to help himself. In this book you are given a chance throughout the year to find for yourself just what kind of work you are doing in arithmetic. You can keep a record of your progress from week to week.

16. Keeping Your Record. (CLASS WORK.) About once a week during the seventh-grade work you will be given the chance to run an arithmetic race **against your own past records.** Your score will be the **rating** which you earn. If you will turn to Art. 17, *Self-Testing Drill No. 1,* you will see that there are 15 examples and problems to be worked. You will be given exactly 15 minutes. After you have finished, and your papers have been corrected, you will look up your rating in the **Standards** given after the problems and examples. Ratings of 7-10 are very good. Ratings of 4-6 are fair or average, but ratings below 4 are poor.

Now look at the **Progress Chart** on page 27. This shows the record of Helen Cole, a seventh-grade pupil in the Emerson School, for all 30 Self-Testing Drills of the seventh grade. Every time that Helen took one of the drills, she recorded her rating on the Progress Chart. At the back of your book you will find a blank chart on which you can keep your own record. **Look at it now and make up your mind to keep your record neatly.**

In order to show that you understand how to keep your record, answer for your teacher the questions on page 26.

QUESTIONS

1. What is the name of the pupil who kept the Progress Chart shown on page 27? How old was she? What school and grade was she in?

2. What was her rating on Self-Testing Drill No. 1? On Drill No. 6? On Drill No. 13? Drill No. 25? Drill No. 30?

3. Did Helen beat her record on Drill No. 10 when she took Drill No. 11?

4. What do you suppose happened on Drills 17 and 18?

5. Did Helen improve very much during the seventh grade?

6. How many examples and problems did Helen work correctly on Drill No. 1 in order to earn the rating, 3? How many would she have had to have right to earn a rating of 6? Of 9?

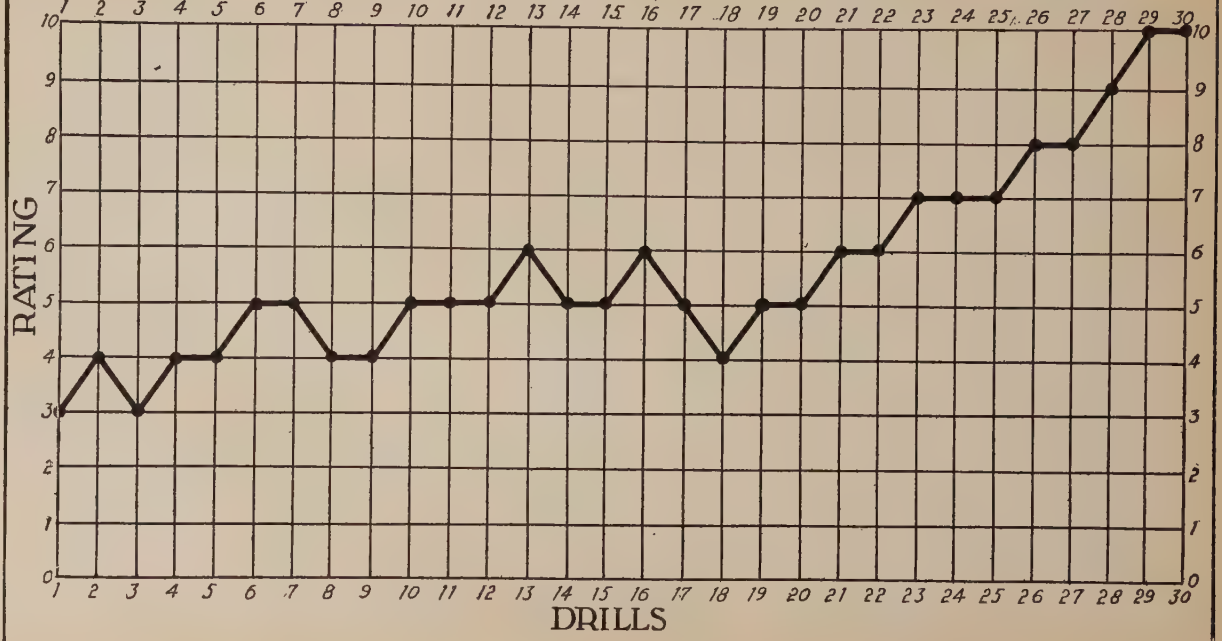
HINT: Her rating on Drill No. 1 was 3. Look in the table of Standards at the end of Drill 1, Art. 17. How many did she need to have correct to earn a rating of 3? Answer the other parts of Question 6 by the use of the table of Standards.

7. On what drills did Helen make her lowest ratings? On what drills did she make her highest ratings?

Now get ready to try Drill No. 1. Read the directions. Be sure that you understand these directions. Wait for the teacher to give the signal "Go!" Work hard and try to make a good record the very first time. After you have finished and your teacher has given you the right answers, find how many you worked correctly and look up your rating in the Standards. Record your rating on your own Progress Chart. Always record your **first** trial on the chart. Be sure to correct examples you get wrong on your first trial. This will help on the next drills.

PROGRESS CHART

Name Helen Cole Age 13 Grade 7 School Emerson



SELF-TESTING DRILL NO. 1

17. Directions. Wait for the signal "Go!" Whenever possible, save copying by placing your paper right below the example. Copy only when necessary.

You will be allowed exactly 15 minutes. Work as fast as you can without making mistakes. Do the examples and problems in order.

Try to make a high rating on your first drill. Rate yourself by the standards at the end of the drill, and mark your record on the Progress Chart.

1. $\frac{1}{2}+\frac{3}{4}+\frac{5}{8}=?$

2. Find $\frac{2}{3}$ of $\frac{1}{2}$.

3. *Subtract:*

1198291
403856

4. Find the difference of 68535.9 and 12794.6.

5. Helen thought that the correct answer to 1,365,919 minus 580,457 was 785,462. Betty said it was 885,462. Which girl was right?

6. *Add:*

47.5
84.0
836.5
16.7
673.8
33.4

7. $33269.6 \div 914 = ?$

8. *Multiply:*

8207
963

9. A discount of 25% is allowed on a bill of $4466.00. Find the amount of discount.

10. *Find the sum:*

2 ft. 7 in.
1 ft. 9 in.
11 in.
2 ft. 3 in.

11. *Multiply:*

4 ft. 9 in.
8

12. 75 is what per cent of 300?

13. Do not work this example, but choose the best of the three printed answers and write it as your answer.

15% discount on a bill of $504 is about *$150 $75 $50.*

14. Find the area of the square shown at the right.

15.* (*a*) $4\frac{1}{8}$ equals what improper fraction?
(*b*) $5 equals how many dimes?
(*c*) 18 gal. = how many quarts?
(*d*) $62\frac{1}{2}\%$ equals what fraction?

*To count as correct, each part of Example 15 must be correct.

STANDARDS

Number correct....	0–4	5	6–7	8	9	†	10	11	12	13–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

†To the Teacher: Here, as in a few other places, the experimental work showed that it was impossible to earn this exact rating. For explanation see the *Teacher's Guide Book.*

Correct all errors at once. If your rating on the first trial does not satisfy you, turn to the *Self-Help Index,* page 471, and find articles which contain the kind of help you need.

PRACTICE IN PROBLEM SOLVING

18. Advice about Problem Solving. (Class Work.) You will find your problem work easier if you follow the next three suggestions.

Suggestion 1. Read the problem several times. As you read it, ask yourself the four questions below.

(*a*) What does the problem ask for?
(*b*) What facts are given?
(*c*) What are the right steps or processes to be used?
(*d*) What would be a reasonable estimate for an answer?

Suggestion 2. Solve the problem and compare your estimated answer with the one actually found by working the problem. See if the two answers are about the same. (Estimating answers in this way will keep you from making foolish mistakes.)

Suggestion 3. Check your work to be doubly sure that you have made no mistakes.

Problem 1 below shows how you can use the three suggestions you have just read. Read Problem 1 carefully two or three times before you begin work.

Problem 1. The Brock and Steele Hardware Co. gives a 3% discount on all bills paid within 10 days from the date of purchase. Mr. E. R. King bought $35 worth of wire fencing. Three days later he sent a check in payment of the bill. How much did he actually pay?

Select the correct answers to each of the four questions given below and on the next page.

1. What is asked for?
   - (*a*) The date Mr. King paid his bill.
   - (*b*) The amount of fencing purchased.
   - (*c*) The rate of discount.
   - (*d*) How much Mr. King paid.
   - (*e*) The amount saved by paying the bill within 10 days.

2. What facts are given?
   - (*a*) Rate of discount allowed for cash within 10 days.
   - (*b*) The amount actually paid.
   - (*c*) The list price of the goods.
   - (*d*) When the bill was paid.
   - (*e*) The amount Mr. King saved.

3. What are the right steps or processes?

(*a*) Find 3% of \$35 and add this to \$35.
(*b*) Find 3% of \$35.
(*c*) Find 3% of \$35 and subtract this amount from \$35.
(*d*) Subtract 3¢ from \$35.
(*e*) Divide \$35 by .03.

4. What would be a reasonable answer (a good estimate)?

(*a*) \$34.97.
(*b*) About \$34.
(*c*) About \$1.05
(*d*) About \$30.
(*e*) About \$25.

You can estimate a reasonable answer like this: 3% means $\frac{3}{100}$. This is about $\frac{1}{30}$. $\frac{1}{30}$ of \$35 is about \$1. \$35 less \$1 is \$34.

Study the work and the check below and see how closely your estimated answer agrees with the correct answer.

$.03 \times \$35 = \$\ 1.05$ $\qquad$ $100\% - 3\% = 97\%$

$\$35 - \$1.05 = \$33.95$ ***Ans.*** $\qquad$ $.97 \times \$35 = \$33.95$ ***Check***

NOTE: You have not learned to check percentage problems in the way shown above. Do you see that this check is really another way of working the problem? Do you also see that it is really a check because it uses different numbers and a different order of steps or processes?

Now think out the way to work Problems 2 and 3. Ask yourself the same questions that are used in Problem 1.

Problem 2. A man had \$375.50 in the savings bank on January 1, 1927. During 1927 he earned \$1800 and put one-third of it into savings. What were his total bank savings on January 1, 1928, leaving out interest?

Problem 3. The city of Springdale paved an alley 300 ft. long and 18 ft. wide. How many square yards of paving were required?

MEASUREMENT OF PROBLEM-SOLVING ABILITY

19. Getting Ready for Problem Scale No. I. (SILENT STUDY.) In your seventh-grade work you are going to take 14 Problem Scales. The next article is *Problem Scale No. I.* Look at Art. 20 right now. Notice the **Standards** on page 34. Notice also that each problem or part of a problem has a number printed at its right. This number tells you how many points you will earn if you solve that problem or part of a problem correctly. In Problem Scale No. I you can earn 70 points by working every problem correctly.

How many points must you earn to do *Superior* work? How many points to do *Average* work? Do you think you can earn enough points to rate as doing *Good* work?

Before starting Problem Scale No. I, turn back to Art. 18 and study again the questions which you should ask yourself each time you work a problem.

PROBLEM SCALE NO. I

20. Problems Using Denominate Numbers. (WRITTEN WORK.) You can earn 70 points by working the ten problems correctly. Find your score by adding the numbers printed at the right of each problem you solved correctly. Rate yourself by the standards at the end of the problems.

Notice that Problem 6 has two parts. Number and letter your answers plainly.

1. The Ball Development Company bought a tract of land containing 1264 acres. The company cut this land up into farms for sale. During the first sale, Jones bought 76 A., White bought 134 A., Smith bought 89 A., Mrs. Brown bought 270 A., and Allen bought 307 A.

How much land did the Ball Development Company have left to sell? (4)

2. How many times as far is it from Boston to London as it is from Boston to New York? The distance from Boston to London is 3641 mi. The distance from Boston to New York is 232 mi. (Answer to one decimal place.) (5)

3. During one morning a saleslady sold the following amounts of cloth: $3\frac{1}{2}$ yd., $2\frac{1}{3}$ yd., $4\frac{5}{6}$ yd., and $5\frac{1}{4}$ yd. What was her total sale of cloth (in yards) for the morning? (5)

4. The Minneapolis baseball team did its spring training in San Antonio, Texas. After the training was over, the men left for Minneapolis. They boarded a train at San Antonio on Monday at 9 A.M. It arrived in Minneapolis the following Wednesday at 10:45 A.M. How long did it take the team to make the trip? (6)

5. In estimating the costs of a hiking trip, an allowance of 40¢ a meal was made for food. In finding the actual costs after the trip, it was found that the cost of food per meal was $32\frac{3}{5}$¢. How much too large was the estimate? (6)

6. The Oakdale farm planted 367 A. of potatoes. The yield was 35,232 bu. The Homestead farm planted 244 A. of potatoes. The yield was 23,058 bu.

(*a*) Which of the two farms above raised more potatoes per acre? (6)

(*b*) How much more per acre? (8)

7. Six men are buying a piece of land upon which they plan to build houses. The land has an area of $5\frac{1}{4}$ A. The men agree to divide the land equally among themselves. How much land will each man have? (7)

8. Find the area of the lot advertised on the sign shown at the right. (7)

9. The gasoline tank of a certain automobile holds 20 gal. The tank was $\frac{3}{4}$ full at the beginning of a trip. It was $\frac{1}{3}$ full at the end of the ride. How many gallons of gasoline were used on this trip? (7)

10. From the roof of a schoolhouse on a western prairie, one can see 7 miles of straight railroad track. How many tons of rails can be seen if there is a double track (4 rails) and each rail is 33 feet long and weighs 95 lb. per yard? (9)

STANDARDS

Superior work:	26–70 points	Slow work:	7–12 points
Good work:	19–25 points	Inferior work:	0– 6 points
Average work:	13–18 points		

THE THIRD USE OF PERCENTAGE

21. Three Uses of Per Cents. (Class Work.) You have already learned the first two uses of percentage. This article shows a good way of thinking and working examples that require a third use of per cents. Make sure that you understand this article.

THE FIRST USE

Example 1. Find 25% of 248.

25% of means ".25 ×," or "$\frac{1}{4}$ of," or "$\frac{1}{4}$ ×."

Example 1 asks you to find a **given per cent of a number.**

The Work

$$\begin{array}{r} 248 \\ \times .25 \\ \hline 62.00 \end{array} \quad \text{or} \quad \frac{1}{4} \times \cancel{248}^{62} = 62$$

THE SECOND USE

EXAMPLE 2. 62 is what per cent of 248?

"What per cent of" means "what fraction of."

Notice that in Example 2 you are asked to find **what per cent one number is of another.**

The Work

$\frac{62}{248}$ or $248\overline{)62.00}$ .25

Change answer from decimal form to per cent form. Why?

THE THIRD USE

EXAMPLE 3. 62 is 25% of a number. What is the number? (What is 100% of the number?)

If 62 is 25% of a certain number, you can find that number by dividing 62 by .25.

The Work

$.25\overline{)62.00}$ 248.

Divide: .25)62.00

Notice that 25% in decimal form is used as the **divisor.**

Notice that in Example 3 you are asked **to find a certain number.** You are **given a number which is a certain per cent of the number you are asked to find.** This is a use of percentage which you have never before learned. Study Examples 1, 2, and 3 again to make sure that you see exactly what the three uses of percentage are and how each is worked.

Important: This year you will do many examples like Example 3 above. In order to do such examples easily and correctly, you must be able to do two things:

1. Change any given per cent to decimal form (as 25% = .25).
2. Divide decimals correctly.

If you need to review these two things, turn to the Self-Help Study and Practice Lessons, pages 438 and 450-451.

The first use of percentage always calls for multiplication. (See Example 1, page 34.)

The second and third uses of percentage can always be worked by division. (See Examples 2 and 3, page 35.)

Some pupils can always tell when they should divide in a percentage example or problem, but they are not sure what number to use as the divisor. Study Examples 4 and 5 below and then do the exercise that follows.

EXAMPLE 4. 320 is what per cent of 800?

$$\begin{array}{r} .40 \\ \frac{320}{800} \text{ or } 800\overline{)320.00} \end{array}$$

$.40 = 40\%$, *Ans.*

Example 4 asks you to find what per cent one number (320) is of another number (800). Explain the work.

EXAMPLE 5. 320 is 40% of a number. What is the number? (Find 100% of it.)

Example 5 asks you to find a number (100%) when you know that 40% of the number equals 320. Explain the work. Does 800 seem a reasonable answer?

$$\begin{array}{r} 800. \text{ } \textit{Ans.} \\ .40\overline{)320.00} \end{array}$$

EXERCISE

For each example below, first talk over in class how you would do the work. Then work the examples.

1. If \$285 is 38% of a certain amount, what is the amount?
2. \$285 is what per cent of \$750?
3. 18 is what per cent of 20?
4. What per cent of 36 is 5.76?
5. 90% of a number = 18. 100% of the number = ?
6. 16% of an amount equals \$5.76. What is the amount?
7. 32% of $N = 182.4$. What does N equal?

22. Five Samples of the Third Use of Percentage. (STUDY PERIOD.) Study the following five samples of the third use of percentage. You will see that this use of percentage means finding 100% of a number when a given per cent of it is known.

Sample 1. 63 is 42% of what number? (Or, what is 100% of the number?)

Think 42% as .42

If 63 is .42 of a number, see if 100% of the number may be found thus: .42)63

150. *Ans.*
.42)63.00
42
210
210
00

(Notice that you use regular division of decimals)

Read the example again. Is 150 a reasonable answer?

Check by seeing what 42% of 150 is.

150
×.42

➡It will help you if you think of the third use of percentage as "hunting for 100% of something." You are given some other per cent to hunt with. You hunt successfully with the given per cent when you **change it to a decimal and use it as the divisor.**

Sample 2. 93.31 is 43% of what number?

Here is a good way to solve examples like Sample 2:

Think: What is 100%, or the whole of the number?

217. (Regular division of decimals)
.43)93.31

Think 43% as .43, the **divisor.**

Check by seeing if 43% of 217 is exactly 93.31.

217
×.43

Remember that the per cent which is given is to be used in decimal form as a divisor.

Sample 3 is probably as hard as any you will ever have to work. Be sure that you understand it thoroughly.

Sample 3. 17.4564 is 23.4% of what number?

Think of 23.4% as .234, to be **used as the divisor.**

$$.234\overline{)17.4564} \quad 74.6$$

(Regular division of decimals)

Check to see if 17.4564 is 23.4% of 74.6 by finding 23.4% of 74.6.

$$\begin{array}{r} 74.6 \\ \times .234 \\ \hline \end{array}$$

Sample 4. $66\frac{2}{3}\%$ of a number equals 18. What is the number?

This example can be most easily worked by the use of fractions. What fraction does $66\frac{2}{3}\%$ equal?

Since $\frac{2}{3}$ of the number $= 18$, $\frac{1}{3}$ of it must equal $18 \div 2$, or 9. Then $\frac{3}{3}$ (or the number) equals 3×9, or 27.

> In finding 100% of a number, if the given per cent equals a known fraction it often saves time to use the fraction instead of the per cent.

Sample 5. 32.5 is 125% of what number?

The number to be found must be less than 32.5, since we know that 32.5 is more than 100% of it.

Think 125% as 1.25, **used as the divisor.**

See if 32.5 is 125% of 26 by finding 125% of 26.

$$\begin{array}{r} 26. \\ 1.25\overline{)32.50} \\ 25\ 0 \\ \hline 7\ 50 \\ 7\ 50 \\ \hline \end{array}$$

Show another way in which you could work Sample 5. (HINT: 125% = what fraction?)

23. Changing Per Cents to Fractions. (ORAL WORK.) Give the fractions that are equal to these per cents.

1. 25%	6. $66\frac{2}{3}\%$	11. $87\frac{1}{2}\%$	16. 60%
2. $16\frac{2}{3}\%$	7. 80%	12. $133\frac{1}{3}\%$	17. 120%
3. 20%	8. 125%	13. $83\frac{1}{3}\%$	18. $33\frac{1}{3}\%$
4. $62\frac{1}{2}\%$	9. 75%	14. 40%	19. $112\frac{1}{2}\%$
5. 50%	10. $12\frac{1}{2}\%$	15. $37\frac{1}{2}\%$	20. 150%

24. Practice in the Third Use of Percentage. (WRITTEN WORK.) Write each example in the form you would use for working it. Change decimal points carefully. Then work and check every example. If you have trouble, study Art. 22 again.

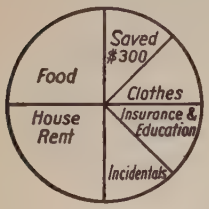
EXERCISE A

1. 135 is 45% of what number?
2. If 4 = 25% of a number, then.....is 100% of it.
3. \$2.40 = 12% of.....
4. 10 ft. is $33\frac{1}{3}\%$ of.....
5. 12 mi. is 8% of.....
6. $12\frac{1}{2}\%$ of a number is 24; 100% of it is.....
7. 66 = $37\frac{1}{2}\%$ of a number; so..... = 100% of it.
8. 20 is $2\frac{1}{2}\%$ of what number?
9. 36 is $16\frac{2}{3}\%$ of a number; 100% of it is.....
10. 12 is $37\frac{1}{2}\%$ of what number?

11. The team lost 4 ball games and won the rest. Games lost are represented by the shaded part of the diagram.

(*a*) How many games were played?
(*b*) What per cent were lost?
(*c*) What per cent were won?

12. A girl did 16 problems correctly, which was 80% of the number she tried. How many did she try?



13. This chart shows how Mr. Brown spent his salary in 1927. The whole chart represents his total salary.

(*a*) What % did he save?
(*b*) What % did he spend for food?
(*c*) How much rent did he pay?
(*d*) How much did he earn?

14. Do all the next questions ask the same thing?
(*a*) Of what number is 42 exactly $66\frac{2}{3}\%$?
(*b*) 42 is $66\frac{2}{3}\%$ of what number?
(*c*) $42 = .66\frac{2}{3}$ of what number?
(*d*) 42 is $66\frac{2}{3}\%$ of a number; what is 100%, or the whole of the number?
(*e*) 42 is $\frac{2}{3}$ of what number?

EXERCISE B

1. 52 is 80% of.....

2. 37.4 is 68% of.....

3. If 98 is 35% of a number, what is 100% of it?

4. 46% of $N = \$20.70$; 100% of $N =$.....

5. 125% of a number is 25. What is the number?

6. 99 is 18% of what number?

7. Of what number is 204 exactly 30%?

8. 105 is 84% of what number?

9. 450 is 20% of a number. Find 100% of the number.

10. 66 is 120% of what number?

25. Rapid Drill. (ORAL WORK.) The little examples in this drill will help you to make a good rating on Self-Testing Drill No. 2. Spend a few minutes in study and be ready with the answers. In division examples give only the first quotient figure.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)
1.	$\begin{array}{r} 12 \\ -4 \\ \hline \end{array}$	$1\frac{1}{4} = \frac{?}{4}$	$10+5+5$	How to find "average"?	$\begin{array}{r} 96 \\ \times 9 \\ \hline \end{array}$
2.	$\frac{2}{5}+\frac{1}{3}$	$39\overline{)277}$	$17+5+4$	$12-6$	*Reduce:* $11\frac{11}{10}$
3.	$\frac{5}{12}\times\frac{24}{25}$	$1.6+.2+.7$	50% of \$525	$47\overline{)375}$	What is a discount?
4.	$\begin{array}{r} 6 \\ -4\frac{3}{5} \\ \hline \end{array}$	$17+6+6$	$\begin{array}{r} 17 \\ -9 \\ \hline \end{array}$	$320 \div 1000 = ?$	$\begin{array}{r} 14 \\ \times 3 \\ \hline \end{array}$
5.	$1.1-.7$	Divide .036 by 6	$.0016 \times 100 = ?$	$\frac{5}{8} \div 2\frac{1}{2}$	Area of a $\triangle = ?$
6.	$68\overline{)612}$	$\begin{array}{r} 902 \\ \times 2 \\ \hline \end{array}$	What is a square?	What % of 7 is $3\frac{1}{2}$?	4 pk. 16 qt. = ... pk.
7.	$\begin{array}{r} 12 \\ -3 \\ \hline \end{array}$	$\frac{2}{3}$ = what decimal?	$\begin{array}{r} 59 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 66.0 \\ -31.3 \\ \hline \end{array}$	$16\frac{2}{3}\%$ of 96
8.	$\begin{array}{r} 68 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 850 \\ -560 \\ \hline \end{array}$	$12\frac{1}{2}\%$ = what fraction?	Meaning of "per cent"?	6000 lb. = ...T.

SELF-TESTING DRILL NO. 2

26. Directions. Wait for the signal "Go!" Wherever possible, avoid copying by placing your paper right below the example.

Time allowed: 15 min. Work as fast as you can without mistakes. Do the examples and problems in order.

Try to make a better record than you did on Drill 1. Find your rating in the standards at the end of the drill, and mark your record on the Progress Chart.

1. *Subtract:*

1067298
476239

2. *Subtract:*

804852
128467

3. One summer James earned \$327. He saved 50% of it. How much did he save?

4. *Subtract:*

9566.27
5092.68

5. *Subtract:*

$21\frac{1}{4}$
$17\frac{3}{4}$

6. $\frac{2}{3} \div \frac{1}{4} = ?$

7. $392\overline{)277536}$

8. Mr. White receives as his salary $12\frac{1}{2}\%$ of the amount of his sales. On a western trip he sold \$6118.00 worth of goods. Find his earnings from these sales.

9. *Multiply:*

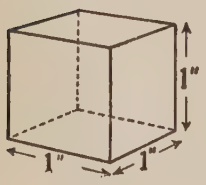
359.6
63.9

10. *Subtract:*

8 qt.
2 qt. 1 pt.

11. What per cent of 76 is 57?

12. What is the volume of the figure shown at the left?

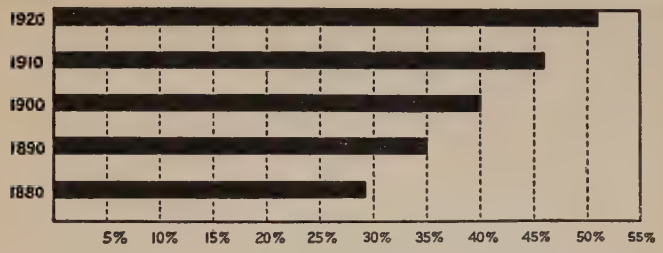


13. *Find the average:*

5084
491
6572
5976
9489

14. $2\overline{)6 \text{ pk. } 3 \text{ qt. } 1 \text{ pt.}}$

PERCENTAGE OF PEOPLE LIVING IN CITIES, BY TEN-YEAR PERIODS



15.* Answer the following questions by means of the graph above.

(*a*) What per cent of the people lived in cities in 1880? In 1900? In 1920?

(*b*) About how many times as large a per cent of the people lived in cities in 1910 as did in 1880?

*To count as correct, each part must be correct.

STANDARDS

Number correct.....	0–3	4	5	6	7	8	9	10	11	12–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

It will pay you to study your mistakes so that you know exactly why you made them.

CONTINUING WITH PERCENTAGE

27. Problems Requiring the Third Use of Percentage. (WRITTEN WORK.) The problems below give you a chance to show that you have mastered the third use of percentage, which you have studied in Arts. 21 and 22. All pupils doing good seventh-grade work should be able to work these problems without having to ask anyone for help.

1. A factory manufactured 600 chairs in one day. The manager said this was only 80% of maximum production.

How many chairs could this factory make a day if it was working at maximum production? (When you get the maximum production, check it by seeing if 80% of it does equal the 600 chairs, as stated in the problem.)

2. Mr. Stevens received a check for $404 as his monthly pay as a salesman. He is paid 15% of the amount of his sales. If he was paid the correct sum of money, what was the amount of his sales? (When you get the amount of his sales, find 15% of it. If your work is correct, the 15% of his sales should equal what sum of money?)

3. A principal reports that there are 897 pupils in his school. The building is over-crowded, because the number of pupils is 130% of the number for which the schoolhouse was planned. The building was planned to hold how many pupils? (Can you think of a way to check your answer to help make sure that it is correct?)

4. During the August furniture sale a sofa was marked at $140. It was advertised that this was 80% of the regular price. Mrs. Jones noticed that the regular price of the sofa was still marked on the price tag. What was the regular price if the marked-down price was "as advertised"?

5. A man had an income of $300 a year from money he had saved. His money was earning 6% interest. How much money had he saved?

6. The Commercial Club of Elmwood reports that the population is now 10,287 and that the town has grown 50% in population in the last 5 years. If this report of the Commercial Club is correct, what was the population of the town 5 years earlier? (HINT: Think of 10,287 as 150% of the population 5 years ago.)

28. Important Drill on the Three Uses of Percentage. (Oral Blackboard Work.) Do not take time to work the examples in this article, but study them so that you can tell and also show on the blackboard exactly how you would work them. The answers for Samples A, B, and C suggest how to do the work.

(*A*) Find 45% of 687.

Ans. I would multiply 687 by the 45% expressed as the decimal forty-five hundredths. (The work you would put on the blackboard is shown at the right.)

Multiply:
687
.45

(*B*) \$88.32 is what per cent of \$138?

Ans. I would think: 88.32 is what fraction of 138? It is $\frac{88.32}{138}$. This means 88.32 divided by 138. The result will be in decimal form, which must be changed to per cent form.

(*C*) If 56 is 13% of a number, what is the number?

Ans. I would hunt for 100% by using 13% in decimal form as the divisor. I would divide 56 by thirteen hundredths (or point 13).

.13)56

EXERCISE

1. 65 is what per cent of 809?

2. What is 56% of \$3702?

3. If I know that 774 is 42% of a number, how can I find the number?

4. When 957 is known to be 6% of a certain number, in what way can 100 per cent, or the whole of the number, be found?

5. By looking at the numbers I know that 34 is a rather small per cent of 6700; but how can I tell exactly what per cent 34 is of 6700?

6. Anyone can see that 4% of 855 is a rather small number, but how can I find out accurately what 4% of 855 is?

7. Most seventh-grade pupils could tell by looking at the numbers that if 60 is $5\frac{1}{2}\%$ of a number, 100% of the number (or the number itself) is about 1200. Can you show how to find the exact number of which 60 is $5\frac{1}{2}\%$?

8. Show how to find the answers to the three examples below by writing the work to be done for each one.

(*a*) 846 is 45% of what number?
(*b*) 705 is what per cent of 1580?
(*c*) What is $4\frac{1}{4}\%$ of \$576?

9. Show the work to be done in solving these three examples:

(*a*) Find $7\frac{3}{4}\%$ of 440.
(*b*) Find what per cent 356 is of 8300.
(*c*) Find 100% of a number when 64% of it is known to equal 1560.

10. Tell what percentage facts each of the examples below will give you:

(*a*) $465 \times .06$ (*b*) $\frac{57}{465}$ or $465\overline{)57.00}$ (*c*) $.43\overline{)5875}$

11. Mr. Adams bought an automobile for \$1150. Two years later he turned it in on a new car, and was allowed \$400 for it. What per cent of the original cost of the car was he allowed?

12. The United States produces about 62% of the copper supply of the world. In 1924 there were 955,745 tons mined in the U. S. About how many tons of copper were mined in the world in 1924?

CHECKS WITH INTEGERS

29. Ways of Checking for Accuracy. (CLASS WORK.) You have already learned ways of checking addition, subtraction, multiplication, and division of whole numbers. Other ways of checking these processes are often used.

CHECKS FOR ADDITION

Here is another way to check addition:

463	***The Check***
946	23 ←Sum of the units added separately.
931	21 ←Sum of the tens added separately.
69	29 ←Sum of the hundreds added separately.
724	3133 ←Sum of all columns.
3133	

Why, in the check, is the 1 in the 21 put under the 2 rather than under the 3?

When the example contains many large numbers, you can check by dividing the example into two parts.

8901	***The Check***
6248	
3913	
5074	
8934	24136
1176	
289	
4167	
38702*	14566
	38702*

In the sample at the left, the dotted line below the 5074 shows the example divided into two parts for checking.

The sum of the upper half of the example is 24,136. The sum of the lower half is 14,566. 24,136+14,566 =38,702.

The sum in the example when first added was also 38,702. Thus the check shows that the work is correct.

A CHECK FOR SUBTRACTION

Some pupils who like to "play" with numbers will find this check interesting.

Subtract:	*Check*
62073	62073
48129*	−13944
13944	48129*

Do you see why this check will work if you have subtracted correctly?

Try this check on a few examples "just for fun."

Caution: Some pupils, when they write their check, make this mistake: They write the result of their check for the answer of their example. Remember that the check is not the answer; it is only a way to help make sure that the answer is correct.

Explain the mistakes in the next examples.

$43189 - 21686 = ?$ *Ans.* 43189. $21582 - 11094 = ?$ *Ans.* 11094.

A CHECK FOR MULTIPLICATION

It is possible to check multiplication by using division. Divide the product by the multiplier and, if the work is correct, you will get the multiplicand as your check answer (Check I). Divide the product by the multiplicand, and you will get the multiplier (Check II).

The Work	*Check I*		*Check II*
483*	483*		42*
42*	42)20286		483)20286
966	168		1932
1932	348	or	966
20286	336		966
	126		
	126		

The division check for multiplication is not used very often. Checking by reversing multiplier and multiplicand is the one to depend on in almost all of your work.

Work the examples below. Use the new ways of checking that you have just studied.

1. *Add:*	2. *Multiply:*	3. *Subtract:*	4. *Add:*
189	582	34682	708
51	84	5890	82
40			9259
874			4
668	5. *Subtract:*	6. *Multiply:*	720
389	14787	641	75
19	4798	74	538

30. A Word on Checking. (Silent Reading.) After all, what is the main reason for checking? Is checking insisted upon just to "make work" in arithmetic, or is there a real use for it? In checking your work, do you very often find that you have made a mistake? Has checking saved you from mistakes? Do you find checking a useless bother, or does it make you feel more confident of your work because you have "hunted down" some mistakes?

You can think of checking as **self-criticism.** None of us is so perfect that he never makes mistakes. By checking one can usually find his mistakes and correct them.

The following quotation is from a composition written by a seventh-grade pupil. The subject of the composition was: "Why It Pays to Check."

I do not check because I like to. I do not check because my teacher makes me. But if I don't check and then hand in a wrong answer, I have to correct the mistake anyway, and I get a low

MANAGER: The new machines we are planning will cost us less than $1200.
PRESIDENT: Are you sure they will not cost more?
MANAGER: Yes, I have checked all the figures carefully.

mark for my work. So I check to make sure I have right answers; then I get better marks and do not have to do my work over again.

Does the composition express your feeling about why checking pays? Give any other ideas you may have about why it pays to check. The picture may suggest some.

SELF-TESTING DRILL NO. 3

31. Directions. Wait for the signal "Go!" Wherever possible, avoid copying by placing your paper right below the example.

Time allowed: 15 min. Work as fast as you can without mistakes. Do the examples and problems in order.

Try to beat your record on Drills 1 and 2. Find your rating on the drill, and mark it on the Progress Chart.

1. *Add:*

987
165
797
432
458
401

2. *Subtract:*

1551391
946719

3. $3\frac{1}{2} - 2\frac{1}{4} = ?$

4. *Subtract:*

1495713
608274

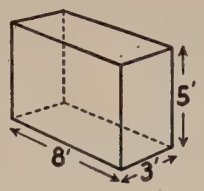
5. $4\frac{1}{2} \div 3\frac{3}{5} = ?$

6. Divide 672,768 by 768.

7. John said the difference of 41.4377 and 12.3869 was 19.0508. If he was wrong, give the correct answer.

8. Multiply 486.1 by 210.5.

9. Find the volume of the solid shown at the right.



10. *Subtract:*

4 yd. 2 ft.
1 yd. $2\frac{1}{2}$ ft.

11. What is 150% of 8322?

12. 5)3 T. 1000 lb.

13. *Find the average:*

349
298
20
691
619
255

14. 168 is what per cent of 84?

15. Find the interest on \$2065 for 9 months @ $3\frac{1}{2}$% per annum.

(*Annum* is a Latin word meaning "year.")

STANDARDS

Number correct	0–2	3–4	5	*	6	7	8	9	10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Are you correcting mistakes? Which examples did you miss? By using the *Self-Help Index*, page 471, you can find articles that will help you strengthen your "weak spots" in arithmetic.

PERCENTAGE

32. Judging in Percentage. (Oral Work.) It does little good to decide correctly that the way to solve a problem is to multiply and then make mistakes in the multiplication.

On the other hand, correct multiplication is of no use if the way to solve the problem is by dividing rather than multiplying. In arithmetic one must do two things: (1) Decide correctly **what** to do, (2) Do the work **accurately.**

In working percentage examples and problems pupils often do their calculation correctly, but choose the wrong way of solving the example. Thus, in the example: "If \$191.54 is 62% of a number, find the number," a pupil might do the work as shown at the right.

Here the pupil has multiplied correctly, but he should not have multiplied at all! What should he have done?

\$191.54
.62
38308
114924
\$118.7548 or \$118.75 *Ans.*

The work below gives you practice in deciding what to do in solving percentage examples. Right and wrong ways of solving each example are given. Choose the correct way.

1. Find 84% of 934.

(*a*) $.84\overline{)934}$ (*b*) 934×84 (*c*) $\frac{84}{934}$ or $934\overline{)84.00}$ (*d*) $934 \times .84$

2. I want to find the whole or 100% of a certain amount. I know that $12\frac{1}{2}\%$ of it is \$75.

(*a*) $1.25\overline{)75.00}$ (*b*) 75 or $75 \times \frac{1}{8}$ $\times .125$ (*c*) $.125\overline{)75.000}$ (*d*) $75 \times 12\frac{1}{2}$

3. 965 is what per cent of 450?

(*a*) $\frac{450}{965}$ or $965\overline{)450.00}$ (*b*) $\frac{965}{450}$ or $450\overline{)965}$ (*c*) 965×450 (*d*) $965 \times .45$

4. What is $\frac{1}{2}$ of 1% of \$764?

(*a*) $764 \times .005$ (*b*) 764 or $764 \times \frac{1}{2}$ $\times .50$ (*c*) $.005\overline{)764}$ (*d*) $764 \div \frac{1}{2}$

5. If 896 equals 100% of a number, what per cent of the number does 316 equal?

(*a*) 896 ×1.00 (*b*) 316 ×1.00 (*c*) $\frac{316}{896}$ or $896\overline{)316.00}$

(*d*) $\frac{896}{316}$ or $316\overline{)896.00}$

6. If $6\frac{1}{2}\%$ of a number equals 359, what is the number?

(*a*) $.65\overline{)359.00}$ (*b*) $.065\overline{)359.00}$ (*c*) 359 ×.065 (*d*) 359 ×.65

7. 906 is what per cent of *N* if 1263 is 100% of it?

(*a*) 1263 ×90.6 (*b*) $\frac{906}{1263}$ or $1263\overline{)906.00}$

(*c*) $\frac{1263}{906}$ or $906\overline{)1263.00}$ (*d*) 906 ×12.63

8. 100% of a certain amount equals what, if 3% of the amount is known to be $114?

(*a*) 114 ×.03 (*b*) 114 ×1.03 (*c*) $114\overline{)3.00}$ (*d*) $.03\overline{)114.00}$

9. A problem tells us that $456 is $4\frac{3}{4}\%$ of a certain amount and we must find the amount.

(*a*) 456 ×.0475 (*b*) 456 ×475 (*c*) $.0475\overline{)456.00}$ (*d*) $456 \times 4\frac{3}{4}$

10. We are asked to find 250% of 362.

(*a*) $\frac{250}{362}$ or $362\overline{)250.00}$ (*b*) 362 ×.25 (*c*) 362 ×25 (*d*) 362 ×2.5

If you found this work hard the first time you tried it, do not "trust to luck" that you will know what to do next time. Study Arts. 21 and 22; then try Art. 32 again.

PROBLEM SCALE NO. II

33. Problems Using Common Measures. (WRITTEN WORK.) You can earn 70 points by working every problem correctly. Find your score by adding the numbers printed at the right of each problem you worked correctly.

Spend a few minutes reviewing Article 18 before you begin working these problems.

Rate yourself by the standards at the end of the article.

1. I stored 38 gal. of gasoline in a barrel. Some of it was lost because of a small leak. When I measured the gasoline for use, I had only 36 gal. 3 qt. The rest had leaked out. How much had I lost because of the leak? (4)

2. James gathered a bushel of hickory nuts one fall and sold some of them to neighbors. Mrs. Callen bought a peck, Mrs. Rand bought 2 qt., Mrs. Smith took 1 qt. 1 pt., Mrs. Jones bought 3 qt., Mrs. Weller bought 1 peck, 2 quarts, and Mrs. Libby took 2 quarts. How many pecks, quarts and pints did he sell in all to these neighbors? (6)

3. How many quarts and pints of nuts did James have left after his sales reported in Problem 2? (7)

4. The winner of a cross-country run started at exactly 10:13 A.M. He crossed the finish line at exactly thirty-one minutes six seconds past ten A.M. What was his time for the run? (6)

5. In planning his house Mr. Alston found that he needed a hole 9 ft. deep, 20 ft. wide, and 30 ft. long for the cellar. How much earth had to be removed in order to make this hole? Give your answer in cubic yards. (7)

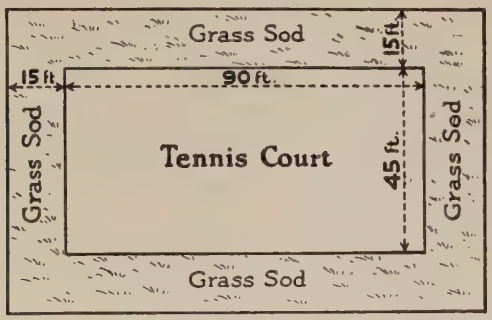
6. When the hole for the cellar was partly done, some boys calculated the amount of earth yet to be taken out.

They estimated that 1800 cu. ft. had been taken out. Using this estimate, find the amount of earth yet to be removed. (7)

7. Three boys collected maple sap. They worked together and agreed to divide the sap equally. They collected 16 gal. 3 qt. in all. What was each boy's share? (7)

8. A workman was strengthening the frame of an airplane. From a piece of steel he cut four braces of the following lengths: 2 ft. 9 in.; 1 ft. 3 in.; 11 in.; 3 ft. 7 in. He wasted 2 in. in fitting the braces. Find the total length of steel actually used in strengthening the plane. (8)

9. A good estimate of the area of forests in the United States is 159,800,000 acres. How many square miles does this area equal? (1 sq. mi. = 640 A.) (8)



10. The picture above is a rough drawing of a tennis court that the Girl's Athletic Club is building. They want to put grass sod around the court as shown. Find the cost of the sod at 54¢ a square yard. (10)

STANDARDS

Superior work:	19–70 points	Slow work:	6–7 points
Good work:	14–18 points	Inferior work:	0–4 points
Average work:	8–13 points		

CHECKS FOR FRACTIONS AND MIXED NUMBERS

34. Accurate Work in Fractions. (STUDY PERIOD.) Checks for fraction work are not so definite as the ones for whole numbers. The following suggestions, however, are very useful.

ADDITION OF FRACTIONS

A good thing to do in adding fractions to make sure your work is correct is to ask yourself certain questions as you look over the work after finishing it.

Ask these questions in looking over your work:

$4\frac{1}{2} \rightarrow \frac{6}{12}$
$2\frac{2}{3} \rightarrow \frac{8}{12}$
4
$\frac{1}{4} \rightarrow \frac{3}{12}$
$3\frac{5}{6} \rightarrow \frac{10}{12}$
$2\frac{7}{4}$ $\quad \frac{27}{12} = 2\frac{3}{12} = 2\frac{1}{4}$
$15\frac{1}{4}$ *Ans.*

1. Have I used the correct common denominator?
2. Is *every* fraction changed correctly?
3. Have I added all the numerators? ($\frac{27}{12}$)
4. Have I reduced correctly? ($\frac{27}{12} = 2\frac{1}{4}$)
5. Have I added the sum of the fractions ($2\frac{1}{4}$) to the whole numbers?

SUBTRACTION OF FRACTIONS

Check by adding as in whole numbers. Thus:

Subtract:

(*A*) 4 $\quad$ (*B*) $12\frac{1}{3}$

Add $2\frac{1}{2}$, $1\frac{1}{2}$ *Ans.* $\quad$ *Add* $7\frac{7}{15}$, $4\frac{13}{15}$ *Ans.*

$3\frac{2}{2}$, or 4. (*Check*) $\quad 11\frac{20}{15} = 12\frac{5}{15} = 12\frac{1}{3}$. (*Check*)

It is not necessary to write down all of the check, but it is wise to **think through** the check to make sure that no mistakes have been made.

MULTIPLICATION OF FRACTIONS

Check multiplication of fractions by asking yourself the following four questions as you look over your work after it is finished:

$$2\tfrac{2}{7}\times 2\tfrac{1}{10}=$$

$$\frac{\cancel{8}^{\,1}}{\cancel{7}_{\,1}}\times\frac{\cancel{21}^{\,3}}{\cancel{10}_{\,5}}=\frac{24}{5}=4\tfrac{4}{5}\ \ Ans.$$

1. Did I change mixed numbers correctly?
2. Did I cancel correctly?
3. Did I multiply correctly?
4. Did I reduce correctly?

DIVISION OF FRACTIONS

Check division of fractions by asking yourself these questions as you look over your work after you have completed it:

$$\tfrac{5}{6}\div 1\tfrac{7}{8}=\tfrac{5}{6}\div\tfrac{15}{8}=$$

$$\frac{\cancel{5}^{\,1}}{\cancel{6}_{\,3}}\times\frac{8}{\cancel{15}_{\,3}}=\frac{4}{9}\ \ Ans.$$

1. Did I change mixed numbers correctly?
2. Did I invert before I canceled?
3. Are the canceling and multiplying done correctly?
4. Was the answer properly reduced?

Using the checks which are given in this article will help you do more accurate work with fractions and mixed numbers. Solve the examples in the exercise below and then check each example carefully.

EXERCISE

1.

$4\frac{3}{5}$

$1\frac{1}{3}$

$\frac{4}{5}$

3

$4\frac{2}{3}$

2.

$4\frac{2}{7}\times 2\frac{1}{3}$

3.

$\frac{2}{3}\div 1\frac{3}{8}$

4. *Subtract:*

$7\frac{3}{4}$

$4\frac{7}{8}$

5.

$15\frac{1}{6}$

$2\frac{3}{4}$

23

$31\frac{5}{6}$

$2\frac{1}{2}$

6.

$1\frac{2}{5}\div\frac{4}{7}$

7.

$7\frac{3}{4}\times\frac{3}{5}$

8. *Subtract:*

$82\frac{2}{3}$

$59\frac{1}{5}$

35. Rapid Drill (ORAL WORK.) Many pupils like to study the following examples in order to give correct answers quickly when called upon. Study for at least five minutes.

In division examples give the first quotient figure only.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	*Add:* 4, 5, 6, 3	107 −57	89)269	56 ×9	$7\frac{1}{2} = \frac{?}{2}$	124 −75
2.	78)156	86 ×6	*Add:* 4, 7, 4, 5	$\frac{1}{2}$ of N = ...% of N	103 −88	*Reduce:* $\frac{7}{2}$ =
3.	.5 of N = ...% of N	132 −53	*Add:* 4, 4, 4, 3	93)661	1 sq. ft. = ... sq. in.	83 ×9
4.	81 ×4	92)368	$\frac{1}{2} = \frac{?}{12}$	*Add:* 2, 8, 7, 6	92)756	993 −346
5.	126 −88	*Add:* 1, 2, 7, 3	53 ×6	$\frac{5}{8} \div 7 = \frac{5}{8} \times$	108 −18	76)228
6.	59)556	Rule for area of rectangle?	42 ×4	32 −16	9+10+4	62 ×5
7.	*Add:* 6, 6, 5, 9	79 ×0	16 ft. = ...yd.	89)89	14% = what decimal?	63 −55

CHECKS FOR PERCENTAGE

36. Percentage Checked Like Decimals. (STUDY PERIOD.) In percentage examples we can check just as we do in whole numbers or in decimals. Thus:

(*A*) Find 142% of 624.

624	1.42
1.42	624
1248	568
2496	284
624	852
886.08 *Ans.*	886.08 *Check*

(*B*) \$616.20 is what % of \$948?

$\frac{616.20}{948}$ or $948\overline{)616.20}$

.65

.65 = 65% *Ans.*

\$948
.65
4740
5688
Check \$616.20

(*C*) If 198 is 24% of a number, what is the number?

(Hunt for 100%, using 24% in decimal form as a divisor.)

825 *Ans.*

$.24\overline{)198.00}$

192
60
48
120
120

825
.24
3300
1650
198.00 *Check*

➽This check helps us to see the relation among the different uses of percentage. If the work is correct, 100% of a number = 825 when 198 = 24% of the number. The check shows that 24% of 825 is 198. Do you see how the different ways of using percentage fit into each other?

(*a*) 198 is 24% of what number? 825 *Ans.*
(*b*) 198 is what per cent of 825? 24% *Ans.*
(*c*) Find 24% of 825. 198 *Ans.*

SELF-TESTING DRILL NO. 4

37. Directions. Wait for the signal "Go!" Wherever possible, avoid copying by placing your paper right below the example.

You will be allowed exactly 15 minutes. Work as fast as you can without making mistakes. Do the examples and problems in order.

Try to beat all your past records. Find your rating in the standards at the end of the drill, and mark your record on the Progress Chart.

1. Write the correct answer if the printed answer is wrong.

Subtract: 439.457
407.568
31.889

2. *Subtract:*
449458
281239

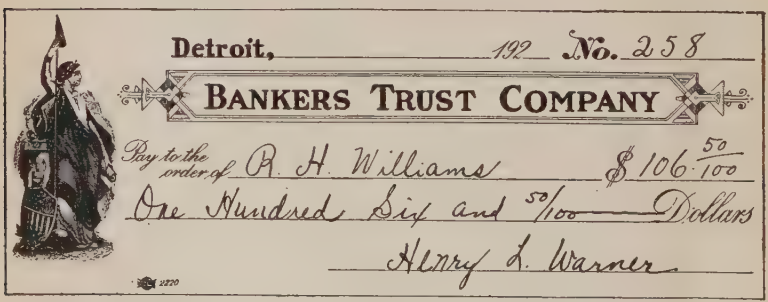
3. *Add:*
$3\frac{1}{2}$
$\frac{3}{8}$
$4\frac{7}{16}$

4.
$7\frac{2}{5} \times 3\frac{1}{8} = ?$

5. *Multiply:*
9714
569

6. *Multiply:*
37.59
175.2

7.
$168\overline{)8.3160}$


Detroit, 192 No. 258
BANKERS TRUST COMPANY
Pay to the order of R. H. Williams $106 50/100
One Hundred Six and 50/100 Dollars
Henry L. Warner

8.* (*a*) What is missing on the check? (*b*) Where must Mr. Williams write his name when he cashes the check?

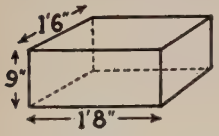
9. *Add:*

4 gal. 3 qt. 1 pt.
5 gal.
2 gal. 1 qt. 1 pt.
2 qt. 1 pt.
6 gal. 1 qt.

10. Is 4.27×6.9 about 2800, 28, or 2.8?

11. 3258 is what per cent of 36,200?

12. My savings account, earning 4%, pays me $32.08 per year. How much money do I have in savings?



13. *Multiply:*

3 yd. 2 ft. 4 in.
5

14. *Find the average:*

56879
67887
47997
38262

15. Find the sum of the areas of the six sides or surfaces of the box shown above. The box is in the shape of a rectangular solid.

*To count Example 8 correct, each part must be correct.

STANDARDS

Number correct	0–1	2	3	4	5	*	6	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

ELECTRIC AND GAS METERS

38. The Electric Meter. (Silent Reading.) Most homes now are lighted by means of electricity. The electric companies charge for the exact amount of electricity used. In order to do this, there must be some sort of measuring instrument which will record the amount of electric current used month by month. For this purpose **electric meters** are used.

There are many forms of electric meters, but all are read in a similar way. The meter is provided with three, four, or more dials like those in the picture below.

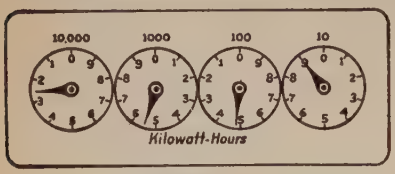


Fig. 1
(Each division on the "10" dial equals 1 kilowatt-hour.)

The dial farthest to the right (the "10" dial) shows ten divisions, much like a clock. Each division represents one **kilowatt-hour** (the unit by which electricity is measured). When the hand on this dial moves all the way around the circle (makes a complete circuit) from zero to zero, the hand on the "100" dial moves from 0 to 1.

In the same way, when the hand on the "100" dial makes one complete circuit from 0 to 0, the hand on the "1000" dial moves from 0 to 1. A complete circuit of the hand on the "1000" dial moves the hand on the "10,000" dial from 0 to 1.

Do you see that the reading of an electric meter calls for work much like decimal numbers? Look at the picture again. The first dial at the left registers **thousands** (of kilowatt-hours); the next dial, **hundreds;** the next, **tens;** and the right hand dial, **ones.** The meter now reads:

"2 thousands, 5 hundreds, 4 tens, and 9 ones," or, simply, "2549 kilowatt-hours."

You should notice that we read each dial by taking the number which the hand is now pointing at or **has just passed by.** Notice also that the numbers on some of the dials run like those on a clock, but the numbers on other dials run in the opposite direction.

The Light Company usually sends a man once a month to read the meter. It is a good plan for some one in the

family to read the meter at the time the "meter man" comes in order to check the reading by the company. To find the number of kilowatt-hours used in any month, the meter must be read twice; that is, on the first of that month and on the first of the following month. The difference in the readings gives the number of kilowatt-hours used during the month.

You may wonder why the dial at the extreme right is marked "10" but you are told to read it as **ones.** In the type of meter shown in Fig. 1 each division of the right hand dial represents one kilowatt-hour. In making one complete circuit, the hand passes over ten such divisions. In this case the label above the dial shows the number of kilowatt-hours registered by one complete circuit of the dial.

Some of you, if you examine the electric meters in your own home, will find the dials similar to those in Fig. 2 or Fig. 3.

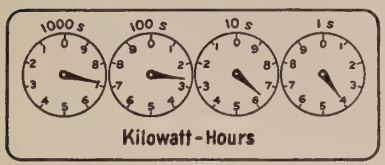


Fig. 2

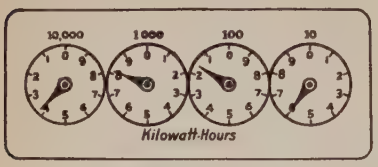
Fig. 3

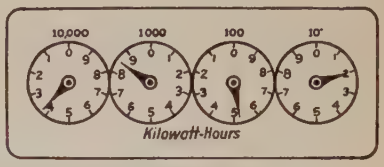
These different labelings on the dials do not change the reading of the meters. The rule is simply to write down the numbers on the dials in order from left to right. That is, for the two dials shown in Figures 2 and 3, the readings are: Fig. 2, 7264 kilowatt-hours; Fig. 3, 5093 kilowatt-hours.

The electric meter shown in Fig. 1 is perhaps the most common form. The meters which you will be asked to read in this book are all of this type.

39. Problems on Electric Meters. (Written Work.) To work these problems you will need to keep in mind the facts in Article 38.

1. Find how many kilowatt-hours were used during July, 1927, if the readings for July 1 and August 1 were as shown on the meters below.





(Each division on the "10" dial equals 1 kilowatt-hour.)

Reading on July 1 Reading on August 1

2. How much was the electric-light bill for July if the rate was 9¢ per kilowatt-hour?

3. Suppose that on January 1, 1927, an electric meter read 6-7-3-5 and on February 1, 1927, the reading was 6-9-1-8. What should have been the amount of the bill if the rate was $12\frac{1}{2}$¢ per kilowatt-hour?

4. Electric-light companies often have what is called a "sliding scale" by which they charge for electricity. Suppose the company in Problem 3 used this "sliding" scale:

First 100 kilowatt-hours	10¢ per k.w.h.
Second 100 kilowatt-hours	6¢ per k.w.h.
Above 200 kilowatt-hours	5¢ per k.w.h.

What would have been the amount of the bill?

5. At a 5% discount for cash, find the amount saved in Problem 4 by cash payment.

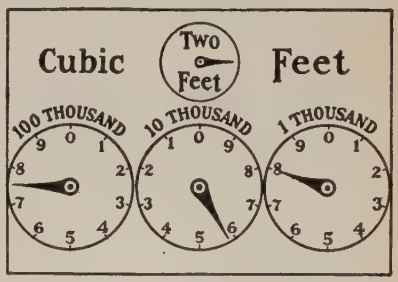
6. In meters like those above how far around the dial does the 10's hand turn to make the hand on the 100's dial move one division (for example, from 6 to 7)?

7. How many times does the hand on the 10's dial turn around while the hand on the 1000's dial moves from 0 to 0?

8. How many times has the hand on the 10's dial turned around when the reading is 7-6-4-3?

40. The Gas Meter. (Silent Reading.) Gas meters are similar to electric meters except that they read in terms of cubic feet. The cubic foot is the unit of measurement of gas.

The drawing of a gas meter at the right shows the arrangement of the dials.



Notice the three large dials in a row. They are labeled from left to right, "100 thousand," "10 thousand," and "1 thousand."

Remember these facts:

The 100 thousand dial registers **tens of thousands** of cu. ft.
The 10 thousand dial registers **thousands** of cu. ft.
The 1 thousand dial registers **hundreds** of cu. ft.

The dial farthest to the left is marked "100 thousand," but you read it as **tens of thousands.** This is because each division of the dial represents ten thousand cu. ft. of gas. Since there are ten divisions on the dial, one complete circuit of the hand represents 10 times 10,000 or 100,000. That is why the left hand dial is called the "100 thousand" dial.

The meter above reads 7-5-8, or 75,800 cu. ft., as shown at the right.

Notice that two zeros have been written after the 8.

70,000
5,000
800
75,800

The following rule will always work for gas meters with three dials labeled like those in the picture on page 65:

1. Write the numbers which the hands point at or have just passed on each dial from left to right.

2. Annex two zeros to the right of the three numbers just read from the dials.

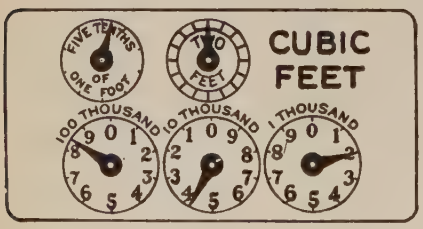
The smaller dial marked "Two Feet" is called the **test dial.** You do not need to understand how this smaller dial works in order to obtain a correct reading.

Remember:

The 100 thousand dial registers **tens of thousands** of cu. ft.
The 10 thousand dial registers **thousands** of cu. ft.
The 1 thousand dial registers **hundreds** of cu. ft.

41. Problems about Gas Meters. (Written Work.) Be sure you know the facts in Article 40 before you try these problems.

1. Read the gas meter shown in the diagram at the left below.

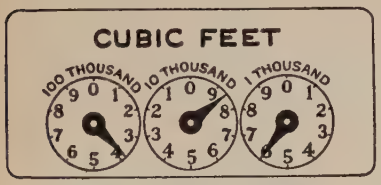


2. If the gas recorded on the meter shown in Problem 1 costs $1.25 per 1000 cubic feet, what was the total cost of the gas?

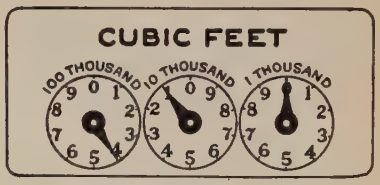
3. The dials of a gas meter read 6-7-4 on March 1, 1927, and 8-9-1 on April 1, 1927. What should have been the amount of the gas bill for March at $1.60 per 1000 cubic feet?

4. Draw a gas meter and have the hands show a reading of 63,700 cu. ft.

5. Mr. Frederick Browne received a bill for gas used during July which charged him with 1400 cu. ft. The readings of his meter were as shown below. Was the amount charged correct?



READING ON JULY 1, 1927



READING ON AUGUST 1, 1927

SELF-TESTING DRILL NO. 5

42. Directions. Wait for the signal "Go!" Wherever possible, avoid copying by placing your paper right below the example.

You will be allowed exactly 15 minutes. Work as fast as you can without making mistakes. Do the examples and problems in order.

Try to beat all your past records. Find your rating in the standards at the end of the drill, and mark your record on the Progress Chart.

1. *Add:*

761
889
437
926
423

2. *Subtract:*

621.924
603.785

3. *Subtract:*

1075572
849759

4. *Add:*

$\frac{1}{6}$
$\frac{1}{2}$
$\frac{7}{12}$

5. *Multiply:*

$2\frac{1}{2}$
$6\frac{2}{5}$

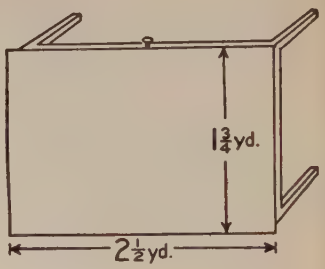
6. $549\overline{)2904.21}$

Go on to the next page.

7. The Wilson School Improvement Club earned \$794 during the year. 75% of this was saved. How much was saved?

8. Multiply 9073 by 748.

9. What is the area of the table top shown in the drawing at the right?



10. *Multiply:*

2 lb. 3 oz.
6

11. Is $376 \div \frac{1}{20}$ about 7500, 750, or 180?

12. What is the total of these weights?

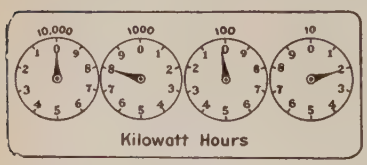
2 lb. 10 oz.
4 lb. 8 oz.
1 lb. 14 oz.
4 lb. 9 oz.

13.* (*a*) $\frac{26}{3}$ equals what mixed number?

(*b*) 14 pt. = ... qt.

(*c*) 29 yd. = ... ft.

(*d*) \$5 = ... ¢



14. If 325 equals 100% of a number, what per cent of the number is 26?

15. Read the electric meter shown at the left.

*To count Example 13 correct, each part must be correct.

STANDARDS

Number correct	0–2	3–4	5	6	*	7	8	9	10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

PROBLEM SCALE NO. III

43. Problems Using Decimals and Per Cents. (WRITTEN WORK.) You can earn 55 points by working every problem correctly. Find your score and rate yourself as directed in Art. 19.

The suggestions on problem solving in Article 18 should help you score higher on this Problem Scale.

1. In a certain part of the country, the amounts (in inches) of rainfall for the first 6 months of the year were as follows: .66, 1.53, 4.16, 2.26, 5.08, 6.45. How much rain fell during the first half of the year? (4)

2. During the last half of the same year the amounts (in inches) of rainfall by months were as follows: 4.25, 1.44, 4.83, 1.45, 2.03, 1.90. How much rain fell during the last half of the year? (4)

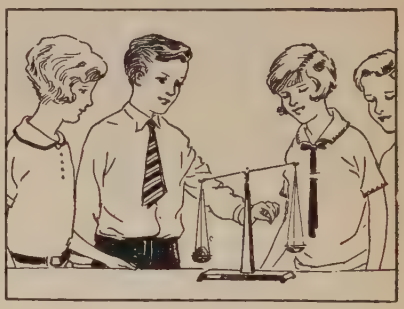
3. During which half of the year did the more rain fall and by how much? (See Probs. 1 and 2 above.) (5)

4. The annual report of the American Telephone and Telegraph Company for the year 1924 shows that for 1922 its income from ordinary telephone calls amounted to $374,718,580. Its income from toll (long-distance) charges was $163,097,837.

Its income from other sources was $9,012,656. The "total operating income" is the sum of the above three items. Find the total operating income for the year 1922. (4)

5. The same report shows that the income for 1923 was as follows: ordinary phone calls, $412,009,426; toll charges, $178,427,051; other sources, $11,153,311. Find the total income for 1923 from the three items given. (4)

6. Using the data (the facts) given in Problems 4 and 5, find the per cent of increase in total income for the Telephone Company for the year 1923 over the year 1922. (Report your answer to the nearest whole per cent.) (8)



7. A science class wanted to find out how much loss in weight took place when ordinary dirt taken from the school playground was thoroughly dried. The class took 54 cu. in. of soil from the playground. It weighed 1.94 lb. The class dried this soil thoroughly and weighed it again. It then weighed exactly 1.459 lb. How much had the soil lost in weight? (5)

8. The best score that can be made on a certain reading test is 128 points. Helen made a score of 65. What per cent of the best possible score did she make? (Carry answer to one decimal place.) (7)

9. Mary made a score of 85 on the same test. What per cent of the best possible score did she make? (Carry result to one decimal place.) (7)

10. Esther made 104 points on the same test. What per cent of the best possible score did she make? (Carry result to one decimal place.) (7)

STANDARDS

Superior work:	31–55 points	Slow work:	12–18 points
Good work:	24–30 points	Inferior work	0–11 points
Average work:	19–23 points		

SUMMARY

44. General Review. (Oral Work.) Study the work in this article so that you can give correct answers when you are called upon.

1. What question using percentage would be answered by working this example? $456 \times .07$

2. What question using percentage would be answered by working this example? $\frac{98}{389}$ or $389\overline{)98.00}$

3. What question using percentage would be answered by working this example? $.64\overline{)470.00}$ or $.64\overline{)470.00}$

4. What question using percentage would be answered by working this example? $\frac{745}{480}$ or $480\overline{)745.00}$

5. What question using percentage would be answered by working this example? $\$644 \times .045$

6. What question in percentage would be answered by working this example? $.875\overline{)632.000}$

7. Do not work the examples in 1 to 6 above, but study them in order to get a good estimate for each answer. (Remember that "good estimate" does not mean a mere guess.)

8. What ratings have you made on the five Self-Testing Drills you have taken?

9. Do you know how to use the Self-Help Index? Suppose you need some practice on addition of integers. On what pages can such drill be found?

10. Find articles in this chapter which explain how to solve such examples as: (*a*) If 68 is 34% of a number,

what is the whole, or 100% of the number? (*b*) Find the whole when you know that 8% of it equals 809.

11. Be ready to show three checks for addition.

12. Be ready to show checks for subtraction, multiplication, and division.

13. In what unit is gas measured? Electricity?

14. When the dials on an electric meter read 3-4-8-1, how many kilowatt-hours of electricity have been used?

15. How many cubic feet of gas have been used when the readings of a gas meter are as follows?

100-*thousand dial*	10-*thousand dial*	1-*thousand dial*
2	6	3

CHAPTER III

FACTORS. CHECKING AND SAVINGS ACCOUNTS. MAKING CHANGE. BUDGET MAKING

45. The Nature of Good Seventh-Grade Work. (SILENT READING.) Much more is expected of seventh-grade pupils than of pupils in the lower grades. It is the desire of most pupils in the seventh grade to make real progress during the year. Real progress can come in several ways, such as:

(*a*) Making all written work neat and easy to read.

(*b*) Doing as much of the work as possible **oneself** instead of depending on the teacher for help on every "hard spot."

(*c*) Checking written work so thoroughly that mistakes in calculating will be unusual.

(*d*) Reading problems carefully so that problem solving may be well done independently of the teacher.

(*e*) Keeping watch for little "tangles" which often cause trouble to good as well as to poor workers.

FACTORS

46. The Meaning of Factors. (CLASS WORK.) When you multiply, you have certain names for the numbers you use. You already know what these numbers are called. Sometimes it is convenient to have one name for both the multiplicand and the multiplier. This name is **factor.** A factor is any one of the numbers which are multiplied together to get a product. Thus, $3 \times 5 = 15$. Here 3 is a factor. 5 is also a factor. We usually speak of a factor together with the product; so we say that 3 and 5 are factors of 15.

Sometimes we multiply more than two numbers together. Thus, $3 \times 5 \times 7 = 105$. Here we say that 3, 5, and 7 are factors of 105.

When two or more numbers are multiplied together, each of the numbers is a factor of the product.

Of course a product may have more than one set of factors. For instance, $4 \times 3 = 12$ and $2 \times 6 = 12$. 4 and 3 are factors of 12, since their product is 12. 2 and 6 are also factors of 12, since their product is likewise 12.

Can you give the factors for these products? Some of them have more than one set.

1. 21 2. 20 3. 36 4. 70 5. 48 6. 25

If you know a product of two factors and one of the factors, it is easy to find the other factor. Suppose you want to solve this example: 7 is one factor of the product 21. What is the other factor? Of course you know that $7 \times 3 = 21$. If you divide 21 by 7, does this give you the other factor? If you know a product and one of its two factors, you can always find the other factor by dividing the product by the one you know.

Product divided by one factor gives the other factor.

Use the rule and find the missing factors below.

7. $3 \times ? = 6$
8. $4 \times ? = 8$
9. $? \times 5 = 25$
10. $9 \times ? = 72$
11. $? \times 12 = 48$
12. $11 \times ? = 99$
13. $20 \times ? = 100$
14. $15 \times ? = 45$
15. $? \times 16 = 64$

47. Some Facts About Multiplying and Dividing. (SILENT READING.) Perhaps you have gotten into the habit of thinking that the result in multiplication is always larger than either factor, and that when you divide, the answer is always smaller than the number you divided.

It is true that if you multiply two numbers larger than 1, the product is larger than either of the factors. Thus:

$3 \times 4 = 12$ $4\frac{1}{4} \times 6\frac{1}{3} = 26\frac{1}{12}$ $3 \times 4.5 = 13.5$ $2.2 \times 7.8 = 17.16$

The product in each case is larger than either factor.

But when one of the factors is a number less than 1 (a fraction or a decimal) the product is no longer larger than either factor. It is smaller than the multiplicand, but larger than the multiplier. Study the examples below.

$\frac{1}{2} \times 4 = 2$ $\frac{7}{8} \times 32 = 28$ $.6 \times 12 = 7.2$ $.08 \times 13 = 1.04$

When you multiply two fractions or decimals, the product is **smaller** than **either** of the factors. Thus:

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$ $.2 \times .3 = .06$ $.6 \times .12 = .072$

Remember: If the two factors are greater than 1, the product is larger than either factor. If one of the factors is less than 1 (a decimal or a fraction), the product will be smaller than one factor and larger than the other. If both factors are smaller than one, the product is smaller than either factor.

Usually we think that division means getting an answer smaller than the number divided. This is true when we divide by numbers larger than 1. Thus:

$32 \div 4 = 8$ $20 \div 2\frac{1}{2} = 8$ $4.6 \div 2.3 = 2$ $\frac{1}{2} \div 4 = \frac{1}{8}$

Each answer is smaller than the number divided.

But what happens when we divide a number larger than 1 by a fraction or a decimal? Look at these examples: $8 \div \frac{1}{4} = 32$. $15 \div \frac{1}{3} = 45$. $12 \div .6 = 20$. You see that the answer is **larger** than either of the numbers you used. The

same is true when both the numbers are fractions or decimals (numbers less than 1). Study the examples below:

$\frac{3}{4} \div \frac{1}{4} = \frac{12}{4} = 3$ $\frac{3}{5} \div \frac{2}{5} = \frac{15}{10} = 1\frac{1}{2}$ $.48 \div .4 = 1.2$

Remember: Divide by numbers larger than 1 and your answer will be smaller than the number divided. But if the divisor is a number less than 1 (a fraction or a decimal), the answer will be larger than either the divisor or the dividend.

When you multiply or divide by 1, is your answer larger than, equal to, or the same size as the number you are multiplying or dividing? If you multiply by zero is your answer larger than, smaller than, or the same size as the number multiplied?

Look at Example 1 in Article 48. Should the estimate be larger or smaller than either of the factors? Look at Example 12. Should the estimate be larger than 14 or smaller?

48. Practice in Estimating. (WRITTEN WORK.) At the left below are examples. At the right are estimated answers. Write the example number and beside it the letter of the estimate that you think is reasonable for the example. The estimate for Example 1 is (*k*).

Examples	*Estimates (not in order)*
1. 8.4×101	(*a*) 2.00
2. $2.4\overline{)4.968}$	(*b*) 45
3. $6788 - 4938$	(*c*) 1200
4. $9574 \div 103$	(*d*) 1− (a little less than 1)
5. 240×11	(*e*) 95.74
6. $15.17 \div \frac{1}{3}$	(*f*) 4000
7. $15.17 \times \frac{1}{3}$	(*g*) 14+ (a little more than 14)
8. $\frac{7}{8} + \frac{1}{9}$	(*h*) 340
9. $4927 - 932$	(*i*) 2400

10. 347
600
75
214

11. 17×21

12. $14 \div 1\frac{1}{12}$

(*j*) 1800

(*k*) 840

(*l*) 5

49. Rapid Drill. (ORAL WORK.) These rapid drills will help you make good scores on the Self-Testing Drills. In division examples give only the first quotient figure.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)
1.	$83\overline{)746}$	$111 - 87$	$2\overline{)5}$ qt.	69×7	$2\frac{5}{12} + 3\frac{1}{3}$
2.	$10 + 5 + 2$	59×9	*Reduce:* $5\frac{12}{4}$	$69\overline{)552}$	.165 = what %?
3.	$\frac{10}{3} \times \frac{6}{20}$	$13 + 9 + 9$	72×8	$8 + 5 + 9$	$1226 - 340$
4.	$1\frac{5}{8} - \frac{3}{4}$	10% of \$255.50	$19\frac{3}{4}\%$ = what decimal?	$943 - 778$	$98 + 9$
5.	96×6	$87\frac{1}{2}\%$ of 56 = ?	$48\overline{)283}$	46 sq. ft. = ... sq. yd.	$\frac{2}{5} \times 90$
6.	$.0101 \times 100 = ?$	$1.7 - .8$	$10 \div 1000$	$2 \times 1\frac{1}{4}$	$\frac{1}{8}$ = what %?
7.	$16 + 6 + 2$	If 90 = 100% N, 45 = ...% N.	*Define:* Right triangle	$28 + 7 + 9$	$37\frac{1}{2}\%$ = what fraction?

SELF-TESTING DRILL NO. 6

50. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Be sure to correct mistakes.**

1. *Subtract:*

158064.1
60935.7

2. *Add:*

6280
351
8231
1535
8435

3. What is 1% of 1673?

4. *Find the sum:*

3 bu. 1 pk.
4 bu.
2 bu. 3 pk.
1 bu. 2 pk.

5. Find the difference of 1,282,270 and 373,435.

6. *Add these numbers:* $2\frac{2}{3}$, $\frac{1}{2}$, $3\frac{7}{12}$.

7. $837\overline{)790.965}$

8. *Multiply:*

1.821
4087

9. Would the interest on \$450 @ 10% for 5 years be \$2500, \$225, or \$22.50?

10. Find the product of $3\frac{1}{3}$ and $6\frac{3}{5}$.

11. *Multiply:*

3 sq. yd. 6 sq. ft.
7

12.* (*a*) 1.75 = ...%
(*b*) 126 in. = ...yd.
(*c*) $.66\frac{2}{3}$ equals what fraction?
(*d*) 32 in. = ...ft.

13.* (*a*) Check the statement (bill) at the top of the next page for errors in adding. If the amount \$30.05 is correct, write "correct" as your answer. If it is wrong, write the correct amount.

(*b*) What was omitted when the bill was made out by the company?

*To count Examples 12 and 13 correct, each part must be correct.

ATLANTA, GEORGIA April 1, 1928

AMERICAN SUPPLY COMPANY

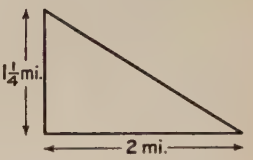
SOLD TO

March	3	1 Table	26	75		
	16	1 Hammer	2	25		
	22	1 Battery for flash light		35		
	29	1 Ball twine		10		
	31	2 Doz. screws		60		
					30	05

Received payment
American Supply Co.
Per H. D. S.
4/10/28

14. 3800 = 100% of N.
228 = what % of N?

15. The drawing at the right represents a city park. What is its area?



STANDARDS

Number correct	0–2	3	4	5	6	7	8	9	10	11–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

THE ESSENTIALS OF CHECKING ACCOUNTS

51. The Purpose of a Bank Checking Account. (SILENT READING.) Often it is inconvenient to pay bills with actual money. It is sometimes quite unsafe and therefore poor business practice to carry large sums of money in one's pockets. The chances of losing money are very great. In paying bills by mail it is not wise to send silver coins or paper bills. For these reasons we **deposit** money in a **checking account** at a bank. The bank will pay the money for us from our account when we

write out an order for it to do so. This order is called a **check.** The check can be taken to the bank by the person to whom it is to be paid, and he can get his money.

RECORDS OF A CHECKING ACCOUNT

52. Keeping a Checking Account. (CLASS WORK.) When one deposits money in a checking account, four careful records are made. Two of these records are made by the bank, and two are made by the **depositor** (the one who puts the money in the bank).

DEPOSITED WITH
First National Bank,
FOR ACCOUNT OF
E. H. Black
STANTON, IOWA Nov. 14 1927

CURRENCY	142	
SILVER	8	74
GOLD		
CHECKS AS FOLLOWS:		
F. B. Sams	23	16
Arthur Ellis	17	35
E. P. Danskin	49	83
J. A. Brock	17	04
TOTAL $	258	12

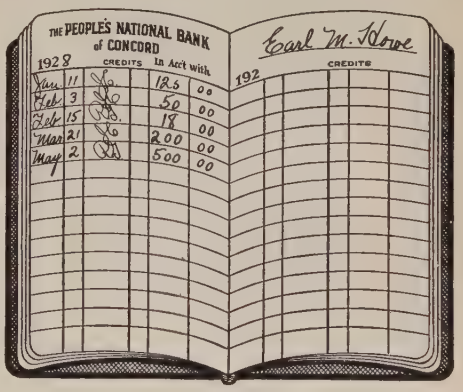
RECORD 1. **The deposit slip.** When you deposit money, you first make out a statement of the amounts you are depositing. This is done on a **deposit slip** like the one shown at the left. These are found in pads on the writing desks of the bank. On the slip you must indicate whether the amounts you are depositing are in **currency** (paper money), silver, gold, or checks. The bank keeps this slip as its record of what you have deposited.

Answer these questions about the deposit slip shown:

1. Who deposited the money?
2. In what bank was the money put?
3. How many checks were deposited?
4. How much did Mr. Black deposit in currency (paper money)?
5. What was the total amount Mr. Black deposited?

RECORD 2. **The deposit or pass-book.** Most banks give a customer a little book in which every deposit is recorded. The record of deposits in this book is made by the bank, rather than by the customer. This helps to avoid errors.

The customer keeps his own deposit or pass-book, and should bring it to the bank every time he makes deposits. If the customer forgets his deposit book, he can make out two deposit slips. One of these the bank clerk will receipt (write his name or initials on) and give to the customer, who can later bring it to the bank with his pass-book and have the amount entered in the book.



RECORD 3. **The monthly statement.** The bank keeps for each customer a record of all deposits, of all money paid out because checks have been **cashed,** and the amount left (the **balance**). At the end of the month the bank prepares a statement of all deposits and amounts paid, with their dates. This statement shows the balance for each day on which checks have been cashed or deposits have been made. It also shows the balance on deposit at the end of the month. It is called the **monthly statement.** Monthly statements are sometimes mailed to the depositor, or he may call for them.

On the next page is shown a picture of a monthly statement. Study it and answer the questions below it.

IN ACCOUNT WITH

MERCHANTS NATIONAL BANK

BOSTON, MASS.

Statement of your account to Jan. 31, 1928

Name Austin C. Phipps

Address 1014 Tremont St.

Boston, Mass.

Old Balance	Date	Checks	Date	Deposits	Date	Balance
143.52	Balance brought forward: 143.52					
	Jan. 5	26.00	Jan. 5	285.26	Jan. 5	360.78
		42.00				
	Jan. 17	15.30			Jan. 17	207.90
		95.00				
		42.58	Jan. 19	7.25	Jan. 19	215.15
	Jan. 27	6.20	Jan. 27	59.30	Jan. 27	256.50
		11.75				
	Jan. 28	10.00			Jan. 28	246.50
					Jan. 31	246.50

1. Whose monthly statement is shown above?

2. How many of Mr. Phipps's checks did the bank cash on January 17? What was the amount of each check?

3. How many times did Mr. Phipps deposit money during January? What were the amounts?

4. How much did Mr. Phipps have left in the bank (his balance) on January 17?

5. What was his balance at the end of the month?

6. How much money did Mr. Phipps have in the bank on December 31, 1927?

RECORD 4. **Checks and check stubs.** The fourth record is kept by the customer himself. The check-book, which contains blank checks, also has spaces for recording the amount of each check, deposits as they are made, and the "balance" at the bank. The blanks for recording these facts are called **stubs** or **check stubs.** Each check has its own stub. When a check is used, its stub, if properly filled out, will show important facts.

Study the picture below. Notice that the stub and check are Number 8.

Answer these questions about Check No. 8 and Stub No. 8:

1. In what bank does Mr. Severance deposit his money?

2. To whom did he pay the money? How much was it? What did he pay it for?

3. When did he write the check?

4. What facts about Check No. 8 are shown on Stub No. 8?

5. Does Stub No. 8 tell how much money Mr. Severance had left in the bank after he paid the $85.00 to Mr. Macdonald? How much did he have left?

STUB

No 8 $ 85 00/100
Dec. 31, 1927
To A. J. Macdonald
For Rent

	DOLLARS	CENTS
BAL BROT FORD	200	00
AMT DEPOSITED		
TOTAL		
AMT THIS CHECK	85	00
BAL CARD FORD	115	00

CHECK

NATIONAL BANK OF WOODLAWN : 328 :

CHICAGO, ILL. Dec. 31, 1927 No. 8

PAY TO THE ORDER OF A. J. Macdonald $ 85 00/100

Eighty-five and 00/100 DOLLARS

W. E. Severance

On the next page are pictures of Stubs 7, 8, and 9. Study these carefully for at least 2 minutes and answer the questions about them.

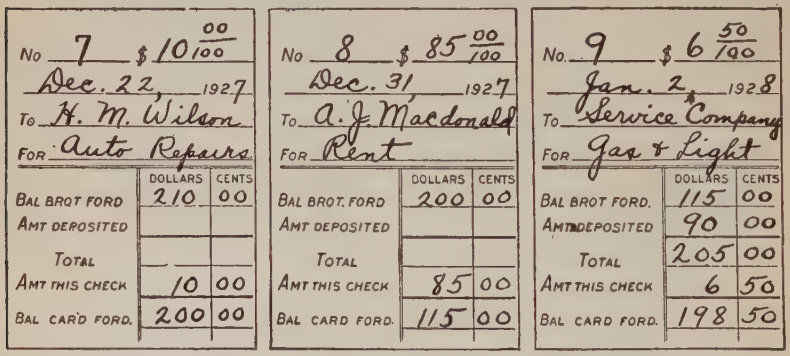
1. For what did Mr. Severance pay Mr. Wilson $10.00?

2. How much did he have in the bank before he paid the $10? How much after he paid it?

3. Does any of these stubs show that Mr. Severance had deposited money in the bank? Which stub? How much money did he deposit? Can you find how much money he had after he made the deposit?

4. What is done with the balance on each stub?

You see that stubs are valuable records of your account at the bank. They tell you the important facts about each check you write, the amounts you deposit, and the amount you have in the bank at any time.



No 7	$10 00/100
Dec. 22, 1927	
To H. M. Wilson	
For Auto Repairs	

	DOLLARS	CENTS
Bal Brot Ford	210	00
Amt deposited		
Total		
Amt this check	10	00
Bal card ford.	200	00

No 8	$85 00/100
Dec. 31, 1927	
To A. J. Macdonald	
For Rent	

	DOLLARS	CENTS
Bal brot. ford	200	00
Amt deposited		
Total		
Amt this check	85	00
Bal card ford.	115	00

No. 9	$6 50/100
Jan. 2, 1928	
To Service Company	
For Gas & Light	

	DOLLARS	CENTS
Bal brot ford.	115	00
Amt deposited	90	00
Total	205	00
Amt this check	6	50
Bal card ford.	198	50

Summary of records for a checking account. The following are the main points about checking accounts:

1. **Deposit slip.** Made out by the depositor, but retained by the teller. The bank's record of the customer's deposit.

2. **Deposit book.** Record of all deposits. The deposits are recorded by a bank clerk, but book is kept by the customer.

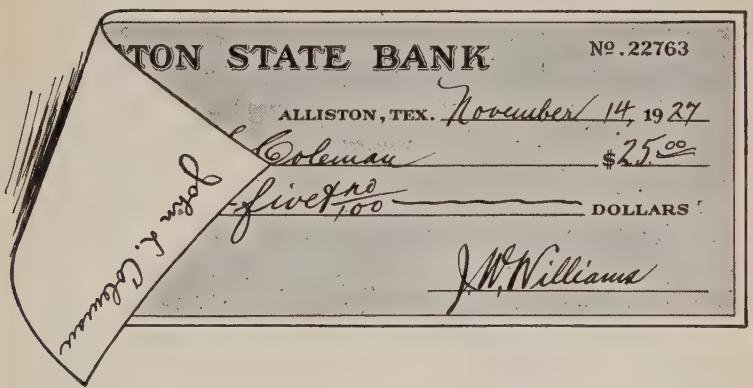
3. **Monthly statement.** All records are made by a bank clerk. Shows all deposits and all amounts of checks paid out, with dates, and the balance at the end of the month. Kept at

the bank until end of month, when the customer receives it for his study and for the correction of mistakes if any have been made.

4. Check stub. Each check has its own stub. Shows amount of check, amount of deposits, and balance. All records on stubs in check-book are made by the customer.

53. The Check Itself. (SILENT READING.) The check is used as money. When you give a check in payment of a bill, the bank will pay the amount stated out of your account when the check is presented for payment, under the following three conditions:

1. The person to whom you made out the check (whose name appears on the line beginning "Pay to the order of") must write his name on the back of it to show that he has received the money. This is called **indorsing** a check. Below is shown how a check is **indorsed** for payment.



A check is often said to be "drawn in favor of" the one whose name appears on the line marked "pay to the order of."

Sometimes a check may show more than one indorsement. That would happen this way: You write a check payable to Mr. King. Mr. King does not cash the check at the bank. He in-

dorses it and gives it to Mr. George Mills, to whom he owes some money. In this case he might write on the back, "Pay to George Mills" and then sign his name. Mr. Mills would then write his name under Mr. King's when he cashed the check.

However, it would not be necessary for Mr. King to write, "Pay to George Mills." He would only have to write his name on the back and give the check to Mr. Mills. But if he wrote, "Pay to George Mills," there would be a record that he had paid the money to Mr. Mills and Mr. Mills would have to write his name on it also.

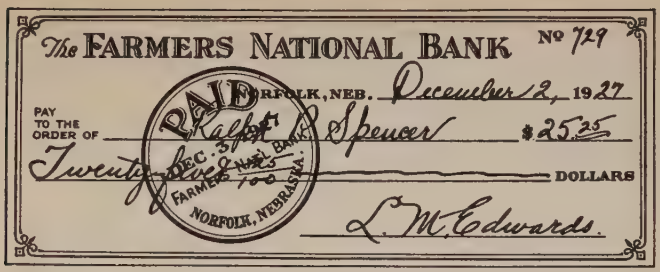
➡When you are given a check, never indorse it until you are ready to cash it. An indorsed check, if lost, might be cashed by the finder unless the bank had been notified that the check had been lost and that payment should be refused.

2. There must be enough money in your account to pay the check.

It is **against the law** to write a check for more money than you have on deposit. The bank will not and should not pay such a check. This is called **over-drawing** your account. By keeping the records on the check stubs carefully, this mistake can be avoided.

3. In order to protect your money on deposit, banks may refuse to cash checks when they are presented by a person who is not known by the bank officials.

When the bank pays from your account the amount written on the check, it **cancels** your check by stamping "Paid" on it. Your **canceled** checks are returned to you by the bank with your monthly statement. They are valuable, because they prove that you have paid the various amounts of money. A picture of a canceled check is shown on the next page.



54. Problems on Checking Accounts. (WRITTEN WORK.) If you have understood Articles 51-53, you should have no trouble with the following problems.

1. Mr. Gray's balance in his checking account on June 1 was $231.76. During June he wrote seven checks for $97.50, $23.00, $4.17, $32.50, $9.87, $45.25, and $21.00, and deposited $132. What was his balance on June 30?

2. On Monday morning a grocer made out a deposit slip for the following: currency, $67.00; silver, $7.43; checks, $10.27, $7.18, $19.73, $35.41, $42.50, $6.25, $51.07. His total on the slip was $171.41. In checking the slip, the teller found a different total. Find the correct total.

3. Gladys Snider had a balance of $37.43 in her checking account on May 1. During the month she made no deposits. Her checks were for $7.93, $6.84, $5.00, $9.00, $3.50, and $4.95. Did she over-draw her account? If so, by how much? If not, what was her balance?

4. On December 17 Mrs. Austin had $42.23 in her checking account. During the next ten days she wrote three checks for $15.00, $17.54, and $13.75, to pay for Christmas purchases. She made a mistake on her stubs, and overdrew her account. For what amount could she have written the last check without overdrawing?

SELF-TESTING DRILL NO. 7

55. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Are you checking the examples of which you are not sure? To do so may help you make a higher score.**

1. *Subtract:*

103639.79
61573.50

2.

$98.71
49.38
75.00
82.99
6.00
61.99

3. *Subtract:*

910800
812284

4. Find the difference of 17 and $9\frac{3}{4}$.

5. *Multiply:*

846.5
8.74

6. A 15% discount is allowed on a bill of $8896. Compute the discount.

7. Divide 2 by $4\frac{1}{3}$.

8. What % of 25 is 6?

9. $504\overline{)3703392}$

10. Subtract 6 bu. 3 pk. from 14 bu. 2 pk.

11. How much interest is earned by a deposit of $8575 at $4\frac{1}{2}$% for 3 yr. 4 mo.?

12. $8\overline{)9 \text{ lb. } 6 \text{ oz.}}$

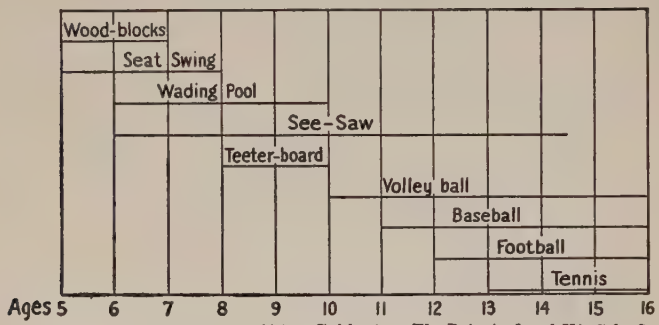
13. How many cubic yards of earth must be removed to provide space for a cement foundation of these dimensions: 4 ft. $\times$ 7 ft. $\times$ 9 ft.?

14.* *Answer the following questions by means of the graph at the top of the next page.*

(*a*) What games are first liked at the age of 6?
(*b*) For how many years is "See-saw" liked?
(*c*) At what age do children begin to like football?

(*d*) At what age does the graph state that children stop playing "teeter-board"?

AGES AT WHICH CHILDREN LIKE CERTAIN GAMES



(After Cubberley: *The Principal and His School*)

15.* Write the numbers and abbreviations which should be on the blanks. Use the rule $A = a \times b$.

A = area
a = height or altitude
b = base

	b	*a*	*A*
(*1*)	9 in.	6 in.	.....sq. in.
(*2*)	.....	8 ft.	40 sq. ft.
(*3*)	9 yd.	.....	27 sq. yd.
(*4*)	8 ft.	16 in.	.....sq. in. or .....sq. ft.

*To count Exs. 14 and 15 correct, each part must be correct.

STANDARDS

Number correct.....	0–1	2	3	4	5	*	6	7	8	9–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

Don't be satisfied with your work on a Self-Testing Drill until you have corrected every mistake. Be sure that you know **why** you made the mistake, so that you will not make the same one again.

BALANCING ACCOUNTS

56. Balancing a Checking Account. (STUDY PERIOD.) When you receive your monthly statement from the bank, you will want to check to see that the stub record of your balance agrees with the bank's record of your balance. During the month you have been adding and subtracting on your check stubs as you have deposited money and written checks. You may have made a mistake in one or a number of these additions and subtractions. Or perhaps the bank clerks have made a mistake in their records of your account. Your record and the bank's record must agree, or you may write a check for more money than you have on deposit. And of course you don't want the bank to make a mistake that would give you a smaller balance than you have.

Careful people always "balance" their account every month. Here is the easiest way to do it: The bank has returned to you a certain number of canceled checks. These have been cashed and deducted from your account. Certain other checks had not yet been presented at the bank for payment when it sent your monthly statement. Therefore the bank has not returned them to you and had not at that time deducted them from your account. The balance on your last stub shows, however, what you have left after **all** checks have been deducted. Your balance and the bank's balance therefore do not agree. You must find what is wrong. This is how to do so:

By comparing your stubs and canceled checks, find which checks have not been returned. Add the amounts on these unreturned checks, and subtract this sum from the bank's balance. Your stub balance should be the same as the amount you get by subtracting. If the two balances do

not agree, you may find your mistake somewhere on your stubs, or the bank may have made an error.

Checks that have not been cashed at the time the monthly statement is sent you are called **outstanding** checks.

EXERCISE

Find whether Mr. Austin's record of his checking account agrees with the bank's record. The black figures show **outstanding** checks.

Balance on monthly statement: \$169.50.
Mr. Austin's balance: \$83.00.

Checks Mr. Austin has written:

\$85.00	\$12.32	**\$ 4.50**	\$14.92	**\$ 3.74**
65.70	25.50	26.68	**3.26**	8.72
6.50	10.00	**68.50**	1.32	19.23

57. Studying Records on Check Stubs. (Class Work.) Sometimes even when all the outstanding checks are allowed for, the two balances still do not agree. When such a disagreement in accounts happens, one should immediately hunt for the error. The bank may have made a mistake, but usually the customer has made the error.

PROBLEM

Miss Ruth Perkins, the head nurse at the Community Hospital, keeps a checking account at the Newton National Bank, Newton, Tenn. Her deposit slips for December, 1927, are as follows:

Dec. 1, 1927, \$275.00.
Dec. 5, 1927, \$72.45.
Dec. 12, 1927, \$15.00.
Dec. 20, 1927, \$25.54.

On the next page are shown the stubs in her check-book for the checks she had written.

No. 46 $5.00
Dec. 1, 1927
To American Relief Committee
For

	Dollars	Cents
Bal. brot ford.	13	50
Amt. Deposited	275	
Total	288	50
Amt. this Check	5	
Bal. card ford.	283	50

No. 47 $25.00
Dec. 1, 1927
To Mrs. Mae Arthur
For Room rent

	Dollars	Cents
Bal. brot ford.	283	50
Amt. Deposited		
Total		
Amt. this Check	25	
Bal. card ford.	258	50

No. 48 $35—
Dec. 5, 1927
To Mrs. R. T. Perkins
For Board

	Dollars	Cents
Bal. brot ford.	258	50
Amt. Deposited	72	45
Total	330	95
Amt. this Check	35	
Bal. card ford.	295	95

No. 49 $17.50
Dec. 7, 1927
To Yetter's
For Clothes

	Dollars	Cents
Bal. brot ford.	295	95
Amt. Deposited		
Total		
Amt. this Check	17	50
Bal. card ford.	278	45

No. 50 $2—
Dec. 12, 1927
To Susie Kirby
For Washington School Play

	Dollars	Cents
Bal. brot ford.	278	45
Amt. Deposited	15	
Total	293	45
Amt. this Check	2	
Bal. card ford.	291	45

No. 51 $40.00
Dec. 19, 1927
To Killian's
For Suit

	Dollars	Cents
Bal. brot ford.	291	45
Amt. Deposited		
Total		
Amt. this Check	40	
Bal. card ford.	251	45

No. 52 $20—
Dec. 20, 1927
To Cash
For Xmas gifts

	Dollars	Cents
Bal. brot ford.	251	45
Amt. Deposited	25	54
Total	276	99
Amt. this Check	20	
Bal. card ford.	256	99

No. 53 $3—
Dec. 20, 1927
To Dr. L. M. Packer
For Tuberculosis Seal

	Dollars	Cents
Bal. brot ford.	256	99
Amt. Deposited		
Total		
Amt. this Check	3	
Bal. card ford.	243	99

No. 54 $100.
Dec. 30, 1927
To Newton Savings Bk.
For Savings dept.

	Dollars	Cents
Bal. brot ford.	243	99
Amt. Deposited		
Total		
Amt. this Check	100	
Bal. card ford.	143	99

Miss Perkins's canceled checks agree exactly with the amounts opposite "Amt. this Check" shown on the stubs.

On January 2d Miss Perkins's statement from the bank was as shown below.

CAPITAL $100,000
Surplus $100,000

IN ACCOUNT WITH

NEWTON NATIONAL BANK

NEWTON, TENN.

Statement of Your Account to Dec. 31, 1927

Name Miss Ruth Perkins

Address 143 Melrose Ave.

Newton, Tenn.

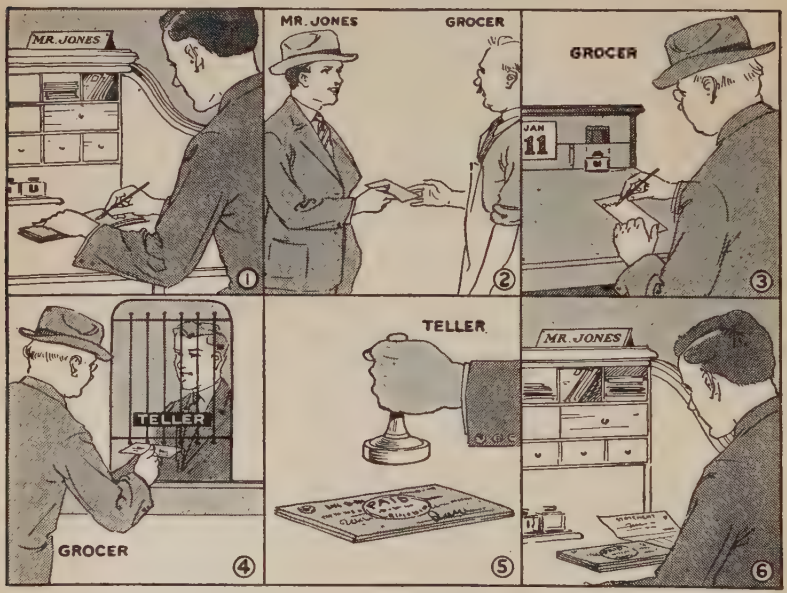
Please examine at once; if no error is reported in ten days, the account will be considered correct.

OLD BALANCE	DATE	CHECKS	DATE	DEPOSITS	DATE	BALANCE
13.50	Balance brought forward			13.50		
	Dec. 1	5.00 25.00	Dec. 1	275.00	Dec. 2	258.50
	Dec. 5	35.00	Dec. 5	72.45	Dec. 5	295.95
	Dec. 7	17.50			Dec. 7	278.45
	Dec. 12	2.00	Dec. 12	15.00	Dec. 12	291.45
	Dec. 19	40.00			Dec. 19	251.45
	Dec. 20	20.00	Dec. 20	25.54	Dec. 20	256.99
	Dec. 30	100.00			Dec. 30	156.99
					Dec. 31	156.99

Miss Perkins, in reviewing her December statement, noticed that the last balance was $156.99. She discovered that the balance on her last stub was $143.99. The balance in her check-book does not equal the balance on the statement. Perhaps an error has been made, or there may be outstanding checks. Find what is wrong.

➡ When a customer cannot locate an error after trying to find it, he should go to the bank and have a bank clerk help him locate it.

58. The Life History of a Check. (Silent Reading.) The drawing below shows the journey that a check takes from the time it is written to pay a bill until it comes back from the bank to the one who wrote it, canceled to show that it has been **cashed.**



Can you explain each of the little pictures? What is the grocer doing in picture 3? What is the teller doing in picture 5? When Mr. Jones filled out the check to the grocer, what else did he fill out?

If you have not already seen real checks, check stubs, monthly statements, deposit books, and deposit slips, ask

one of your parents or some friend of yours who has a checking account to show them to you.

59. Questions about the Checking Account. (ORAL WORK.) From a study of Arts. 51, 52, 53, and 56-58, you should be able to answer every question below.

1. Why do people have checking accounts at banks?

2. What four records of deposits in a checking account are kept?

3. Who writes the record of deposits in the deposit book?

4. Who keeps the records on check stubs?

5. What facts are shown on a properly filled out stub?

6. What facts are shown on a monthly statement?

7. What is to be done if the balance on the monthly statement disagrees with a customer's stub balance?

8. What is meant by "over-drawing" a checking account?

9. Can you name at least 6 items which are shown on a check? (See page 83.)

10. If I pay you a bill of $5.00 by check, what must you do to the check before you can cash it?

11. Find "Bal. Brot For'd $13.50" on page 93. From where was this balance brought?

12. Mr. A wrote 10 checks, which were cashed and canceled by his bank. Who gets these canceled checks?

13. Why is a properly canceled check a receipt of payment?

14. What is an "outstanding" check?

15. Can you give any reason why the amount is written in both words and figures on a check?

PROBLEM SCALE NO. IV

60. Problems Using Facts about Birds. (WRITTEN WORK.) You can earn 75 points by working every problem below correctly. Find your score and rate yourself as directed for Problem Scales I to III.

1. When migrating, certain wild ducks fly at the rate of 47 miles an hour. Some homing pigeons fly at the rate of 55 miles an hour. How much farther will a homing pigeon fly in 5 hours than will a wild duck in the same time? (4)

Use these facts in working Problems 2, 3, and 4: The number of wing beats per minute differs with various birds. A sparrow beats his wings 780 times per minute when flying, a wild duck beats his wings 540 times a minute, and a crow beats his wings but 120 times a minute when flying.

2. How many times as fast does a sparrow move its wings as a wild duck? (4)

3. How many times as fast does a sparrow move its wings as a crow? (4)

4. (*a*) The number of times that a crow moves its wings when flying is what per cent of the number of times that a wild duck moves its? (6)

(*b*) The number of times that a duck moves its wings is what per cent of the times that a sparrow moves its? (6)

5. The golden plover nests in the extreme north of North America and winters in the south of South America. This

bird will fly from Labrador to Guiana, a distance of 1700 miles, without stopping. If it flies at the rate of 35 mi. an hour, how long does it stay in the air on this journey? (6)

6. A swallow has 4 sq. ft. 18 sq. in. of wing area per pound of weight. A pigeon has but 1 sq. ft. 14 sq. in. per pound.

(*a*) Which has the greater wing area—a swallow that weighs $\frac{1}{2}$ lb. or a pigeon that weighs $2\frac{1}{2}$ lb.? (6)

(*b*) How much more? (7)

7. It is estimated that a pound of mosquitoes would have 4 sq. yd. 6 sq. ft. 105 sq. in. of wing area; but butterflies will average 3 sq. yd. 8 sq. ft. 87 sq. in. of wing area per pound of weight. A pound of mosquitoes would have how much more wing area than a pound of butterflies? (7)

8. Of all known kinds of animals about two-thirds can fly. There are about 300,000 kinds of flying insects, 20,000 kinds of flying birds, 600 flying animals, 60 kinds of flying fishes, and 15 other kinds that fly. About how many known kinds of animals altogether are there on the earth? (8)

9. By comparing the speed of common grasshoppers with the speed of automobiles it has been estimated that one kind of grasshopper flies at the rate of 15 miles an hour. At this rate, how far will it fly in thirty seconds? Express your answer in yards. (8)

10. A stork has 122 sq. in. of wing area per pound of weight. Figure the wing area of a stork that weighs 11.34 lb. Reduce answer to sq. ft. and sq. in. (9)

STANDARDS

Superior work:	25–75 points	Slow work:	8–12 points
Good work:	17–24 points	Inferior work:	0– 7 points
Average work:	13–16 points		

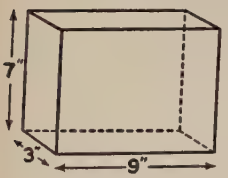
61. Rapid Drill. (ORAL WORK.) Spend 10 minutes in studying the examples before the recitation begins. In division examples, give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	*Add:* 3, 8, 9, 7	*Reduce:* $\frac{18}{4}$ =	158 −79	73 ×6	95)665	30 sq. yd. = ... sq. ft.?
2.	86)708	42 ×9	*Add:* 1, 6, 0, 9	65)197	.125 = ...%	122 −27
3.	579 ×1	145 −46	75)679	97 ×0	*Add:* 5, 7, 3, 6	$\frac{2}{3}$ of N = ...% of N
4.	*Add:* 8, 8, 1, 5	79)474	*Add:* 6, 9, 8, 8	$4\frac{3}{5} = \frac{?}{5}$	122 −95	43 ×4
5.	43)321	91 ×9	114 −48	*Add:* 1, 0, 8, 7	89)445	413 −67
6.	$2 \div 4\frac{1}{2}$ means $2\times$.....	59)240	79 ×4	*Add:* 2, 1, 3, 4	145 −62	2% = what decimal?
7.	116 −77	56)285	21 ×7	Rule for area of △?	*Add:* 6, 7, 7, 3	35)280
8.	176 −79	What is a canceled check?	225 −28	68 ×6	*Divide:* 1 da. 6 hr. by 3	$\frac{1}{4} = \frac{?}{24}$

SELF-TESTING DRILL NO. 8

62. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **What should you do about mistakes on the Drill?**

1. *Subtract:*	2.	3. *Multiply:*	4.
1602475	835)501835	820.6	.476
900689		.512	3.789
			.607
			7.969
5. Find a 2% discount on \$1799.			8.855
			.549



6. What must be added to $14\frac{2}{3}$ to make $28\frac{1}{2}$?

7. Find the volume of the box shown at the left.

8. Check the final balance for July 30, 1927 (\$276.01) as shown on the following bank statement. If the printed balance is correct, write "correct" as your answer. If a mistake has been made, give the correct final balance.

Date	Checks in Detail			Date	Deposits	Date	Balance
	Balance brought forward ☛ 986.15						
July 5	5.85	2.10		July 5	26.38	July 5	994.58
July 6	96.16					July 6	898.42
July 11	1.00	13.64	9.17			July 11	874.61
July 15	2.00	100.50		July 15	85.00	July 15	867.11
July 26	1.85	75.50				July 26	789.76
July 30	513.75					July 30	276.01

9. 1 sq. mi. − 320 A. equals how many acres?

Go on to the next page.

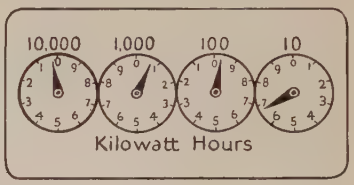
10. \$58.38 was the annual interest paid for a 6% loan. What was the amount of the loan?

11. 2140 = 100% of N. 649 = what % of N?

12. The interest on a certain loan at 5% is \$185.45 a year. What is the amount of the loan?

13.* (*a*) 6 pk. = ____ bu. (*c*) $1\frac{1}{2}$ mi. = ____ ft.
(*b*) 1280 A. = ____ sq. mi. (*d*) 9 bu. = ____ pk.

14. Read the electric meter at the right.



15. 3)6 bu. 2 pk.

*To count Ex. 13 correct, each part must be correct.

STANDARDS

Number correct	0–1	2	3	*	4	5	6	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

THE SAVINGS BANK

63. Savings Bank Accounts. (Class Work.) You have already learned something about checking accounts with banks. There is another kind of bank account which you should understand thoroughly. This is the **savings account.** The savings account is of special interest to you, because probably many of you are now saving money. Every boy and girl should have a savings account, and the bank is the best place to start one.

Many banks allow you to deposit money in what is called their Savings Department, and will pay you interest for the use of your money. The rate of interest on savings is usually 3%, $3\frac{1}{2}$%, or 4%. Many savings banks will allow you to open such an account by depositing \$1.00.

MR. HOWE (To bank official): Mr. Arnold, I want you to meet my son Charles and my daughter Mary. They have each saved a dollar. I told them that I would give them another dollar apiece if they would start a savings account. They decided they would do so.

MR. ARNOLD: That's fine. They are making a splendid start. Wise people always save a part of their money. It is not easy to save, either, because we all see so many things we would like to buy. But there come times when we need a little extra money badly; then we are glad we have some money in savings.

MARY: We learned about interest in school, Mr. Arnold. How much interest does your bank pay?

MR. ARNOLD: We pay $3\frac{1}{2}$ per cent, and you will be surprised how these dollars of yours will grow without your doing anything at all except to save regularly. You see, your money will work for you.

CHARLES: I've got a job delivering papers, and I'm going to save part of that money every week.

The interest on savings accounts is often computed at the end of each six-month period. At that time, the interest due you is added to the **principal** (the amount then on deposit), and both principal and interest earned will draw interest during the following six-month period. When the interest is computed twice a year, the **interest term** is said to be six months.

It will help you to understand savings accounts if you will read the following rules taken from a savings account pass-book (the little book in which the bank records your deposits, withdrawals, and interest earned):

1. Pass-books will be issued to depositors of savings accounts. All deposits and withdrawals must be recorded in this pass-book.

2. At the time of making the first deposit, the one making the deposit must write his or her name in the pass-book and also on a card which is to be retained by the bank.

3. Withdrawals shall be made only upon the presentation of the pass-book by the depositor, in person, or by sending with the book a written order stating the amount to be withdrawn. If such a book be lost or stolen, the person to whom the book was issued must report the loss of the book at once to the bank in writing. If such notice is not given, and the bank should pay in good faith to the one unlawfully in possession of the book, part or all of the money on deposit, the bank shall not be liable for the payment of such amounts a second time.

4. Interest will be paid by this bank on all deposits made, but only for full calendar months during which the money remains on deposit before being withdrawn. Any deposit made within the first five days of the month will be considered as having been deposited on the first day of that month. However, no interest will be paid on fractional parts of a dollar.

5. On the first day of January and on the first day of July, interest will be computed and added to the principal on all deposits entitled to interest under the conditions of section four above.

---------- -- ---------- --------
(Signature of Depositor)

The depositor signs and leaves with the bank a small card which is kept for reference in case of doubt as to the signature (the depositor's name in his own handwriting)

attached to any order for the withdrawal of money he has on deposit.

Answer the questions below to make sure that you have understood the discussion of savings accounts.

1. How many kinds of bank accounts can you name?

2. Give at least one difference between a checking account and a savings account.

3. What are the usual rates of interest on savings accounts according to your text? What is the rate in your town or city?

4. Are the savings banks in your town or city a part of the ordinary banks or are they separate banks?

5. How much money does it take to start a savings account in your local savings banks?

6. What is a pass-book?

7. Do you know what the following words mean? If not, look them up in a dictionary or ask some one who knows. *Depositor, deposit, withdrawal, interest rate, interest term, principal, calendar month,* and *signature.*

COMPUTING INTEREST ON SAVINGS

64. How Interest is Computed on Savings Accounts. (Class Work.) Few men and women really understand how the bank computes the interest on savings deposits. One should know how to compute interest on savings in order to check the bank's figures, since banks sometimes make mistakes.

If you will study all of this article carefully, you will have no trouble in understanding how the bank reckons the amount of interest on your own or your parents' savings accounts. The next page shows how an account is kept in a savings account pass-book. Answer the questions below it.

James L. Montgomery

IN ACCOUNT WITH

THE MINER'S SAVINGS BANK

No. 9864 HAZELTON, PENNSYLVANIA

DATE 1927		WITHDRAWALS		DEPOSITS		BALANCE	
Jan. 3	1927			75		75	
Mar. 3	"			100	50	175	50
May 5	"			50	75	226	25
July 1	"			Interest 3	17	229	42
July 5	"			200		429	42
Aug. 10	"			50		479	42
Sept. 1	"			50		529	42
Sept. 8	"	125				404	42
Sept. 20	"	50				354	42
Dec. 3	"			190	53	544	95
Jan. 2	1928			Interest 8	21	553	16

1. How much did Mr. Montgomery deposit on March 3?

2. What was his balance (total amount in savings) on March 3?

3. What was his balance on May 5? What three amounts added together gave this balance?

4. How much interest did his money earn during the first six months? What was done with this interest?

5. Did Mr. Montgomery draw any money out of his account? If he did, when and what were the amounts?

6. His money earned how much interest the last six months?

On or about July 1, 1927, the bank went over Mr. Montgomery's account to compute the interest his money

had earned during the first six months (see rule 5, page 102.) On an **interest slip** like the one below, the bank clerk put down Mr. Montgomery's balance for each month.

To understand how the interest was computed you must know the rules which savings banks often use. They are given below:

1. Deposits made on or befor the fifth of any month are counted as having been made on the first day of that month.

2. Interest is reckoned only on whole numbers of dollars; the cents are ignored.

3. When one or more withdrawals are made during a month, the interest for that month is figured only on the **smallest** balance on deposit during that month.

INTEREST

Miner's Savings Bank

Depositor No. *9864*

Name *James L. Montgomery* 19*27*

JANUARY	*75*	
FEBRUARY	*75*	
MARCH	*175*	
APRIL	*175*	
MAY	*226*	
JUNE	*226*	
JULY		
AUGUST		
SEPTEMBER		
OCTOBER		
NOVEMBER		
DECEMBER		
Total		
Interest		

4. Deposits made on or after the sixth of a month bear no interest until the first of the following month.

Now answer these questions about interest slips:

1. Why was the February balance $75.00?

2. Why was the March balance on the slip $175.00 instead of $175.50 as shown in statement on page 104? (See rule 2.)

3. What balance is shown on the statement on page 104 for August 10? What balance for August would be shown on the **interest slip?** Why? (See rule 4.)

4. Which of the three September balances shown on the statement will be put on the interest slip? Why? (Read the rules again.)

We are now ready to figure the interest on the savings for the first interest period (January 1 to July 1). The rate of interest was 4%. One way to do this would be to find how much interest each monthly balance would earn and add these amounts. The rate for a year is 4%. The rate for one month would be $\frac{1}{12}$ of 4%, or $\frac{1}{3}$% ($4\% \div 12 = \frac{4}{12}\%$ or $\frac{1}{3}\%$; that is, $\frac{1}{3}$ of 1% or .00$\frac{1}{3}$). Thus we would multiply each monthly balance by $\frac{1}{3}$% and add the amounts. This is done below.

INTEREST

Miner's Savings Bank

Depositor No. *9864*

Name *James L. Montgomery* 19*27*

JANUARY	*75*	
FEBRUARY	*75*	
MARCH	*175*	
APRIL	*175*	
MAY	*226*	
JUNE	*226*	
JULY		
AUGUST		
SEPTEMBER		
OCTOBER		
NOVEMBER		
DECEMBER		
3.17 Total	*952*	
3)952 Interest	*3*	*17*

Balance	*Interest*
$ 75 for Jan. @ $\frac{1}{3}$ of 1%	$ .25
$ 75 for Feb. @ $\frac{1}{3}$ of 1%	$ .25
$175 for March @ $\frac{1}{3}$ of 1%	$ .58$\frac{1}{3}$
$175 for April @ $\frac{1}{3}$ of 1%	$ .58$\frac{1}{3}$
$226 for May @ $\frac{1}{3}$ of 1%	$ .75$\frac{1}{3}$
$226 for June @ $\frac{1}{3}$ of 1%	$ .75$\frac{1}{3}$
	3.17\frac{1}{3}$

THE BANK METHOD

Now from the interest slip notice the way the bank computed the interest. The bank added the monthly balances and found the sum to be $952. It divided the $952 by 3 and then pointed off two decimal places, giving $3.17 as the interest due Mr. Montgomery for the six-month period. Do

you see that dividing by 3 and pointing off 2 decimal places is the same as multiplying \$952 by .00$\frac{1}{3}$?

Which way of computing interest seems easier, the bank method or the other method? Do the results of the two methods agree (if you ignore the $\frac{1}{3}$ of a cent)?

If you are quite alert, perhaps you can prove to the class that both methods must give the same result.

Let us now see how the bank reckoned the interest for the second interest term (July 1, 1927, to January 1, 1928). On January 1, 1928, the bank took Mr. Montgomery's account and computed the interest his money had earned during the second six months of the year 1927. On an interest slip the bank clerk wrote the monthly balances. He then found the total of these amounts, divided by three, and pointed off two places.

INTEREST

Miner's Savings Bank

Depositor No. *9864*

Name *James L. Montgomery* 19*27*

JANUARY		
FEBRUARY		
MARCH		
APRIL		
MAY		
JUNE		
JULY	*429*	
AUGUST	*429*	
SEPTEMBER	*354*	
OCTOBER	*354*	
NOVEMBER	*354*	
DECEMBER	*544*	
8.21 Total	*2464*	
3)2464 Interest	*8*	*21*

The slip shows that Mr. Montgomery's savings account had earned \$8.21 during the last six months of the year.

NOTE: The bank method described here works only for interest at 4%. If the rate is 3%, divide by 4 and point off 2 places.

Some savings banks pay interest every six months but compute the interest on the smallest balance occurring at

any time during the six months interest term or period, instead of reckoning the interest month by month. In this case the smallest balance during the interest term is considered as the sum of money on deposit for the full interest term.

Different banks have different rules for figuring interest on savings. You should learn how your savings bank does it. It would be well to appoint a committee to visit the savings banks in your town or city to find just what rules your banks use.

Can you answer the following questions? To do so you will have to study the interest slips and the pass-book and know the rules.

1. How much money did Mr. Montgomery's savings account earn for him during 1927?

2. What did the bank do each time with the interest amounts?

3. Suppose Mr. Montgomery had been so busy on December 3 that he forgot to make the deposit until December 6. How much interest would he have lost?

4. Mr. Montgomery made no deposits during January 1928, nor did he withdraw any money. What was his January balance on the interest slip?

5. How would you find the amount of interest Mr. Montgomery's money earned during the first three months of 1927?

6. How does withdrawing money from a savings account differ from withdrawing money from a checking account?

7. If you lose your savings pass-book, what should you do immediately? Why?

8. What is one's "signature?"

9. What is meant when we say that our money will work for us?

65. Problems about Savings Accounts. (WRITTEN WORK.) In solving the problems in this article, read Arts. 63 and 64 again if necessary.

1. Harrison Emery put $500 into a savings bank which pays 4%, and left the full amount on deposit for two full interest terms of 6 months each. He then drew out both principal and the interest earned. How much money did he draw out?

2. There is an error in the following figures from a savings pass-book. Find and correct the mistake. (The rate is 4% and the interest term is six months. The same rules apply as in Art. 63.)

Date 1927	*Withdrawals*	*Deposits*	*Balance*
Nov. 12	.....	$ 75.00	$ 75.00
Dec. 6	.....	100.00	175.00
Dec. 23	.....	100.00	275.00
Jan. 2, 1928	*Interest*	.76	275.76

3. A school runs a savings fund which it deposits in a down-town savings bank. On December 22, 1926, it deposited $235.35. No withdrawals were made. What interest did the money earn by July 1, 1927? The rate was 3%.

4. Compute the interest and principal due on January 1, 1928, for the following savings account by both the bank method and the other method. The rate is 3%. (See page 107 for the bank method of computing interest at 3%.)

Date	*Withdrawals*	*Deposits*	*Balance*
July 1, 1927	.....	$100.00	$100.00
Aug. 24, 1927	.....	100.00	200.00
Oct. 14, 1927	$25.00	.....	175.00
Nov. 8, 1927	.....	150.25	325.25

5. What was the interest due on July 1, 1927, on the following account if the interest is computed semi-annually (every six months) at 4%?

Date	*Deposits*	*Withdrawals*	*Balance*
Jan. 5, 1927	$500.00	.....	$500.00
Feb. 2, 1927	200.50	.....	700.50
Feb. 21, 1927	.....	$100.00	600.50
April 7, 1927	50.00	.....	650.50
May 19, 1927	175.00	.....	825.50
June 1, 1927	.....	250.00	575.50
June 8, 1927	.....	115.25	460.25

ACCURACY IN MAKING CHANGE

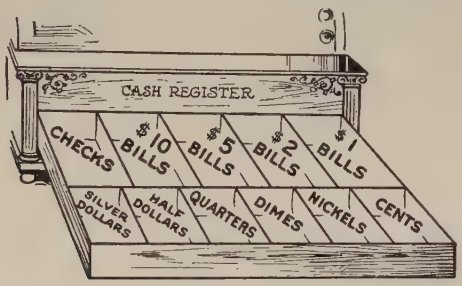
66. A Use of Arithmetic in Making Small Sales. (Silent Reading.) As you know, many stores use a cash register in which the money for sales is kept and from which change is taken. Thus, if the salesman who is shown in the picture on page 112 received a five-dollar bill in payment of a purchase amounting to $2.25, he would put the $5 into the cash register and take out the amount of the change ($2.75) to give back to the purchaser.

In making the change, most clerks **do not** think as follows: $5 given me; sales were $2.25. To get change, subtract $2.25 from $5. This is $2.75; so I must count out $2.75 in change.

If you will watch a clerk the next time he uses a cash register, you will see that he makes change by adding up from the purchase amount to the amount that was given him. Thus, if he must make change from a $5 bill for a $2.25 purchase, he will think: $2.25 and 25 (taking out a quarter from the register) make $2.50, plus 50 (tak-

ing out a half dollar from the register) makes \$3, plus \$2 makes \$5 (taking out two one-dollar bills or one two-dollar bill from the register). Thus, as he has counted up the change, he has also taken it out ready to give back to the purchaser. He will often repeat his count aloud as he gives the money to the customer.

Study the picture of a cash-register drawer below, and think how a clerk would make change for the amounts received and the amounts of sales given below the picture.



(*a*) Sales amount, 19¢; clerk received a quarter.
(*b*) Sales amount, 38¢; clerk received a dollar bill.
(*c*) Sales amount, \$1.35; clerk received a five-dollar bill.

Always begin to make change **from** the amount of purchase. In other words, begin to count up from the sales amount and continue counting until you reach the amount given in payment. Thus, if you were making change for a \$3.15 purchase from a \$5 bill given in payment, you would think:

\$3.15 and a dime make \$3.25,
and a quarter make \$3.50,
and a half make \$4.00,
and a \$1 bill make \$5.00.

67. Making Change. (WRITTEN WORK.) Make a chart like the one below and record the change you would make for the purchases listed below it. The first example is done correctly.

There is often more than one correct way to make change. After finishing the work, talk over in class each answer to see if all have made correct change.

Example	*Amount of Sales*	*Purchaser Gave*	*Cents*	*Nickels*	*Dimes*	*Quarters*	*Halves*	*Dollars*
1	$3.36	$5.00	4		1		1	1

1. Amount of purchase, $3.36. Purchaser gave in payment a $5 bill.

2. Purchase, 40¢. Purchaser gave in payment a half dollar.

3. Purchase, $1.37. Purchaser gave in payment $2.00.

4. Purchase, \$3.60. Purchaser gave in payment \$10.00.

5. Purchase, 17¢. Purchaser gave in payment 50¢.

6. Purchase, \$1.10. Purchaser gave in payment \$1.50.

7. Purchase, \$2.05. Purchaser gave in payment \$2.25.

8. Purchase, \$4.50. Purchaser gave in payment \$4.50.

9. Purchase, \$3.75. Purchaser gave in payment \$5.00.

10. Purchase, 18¢. Purchaser gave in payment 50¢.

SELF-TESTING DRILL NO. 9

68. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Is the line going up on your Progress Chart?**

1. $25 \div 12\frac{1}{2} = ?$

2. Mary said the correct answer to 1,018,232 − 594,357 was 423,875. Floyd said it was 534,875. Who was right?

3. $409\overline{)343151}$

4.
\$6.99
8.86
7.33
4.67
5.99
2.57

5. $5\frac{1}{3} \div 6\frac{2}{3} = ?$

6. How much greater is $22\frac{2}{9}$ than $18\frac{1}{2}$?

7. Find the product of 81.49 and 836.

8. 25% of $N = 206$. 100% of $N = ?$

9. $507 = 33\frac{1}{3}\%$ of N. 100% of $N = ?$

10. What per cent of 1950 is 325?

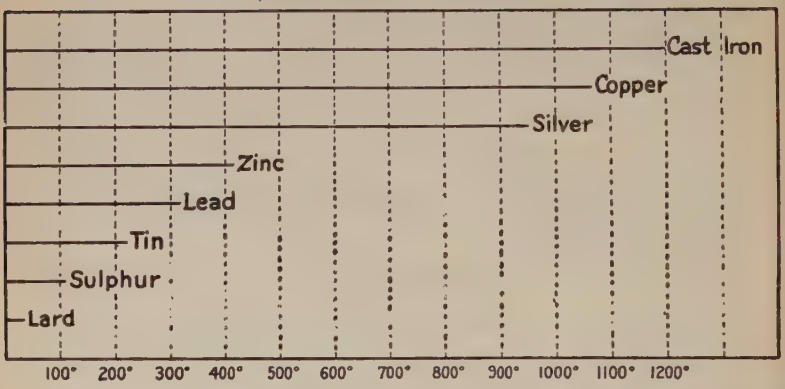
11. Find the difference of 2 pk. 3 qt. and 1 pk. 5 qt.

12. What is the volume of a solid, each of whose three dimensions is one yard?

13. What interest is earned on \$9145 in $1\frac{1}{2}$ yr. at $4\frac{1}{2}\%$?

14. 7 pk. 2 qt. $\div 4 = ?$

MELTING TEMPERATURES OF CERTAIN SUBSTANCES
(IN DEGREES CENTIGRADE)



15.* Answer the following questions by means of the graph above. The graph shows the temperatures at which certain substances melt.

(*a*) What is the melting temperature of sulphur?

(*b*) What metal has about twice as high a melting point as zinc?

(*c*) What metal has a melting point about one-fourth as high as cast iron?

(*d*) About how much lower is the melting point of silver than that of copper?

*To count Ex. 15 correct, each part must be correct.

STANDARDS

Number correct	0–1	2	3	4	*	5	6	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Were there some "hard spots" for you in the drill? The *Self-Help Index*, page 471, will direct you to study material that will clear up things you may not understand.

REVIEW OF PERCENTAGE

69. Mixed Review of the Uses and Applications of Percentage. (ORAL WORK.) You should not have to use your pencil for this article. Study it silently for 10 minutes in order to be ready with correct answers when your teacher calls upon you.

1. Which one of the three questions will be answered by working Example A?

(*a*) What is 64% of 84?

(*b*) What per cent of 84 is 64?

(*c*) If 84 is 64% of a number, what is 100% of the number?

EXAMPLE A

$$\begin{array}{r} 84 \\ \times .64 \\ \hline \end{array}$$

2. Which one of the three questions will be answered by working Example B?

(*a*) What is 96% of 124?

(*b*) 96 is what per cent of 124?

(*c*) If 124 is 96% of a number, 100% of the number = ?

EXAMPLE B

$\frac{96}{124}$ or $124\overline{)96.00}$

3. Which one of the three questions will be answered by working Example C?

(*a*) What is 46% of 136?

(*b*) 46 is what per cent of 136?

(*c*) If 136 = 46% of a number, 100% of the number = ?

EXAMPLE C

$.46\overline{)136.00}$

In the next three examples, choose the correct answer.

4. The process used in finding a per cent of a number is *multiplication* *division.*

5. The process used in finding what per cent one number is of another is *multiplication* *division.*

6. When a certain per cent of a number is given, the process used in finding 100% of that number is *multiplication* *division.*

7. Tell what ought to be on each blank. Read carefully or you will give wrong answers.

(*a*) $16\frac{2}{3}\%$ of 66 is.....
(*b*) 100% of 96 is.....
(*c*) $\frac{3}{4}$ is 50% of.....
(*d*) $33\frac{1}{3}\%$ of 105 is.....
(*e*) 34 is 1% of.....
(*f*) $\frac{5}{4}$ of N =.....% of N
(*g*) 250% of 20 is.....
(*h*) 5 is $\frac{1}{2}$ of 1% of....
(*i*) $24\frac{1}{2}\%$ =.....(decimal)
(*j*) $2\frac{1}{4}\%$ =.....(decimal)
(*k*) 15 is $66\frac{2}{3}\%$ of.....
(*l*) $12\frac{1}{2}\%$ of 7 is.....

8. A discount of 5% for prompt payment on a bill of \$400 would be what amount of money?

9. 6% interest on a loan of \$50 for six months would be what amount of money?

10. A quantity discount of 30% on a bill of \$1500 would be what amount of money?

11. 50 feet of a 200-foot board walk had rotted and had to be repaired. What per cent of the walk had to be repaired?

12. In a certain nursery there are 100 oak, 75 ash, 50 maple, 30 elm, 25 hickory, 15 poplar, and 5 walnut trees.

(*a*) What per cent of the trees are oak?
(*b*) What per cent are ash?
(*c*) What per cent are poplar?
(*d*) What per cent are walnut?

13. A clerk figured a 20% discount on a bill to be \$25. What was the bill before discounting, if he figured correctly?

14. A clerk figured that a 4% discount on a bill was \$2.64. What was the original bill if he figured correctly?

15. A 10% discount on a bill amounted to \$3.75. What was the amount of the bill before it was discounted?

16. You should find the next six examples easy.

(*a*) 8 is what % of 32?
(*b*) 19 is what % of 190?
(*c*) 40 is what % of 20?
(*d*) 6 is what % of 36?
(*e*) 11 is what % of 55?
(*f*) $\frac{1}{4}$ is what % of $\frac{3}{4}$?

17. Per cents can be added and subtracted.

(*a*) 4%+8%=
(*b*) 24%−6%=
(*c*) Find the average of 8%, 6%, and 21%.

70. Problems Using Per Cents. (WRITTEN WORK.) If you have solved all the problems in Art. 69 correctly, these problems should not trouble you.

1. Joseph sold his motorcycle to Richard on the following terms. Price \$80. Payments as follows: 20% cash, 30% in 30 days, and the rest in 90 days.

(*a*) How much did Richard pay at the time he bought the motorcycle? (*b*) How much in 30 days? (*c*) In 90 days?

2. Henry and his father were driving a new automobile from Detroit to their home. When they stopped in a town for supper, Henry's father said they had covered about 80% of their trip. The mileage record on the car read 112 mi. Henry figured how far the total trip would be. What should his answer have been?

3. About what per cent of the year's work in terms of pages of this book will you finish when you complete this page? (HINT: Consider page 386 as the last page of the year's work.)

4. Last year Mr. Wilson earned \$2450. This was 135% of what he spent. How much did he spend? If he saved the rest, how much did he save?

5. This year Mr. Wilson expects to earn $2800. He hopes to save 20% more than he did last year. How much does he hope to save this year? (HINT: You will need to use Problem 4. Is there a fact in Problem 5 you do not need to use?)

6. In solving 24 problems, Esther was wise enough to read 9 of them twice before starting to find the answers. What per cent of the problems did she read twice?

7. James rides to school in a motor bus. He and the driver are good friends, and they often talk over James's school work. Here is a problem the driver gave him. James could not work it, but you should be able to do so.

"I carried 180 passengers yesterday morning. This was 48% of my day's business. How many passengers did I carry yesterday?"

8. Mrs. Wagner loaned $450 to a creamery company. She received $24.75 interest per year. What rate of interest was her money earning for her?

9. Last year William kept an account of all that he earned and meant to record every cent he spent. At the end of the year he balanced accounts. He had earned $196. He could account for all but $4.90 of it. What per cent of his earnings could he not account for?

10. The Bailey family is saving money to buy a radio. Thus far they have saved $42.25. Mr. Bailey said, "This is 35% of what we must pay for the radio." Find the cost of the radio.

11. The principal of the Calhoun school reports that only 75, or 12%, of the pupils of the school have failed to show a gain on their Progress Charts in arithmetic. How many pupils are there in the school?

12. Mr. Travis borrowed \$750 @ 8% for 3 mo. At the end of the 3 months he should have paid back the \$750 and how much interest?

13. There are 256 pupils in the Sidney Lanier School. Approximately 86% of them can swim. How many can swim? Work the problem and tell whether it would be possible for **exactly** 86% to know how to swim.

14. During Thrift Week the pupils in a school in Oklahoma deposited \$746.50 in the school savings bank. 40% of this was deposited by boys and girls who worked after school. These pupils deposited how much money?

INTRODUCTION TO BUDGET MAKING

71. Planning to Save. (Class Work.) It is a good plan to keep an account of all money earned (**income**) and spent (**expenditures**). It is also a good plan to look ahead in order to decide **what you can afford to spend.** Planning accounts ahead is called spending money according to a **budget,** or **keeping a budget.**

Many school systems, city governments, and business concerns spend their money according to a budget. The officials look ahead, see how much money there will be to spend, what different items of expenditure there will be, and then decide how much money can be spent for each item in the budget. The United States Government spends its money on a budget plan.

Many families arrange the expenditures of the home according to this plan of budget making. They decide how much there will be to spend and how much can be spent on each item of family expense.

There are many good ways of planning the items of a budget and listing them under main headings. Here is one way that has been found helpful:

MAIN ITEMS OF A SIMPLE FAMILY BUDGET

Food	*Shelter*	*Clothing*	*Miscellaneous*	*Savings*
Groceries	Rent	Clothes	Church	In savings
Milk and	Taxes	Shoes	Vacations	bank
butter	Cleaning	Repairing	Education	Invested in
Eggs	Fuel	clothes	Amusements	business
Meat	Light	shoes	Doctor	Insurance
	Telephone		Dentist	
	Furniture		Gifts	
	Repairs		Charity	

The main question which must be answered by every business-like man and woman is: How much or what per cent of what I earn can I **afford** to spend for each item in my budget?

A man earning $200 a month cannot afford to pay $100 a month for shelter, but a man earning $500 a month might afford to pay as much as $150 a month, although $125 a month would be nearer the proper amount. The table on the next page is a good one to use in planning a budget. It is based on the expenses of a family of man, wife, and two children. The table shows about how much one can afford to spend on the five main items of a simple family budget like the one given above.

The amount which it costs to support oneself, or one's family, is called the cost of living. The cost of living varies somewhat in different parts of the country; so some changes might have to be made in these estimates to meet local conditions.

BUDGET TABLE FOR A FAMILY OF FOUR

If you earn	*Spend for Food*	*Shelter*	*Clothing*	*Miscellaneous*	*Try to Save at least*
$1200—1450 a yr..	35%	20%	15%	25%	5%
$1500—1750 a yr..	35%	20%	15%	25%	5%
$1800—2350 a yr..	35%	20%	15%	20%	10%
$2400—2950 a yr..	35%	20%	15%	15%	15%
$3000—3500 a yr..	30%	20%	15%	15%	20%

It is not always possible to keep within one's budget in any one year. Severe sickness may force one to spend a great deal for doctor's bills. Rents may be very high, etc. But over a period of years it is wise to keep expenditures within a definite budget.

You do not **have** to remember this information about planning your expenses, but it is wise to have **some plan** of spending your money. When you earn money, decide on a budget, and then stick to it. You will find that by doing this you will get more out of your money. No budget is worth much unless you follow it.

72. Problems In Spending on the Budget Plan. (WRITTEN WORK.) In solving the problems below use the Budget Table at the top of this page. When earnings are given "by the week" or "by the month," change to a yearly basis.

1. If you earn $1200 a year, what is the most you should pay for shelter in a year?

2. If you earn $1200 a year, what is the most you should pay for shelter per month?

3. Mrs. Brown's food bill for August came to $44.50. Did she keep within a fair allowance for food? The family income was $1600 a year.

4. If Mr. Keith earns $2500 a year, how much should he try to save a year?

5. On a salary of $42 a week, Mr. McAndrew allows $25 a month for clothes for his family. Can he wisely afford that much?

6. Mr. and Mrs. E. J. Walker saved $600 last year out of an income of $3000. Was that a fair amount to save?

7. What should Mr. Watson allow for shelter per month if he has an income of $2800 a year?

8. Figure the amounts for the main items of the budget for a year if the family income is $2000 a year.

9. Figure the amount that it is wise to spend **per month** for each item if the family income is $1600 a year.

10. The month's food bills for a year were as follows: $98.75, $110.50, $92.00, $87.80, $94.75, $98.15, $100.25, $97.60, $89.95, $115.35, $95.15, $94.00. Would these expenditures come within a good budget estimate if the family earnings were $300 a month?

PRACTICE IN ESTIMATING

73. Matching Examples and Estimated Answers. (STUDY PERIOD.) At the top of the next page are examples with three estimates for each. Choose the best estimate for the answer of each example, without actually working it. Write your estimates on paper. After the study period is over, the class should agree on which estimate is the best one for each example.

Examples	Estimates		
1. 876×1.01	(*a*) 87687	(*b*) 8.76	(*c*) 876
2. $233 \times 30\frac{2}{3}$	(*a*) 69.9	(*b*) 160	(*c*) 6990
3. 4% of 362	(*a*) 14	(*b*) 1.2	(*c*) 120
4. Find N if $67 = 32\%$ of N	(*a*) 2	(*b*) 50%	(*c*) 200
5. 86 is what % of 21	(*a*) 400%	(*b*) 25%	(*c*) 17
6. $.41\overline{)9.3}$	(*a*) 23	(*b*) 2.3	(*c*) .2
7. $4.63 \times .501$	(*a*) .23	(*b*) 2.3	(*c*) 23
8. If 1.1% of $N = 34$, $N =$	(*a*) 36	(*b*) 360	(*c*) 3600
9. $.032\overline{)601}$	(*a*) 20,000	(*b*) 200	(*c*) 20
10. If $357 = 600\%$ of N, $N =$	(*a*) 60	(*b*) 360	(*c*) 6000
11. $5.1\overline{).1003}$	(*a*) 2000	(*b*) 2	(*c*) .02

PROBLEM SCALE NO. V

74. Problems Using Decimals. (Written Work.) You can earn 60 points by working every problem correctly. Find your score by adding the numbers printed at the right of each problem you worked correctly.

Rate yourself by the standards at the end of the article. Number your answers plainly.

1. The team which won the pennant in the school baseball league had a standing of .803. The lowest team had a standing of .468. What was the difference in standing between the winners and the team at the bottom of the league? (4)

2. Mrs. Bailey keeps an account of her household expenses. Her budget allows her $185 a month for these expenses. Her account book shows the following expendi-

tures for last month: food, $85.95; rent, $40.00; fuel, $6.25; lighting and phone, $3.85; clothing, $15.60; insurance, $5.00; other expenses, $3.17.

How much more or less than her allowance did she spend last month? (5)

3. John wants to buy a sleeping-bag to use next summer. How many hours must he work in order to get enough money to buy the sleeping bag if it costs $12.50 and he receives 30¢ an hour for his work? (5)

4. To the question: "The Baltic Sea is nearest to what continent?" 2400 answers were received. 1756 of these answers were correct. What per cent of these answers were correct? (6)

Use the following facts in Problems 5 to 10:

Mr. Garey kept an accurate account of the cost of rearing a boy to the age of 16. He found that this boy ate during the first sixteen years of his life $2,000 worth of food. $725 was spent for the boy's clothes. The boy's share of the cost of rent was $925. Money spent on the boy's health amounted to $400. Mr. Garey calculated that the cost of recreation for the boy was $150.

5. According to these figures, how much did Mr. Garey spend on the boy in all? (6)

In Problems 6 and 7, carry the answer to one decimal point.

6. What per cent of the total expense of rearing the boy went for food? (6)

7. What per cent went for clothes? (7)

8. If the boy earned 32% of the money spent upon him, how much did he earn? (6)

9. The total cost according to Mr. Garey's calculations, which you found from the facts given above, was for the first 16 years of the boy's life. Find the average yearly cost for the first 16 years. (See Problem 5.) (6)

10. Compute the cost of rearing a girl to 16 years of age if the cost of rearing a boy is $112\frac{1}{2}\%$ of the expense of rearing a girl. (9)

STANDARDS

Superior work:	34–60 points	Slow work:	5–12 points
Good work:	20–33 points	Inferior work:	0– 4 points
Average work:	13–19 points		

75. Rapid Drill. (ORAL WORK.) See how quickly you can do the examples below and at the top of the next page. In division examples give only the first quotient figures.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	$\frac{3}{4}=\frac{?}{12}$	807 ×3	89 −71	150% = what decimal?	4+0+9	85)764
2.	7.3 − 1	*Add:* 5, 2, 5, 3	100)132	45 min. +....min. = 1 hr.	3.6 − .9	951 ×2
3.	13)38	.04 = what %?	$\frac{1}{4}\div 6=$ $\frac{1}{4}\times$....	40 ×0	*Multiply and reduce:* 8 in. ×3	98 −91
4.	45 −28	$\frac{1}{2}\%$ of $900	72)214	3 pk. = ...qt.	*Add:* 6, 10, 4, 8	27)10.8
5.	743 ×2	93)275	7.9 + .7	$\frac{1}{4}\div\frac{1}{4}$	5 = 50% of....	26 −26

	(A)	(B)	(C)	(D)	(E)	(F)
6.	$57\overline{)275}$	*Subtract:* \$5.00 − .85	Budget means what?	44 − 32	509 × 7	*Add:* 6, 2, 3, 4
7.	4.2 − 3.6	$\frac{2}{3} \times \frac{3}{4}$	434 − 100	How do you make change?	3 = ...% of 4	87 × 9
8.	42 − 5	$4.1\overline{)36.9}$	Explain factors	.47 × .5	16 − 7	4 gal. = ...qt.

SELF-TESTING DRILL NO. 10

76. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Are you catching your mistakes by checking your work? You get better ratings if you do.**

1. Find the difference of 168.5418 and 96.0464.

2. *Add:*	3. *Multiply:*	4.	5. *Add:*
957	1072	$6.49\overline{)2329.91}$	$3\frac{2}{5}$
366	4162		6
449			$4\frac{1}{3}$
975			$2\frac{1}{6}$
849			

6. Find the product of $3\frac{7}{12}$ and $2\frac{2}{3}$.

7. Find $\frac{1}{2}\%$ of 723.

8. Is $492.016 \div 4.91$ about: 1000, 100, or 10?

9. What per cent of 336 is 42?

10. *Multiply:*

8 T. 750 lb.
7

11. My share of the profits was \$348.12, or 36% of the total profits. What were the total profits?

12. $428\overline{)287616}$

13. The parallelogram shown at the right has a base of $4\frac{1}{2}$ in. and an altitude of $1\frac{1}{3}$ in. What is its area?

6 sq. ft.	98 sq. in.
4 sq. ft.	112 sq. in.
5 sq. ft.	72 sq. in.

14. What is the total area of the three surfaces whose areas are given at the left?

15. Find the cash on hand for July 1, 1927, in Henry's cash account below.

1927			Receipts		Expenditures	
June	1	Cash from May, 1927	5	75		
	5	New cap			1	25
	10	Mowing lawn		75		
	11	Watering garden		90		
	16	Savings bank			5	00
	17	Cleaning cellar	1	50		
	21	Class dues				35
	25	Movie				25
	26	Working in grocery store	1	75		
July	1	Cash on hand.....				

STANDARDS

Number correct.....	0	1	2	3	4	5	*	6	7–8	9–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

The *Self-Help Index*, page 471, lists articles for study and practice to make hard examples easier the next time you meet them.

SUMMARY

77. General Review. (Oral Work.) Study the exercise below in order to tell what number, word, or phrase should be on each blank when you are called upon.

1. When two or more numbers are multiplied, each of the numbers is a ..... of the product.

2. Checks stamped "Paid" are called ..... checks.

3. Checks that have not been cashed at the time the monthly statement is made out are called ..... checks.

4. The record in the deposit (pass) book is written by the .....

5. $5+0=$ ..... $5-0=$ ..... $5\times0=$ .....

6. To indorse a check means .....

7. A good estimate of $9.1\overline{).4906}$ is .....

8. A deposit slip is given to a depositor when .....

9. The error in the example at the right is .....

10. Two things shown on a monthly statement are..... and .....

$$\begin{array}{r} 265 \\ 204 \\ \hline 1060 \\ 530 \\ \hline \end{array}$$

11. $.94\times894$ would answer this percentage question: .....?

12. The error in the example $\dfrac{26}{4\overline{)824}}$ is .....

13. $\frac{86}{126}$ or $126\overline{)86.00}$ would answer this percentage question: .....?

14. Two factors of 35 are 5 and ..... .

15. The best rating on a Self-Testing Drill I have yet made is .....

16. To "over-draw" one's checking account is .....

17. Five main items for a simple family budget might well be: ....., ..., ..., ..., ...

18. Three things which help in problem solving are: ....., ..., ...

19. You can keep your own records of a checking account on .....

20. Not how much do I want to spend but how much can I ..... to spend is a question all wise people should ask themselves.

21. Interest on savings accounts is usually computed ..... time(s) a year.

22. In reckoning interest on savings, only ..... numbers of dollars are considered; all the ..... are ignored.

23. Bankers often compute interest at 4% for 6 months by dividing by ..... and pointing off ..... decimal places.

24. The correct change from a $1 bill in payment for a 66¢ article might be 4....., 1....., and 1.....

25. The correct change from a $5 bill in payment for $2.35 worth of cloth might be 1....., 1....., 1....., and 2.....

26. Is $.05\overline{).2003}$ about .4, 4, 40, or 400?

CHAPTER IV

MEASUREMENTS. THE FORMULA. THE ARITHMETIC OF COMMISSION

SOME FUNDAMENTAL FACTS OF MEASUREMENT

78. Three Kinds of Measurements. (Class Work.) As you already know, there are three main kinds of measurements. These are:

1. Measurement of Length or Distance. We often find it useful to measure with a high degree of accuracy the distance between two points or two places. **Measurements of distance** would be made in answering such questions as the following:

(*a*) How far is it from New York to Chicago?

(*b*) What is the distance around the world at the equator?

(*c*) What are the dimensions of your classroom?

(*d*) What is the height of the tallest building in your community?

2. Measurement of Area, or Extent of Surface. We need to know with accuracy more facts than those of distance or length. We often need to know the **area** or the extent of surface that is covered by whatever objects we may be thinking about. Measurements of area would be made to answer such questions as the following:

(*a*) Which has the more floor space (area), the assembly room or the halls on the first and second floors of the building?

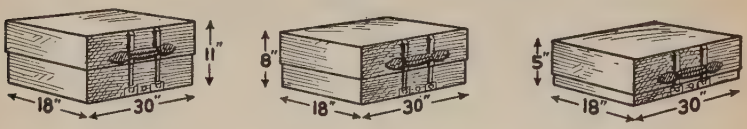
(*b*) How large (how many acres) is Mr. Johnson's farm?

(*c*) What per cent of the earth's surface is water?

3. Measurement of Volume or Extent of Space. In addition to length and surface measurements, we are sometimes interested in measures of volume. Measurements of volume would be necessary in answering such questions as these:

(*a*) What is the capacity of a coal bin with dimensions $8' \times 10' \times 12'$?

(*b*) How many cubic yards of earth must be removed from a level lot in order to make a cellar $10' \times 30' \times 40'$?



(*A*) (*B*) (*C*)

(*c*) What are the volumes of this expanding traveling bag (*A*) when it is extended to its greatest capacity, (*B*) when partially extended, and (*C*) when closed as much as possible?

➠It is important to remember that there are three kinds of measures: (1) those of length or distance, (2) those of area or surface, and (3) those of volume.

Note: The word "capacity" often occurs in speaking of measurements. In one sense **capacity means volume and is expressed in cubic units.** However there are many other uses of the term capacity, such as:

(*a*) The capacity of this freight car is three automobiles. (Note that the capacity of the car in the strict sense of volume would have been stated in cubic feet or cubic yards.)

(*b*) The capacity of this hay barn is 400 tons.

(*c*) The capacity of this schoolhouse is 1200 pupils.

Ordinarily you will have no difficulty in deciding in what sense the word "capacity" is used if you are careful in reading the problems and discussions in this book.

LINEAR MEASUREMENTS

79. Why We Have Several Units of Linear Measure. (SILENT READING.) We need to measure distances of such greatly different lengths that we have several **units of measure,** or **standard units,** for distances. Thus:

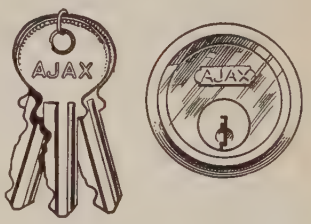
The **inch** is a unit of measure of small distances.

The **foot, yard,** and **rod** are used for longer distances.

The **mile** is the unit of measure for even longer distances.

Of course these units of measure are all related to each other. For example, one linear foot is equal to 12 linear inches. The word **linear** is the adjective form of **line.** There are $5\frac{1}{2}$ or 5.5 yd. in a rod, etc. A table of linear measures will be found on page 470 of this book.

One reason why we have several units of linear measure is convenience. Sometimes we need to make very exact measurements. How could the man who manufactures keys and locks, like those at the right, think accurately about his work if the only unit of linear measure were the foot or yard?



At other times we would find that it would be very bothersome and would cause needless work to measure with so small a unit as the inch. For many of our purposes a large unit only is needed. We could measure the distance around a room in inches, but it is more convenient to use feet, or yards. In this way we can work with smaller numbers. In planning a day's trip in an automobile, we need to know the distance between towns only in miles. We do not need to know that it is 16 mi. 500 ft. 6 in. from the post office at Ashton to the

post office at Bellevue. "A little over 16 mi." is all we would need to know.

To express the area of a large farm in square inches or square feet, or the capacity of a box-car or a large warehouse in cubic inches would require inconveniently large numbers, and call for very tiresome measurements.

80. Changing Distances from One Unit to Another. (CLASS WORK.) In our use of linear measurements we often need to know how to change a distance reported in yards to an equivalent one reported in feet, or a distance recorded in inches to an equivalent one in feet, etc.

Every seventh-grade pupil should be able to do the work below accurately and easily.

Tell what should be on each blank.

1. 1 mi.	=.....ft.	10. 90 in.	=.....yd.
2. 29 in.	=.....ft.	11. 1000 ft.	=.....in.
3. 60 ft.	=.....yd.	12. $\frac{3}{4}$ mi.	=.....ft.
4. 1 rd.	=.....ft.	13. 1 rd.	=.....yd.
5. 3 in.	=.....ft.	14. $\frac{1}{3}$ ft.	=.....in.
6. 9 yd. 2 ft.	=.....ft.	15. 1 mi.	=.....rd.
7. $\frac{1}{3}$ mi.	=.....yd.	16. 16 yd.	=.....ft.
8. 33 ft.	=.....rd.	17. 6 in.	=.....ft.
9. 8 ft. 2 in.	=.....in.	18. 1 mi.	=.....yd.

☞An important fact of linear measurement is that a distance stated in any unit of linear measure can be expressed in any other unit of linear measure.

Thus, a distance in miles can be expressed as a distance in feet, a distance stated in feet can be expressed in inches, etc. These changes can also be made in reverse order.

81. Estimating Distances. (Class Work.) It is useful to be able to **estimate** distances and lengths without measuring them. One can learn to estimate short distances by repeated practice in judging distances and then actually measuring them.

Spend a few minutes in estimating short distances. The following suggestions will be useful to your class.

1. Agree on several distances to estimate. One class agreed on the following:

A straight line drawn on the blackboard.
The distance between the floor and the top of the blackboard.
The width of some door in the room.
The distance between two chalk marks drawn on the floor.

2. Write your estimates.

3. After you have written your estimates, measure each distance carefully.

4. Compare your estimates with the actual distances you found by measuring.

5. Note whether you are inclined to "over-estimate" or to "under-estimate" distances.

➡ If your estimates were very poor, it would be a good plan to practice estimating distances until you can do it fairly accurately.

It is much more difficult to estimate long distances by looking at them. Objects seen at great distances appear very much smaller than they actually are. In looking at mountains it often seems as though they were but a few miles away, when, as a matter of fact, they may be 20 or even 50 miles distant. In the picture on the next page, notice how the telephone poles and other objects look smaller the farther they are away.

Try to estimate some long distances for which the actual lengths can be obtained, as heights of tall buildings or monuments, distances from one farmhouse to another, or from one place to another in your city.

82. Accurate Measurement of Short Distances. (SILENT STUDY.) It is difficult to measure small distances with accuracy. If you measured off an inch with a dull, blunt pencil and a poor ruler, you would probably not get **quite the same** distance that a very skilled draftsman, using a sharp pencil and a high-grade ruler, would get. The following lengths were given by four seventh-grade pupils in measuring this line:

(*a*) $1\frac{1}{8}''$ (*b*) $1\frac{1}{8}+''$ (*c*) $1\frac{3}{16}''$ (*d*) $1\frac{1}{4}''$

Which of the measurements is most nearly correct?

Many of the common things in life are measured with very great accuracy. Parts of machinery often have to be made so carefully that an error of $\frac{1}{100}$ or even $\frac{1}{1000}$ of an inch in measuring them would make the part useless.

Some time when you are asked to study a topic for a report to your class or for a composition in your English work, you will find some very interesting facts about accurate measurements and how they are made by studying topics like the following:

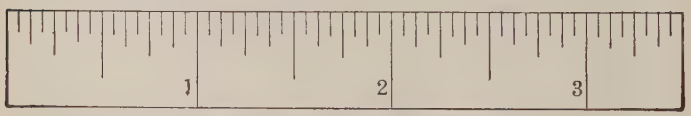
The Accuracy of Measurements of the Parts of an Automobile.
How a Watch Is Made.
How a Fountain Pen Is Made.

For your work in arithmetic the following cautions are worth remembering:

1. Measure more than once and always with great care. Average the results of several measurements.
2. Be sure your ruler or yardstick is straight and true.
3. Use the best ruler or measuring instrument you can get.
4. Work very carefully. Do not let your ruler or pencil slip.

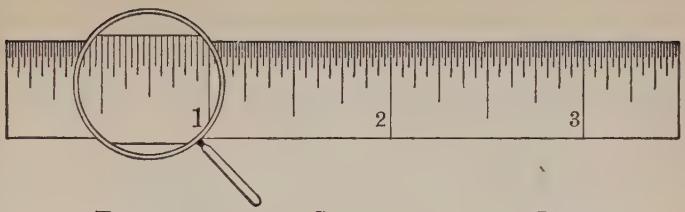
By looking at the rulers below and on the next page, you can see how accuracy of measurement depends upon the measuring instrument used.

HALVES AND QUARTERS OF AN INCH



HALVES, QUARTERS, EIGHTHS, AND SIXTEENTHS OF AN INCH

SIXTEENTHS AND THIRTY-SECONDS OF AN INCH



THIRTY-SECONDS AND SIXTY-FOURTHS OF AN INCH

83. Problems about Linear Measurement. (WRITTEN WORK.) The following problems will help you see some of the many uses that can be made of the linear measures about which you have been studying.

1. What is the exact length (top to bottom) of the paper on which this page is printed?

2. What are the exact dimensions of the New Work sign on page 135?

3. How wide is the ruler shown at the top of this page? Measure the width of the part of the ruler shown under the magnifying glass. What is the difference of the two measurements?

4. Mr. Henry had a hedge planted across 60 feet of the back and along 40 feet of one side of his lot. A gardener furnished the bushes and did the work for 60¢ a foot. How much did it cost Mr. Henry?

5. A western farmer fenced a field 40 rods by 80 rods. He set the posts 15 feet apart. How many posts did he use?

6. An army balloonist made an ascent of 43,580 feet. How many miles high did he go? (Carry answer to two decimal places.)

7. The sailor's mile, often called the "knot", is 1.15 times as long as the mile you have learned. The knot is how many feet long?

DRAWING TO SCALE

84. Scale Drawings. (Class Work.) When an architect prepares the plans for a building or a draftsman draws a design for a machine, he must let some small unit of measure like an inch or a fraction of an inch stand for some larger unit. If he did not do this, he could not get his drawing on a conveniently sized sheet of paper or even on any sheet. He therefore draws "to scale." He lets 1 inch on his drawing stand for 5 feet, 8 feet, 16 feet, or whatever number of feet may be convenient; or he may let $\frac{1}{4}$ inch or $\frac{1}{8}$ inch stand for 1 ft., 2 ft., or 3 ft., as he pleases, and every part of his drawing is made according to this scale. Thus on a sheet of convenient size, he has a complete plan of the building, the machine, or whatever it may be.

You are probably best acquainted with scale drawings from the maps in your geography books. On maps, an inch generally stands for a certain number of miles.

In drawing to scale, keep these things in mind:

(*a*) Choose a scale that will make a drawing of reasonable size on your paper.

If you were drawing a baseball diamond to scale, you would not use the scale 1″ : 1′, for it is 90 feet from home plate to first base. If the scale were 1″ : 1′, your drawing would be at least 90 inches long. This would be too large for your paper.

If you desire to have a map on a page that is 4″ by 6″, the longer side of the area to be mapped would have to be represented by a line not longer than 6″. If the map were to be made on a chart $2\frac{1}{2}$′ by 3′, the longer side of the area to be mapped would have to be shown by a line not longer than 3′.

(*b*) Be sure to get all needed measurements of the object you are going to draw. If one is not careful, he is likely

to have his drawing well along and suddenly find that he has failed to measure certain needed parts of the object.

(*c*) Use a sharp pencil and a good ruler.

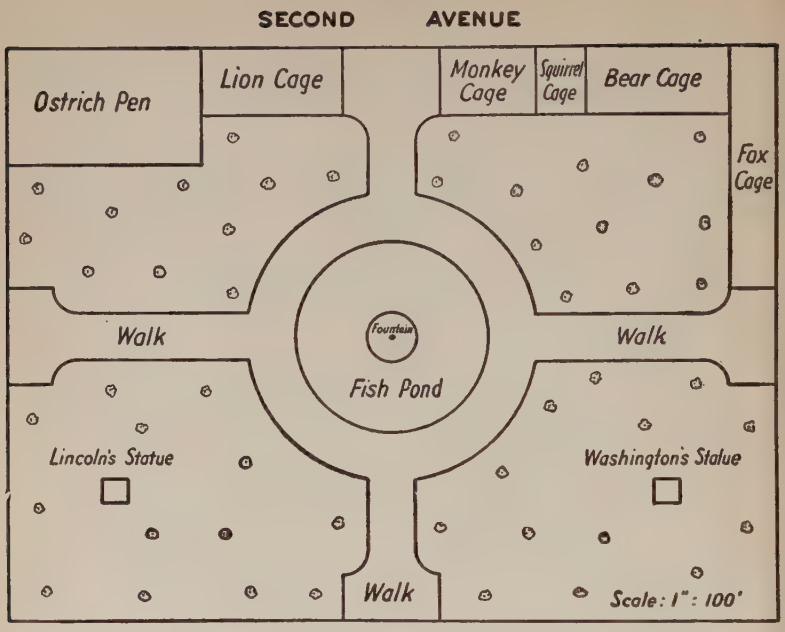
85. Making and Studying Scale Drawings. (WRITTEN WORK.) Every seventh-grade pupil should make at least two good scale drawings. Choose for your drawing any objects that you wish. You should of course select something that you can measure or the measurements of which you can obtain in some other way.

1. Below are some suggestions for scale drawings.

A Floor Plan of Your School.
A First- or Second-Floor Plan of Your Home.
The Map of Some Park.
The Map of a Football Field or of a Tennis Court.
Plan of the School Grounds.
A Bookcase.
The Buildings and Lots of a Farm.
The Front of Your School Building.

2. (OPTIONAL WORK.) The picture on the next page shows how a seventh-grade pupil made a scale drawing of one of the parks in his city. Measure it carefully, and on a piece of paper at least $8\frac{1}{2}'' \times 10''$ in size, make a scale drawing which is an exact copy of the picture on page 140 except that you should make the dimensions of your drawing **exactly twice as large** as those in the picture.

You should make the dimensions of all the objects twice the size shown in your book, but you need not bother to measure the small marks which represent trees and shrubs. These you can guess at and draw approximately as they are shown in your book. Study carefully the suggestions and questions below the picture before you begin your own drawing.



SUGGESTIONS AND QUESTIONS

1. Make your drawing in pencil. Draw the lines faintly at first and then, after you are sure they are all in the right places, go over the lines and make them heavy.

2. Do you think it would be wise to plan what the scale of your drawing will be? When your drawing is made, will the scale still be "1″ : 100′" (as shown in the drawing above)? Do you think the scale of your drawing should read: Scale, 1″ : 50′?

3. Would it be a good idea to find what the dimensions of the drawing of the park are in inches? What will these dimensions be in your enlarged drawing?

4. In the drawing on page 140, how wide is the walk around the fish pond? How wide will it be in your drawing?

5. Do you think it would be a good plan to write all the dimensions and distances as shown in the drawing in your book, and then multiply each by two before you begin to make your enlarged drawing?

Use your completed drawing in answering the following questions:

6. What are the actual dimensions in feet of the various animal cages?

7. What is the diameter in feet of the base of the fountain (the circle in the center of the fish pond)? What is the diameter in feet of the fish pond?

8. What are the actual dimensions of the bases of the two statues (the squares are the bases)?

ORAL PRACTICE ON USEFUL INFORMATION

86. A True-False Test. (Oral Work.) In this article are 12 statements. Some of these are true and some are false. Study them in order to be ready to tell which are true and which are false.

1. 3.5 feet equal 42 inches.

2. The check shows the correctness of the work.

$42178 - 38219 = 3959$ *Ans.*

$$\begin{array}{r} 42178 \\ 38219 \\ \hline 3959 \\ \hline 42178 \end{array}$$ *Check*

3. In adding fourths and sixths, twenty-fourths should be used as the smallest common denominator.

4. There are exactly six errors in all in the two tables of equivalents below.

FRACTION.....	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{1}{8}$
DECIMAL.....	.5	$.33\frac{1}{3}$	7.5	$.16\frac{1}{4}$	.125
PER CENT.....	50%	$33\frac{1}{3}\%$	75%	$16\frac{1}{4}\%$	125%

FRACTION.....	$\frac{2}{3}$	$\frac{5}{4}$	1	$\frac{1}{100}$	$\frac{1}{5}$	$\frac{7}{8}$
DECIMAL.....	$.66\frac{2}{3}$	1.25	1.00	.1	.2	87.5
PER CENT.....	$.66\frac{2}{3}\%$	125%	100%	1%	20%	87.5%

5. A good estimate of the quotient of $.3\overline{).90001}$ is 3.

6. A question in percentage answered by working the example $\frac{64}{748}$ would be "64 is what per cent of 748?"

7. There are 640 A. in a square mile.

8. A rating of 4 on a Self-Testing Drill is very good.

9. In multiplication the product is always larger than any one of its factors.

10. Five of these are not in the best form for answers:

$4\frac{1}{2}$ $\frac{7}{6}$ $8\frac{6}{4}$ $3\frac{1}{3}$ 1 ft. 14 in. 3 qt. 3 pt. $4\frac{6}{10}$

11. A question in percentage answered by working the example $.6 \times 367$ would be, "what is 6% of 367?"

12. A question in percentage answered by working the example $.07\overline{)143}$ would be, "what is the whole number, if 143 is 7% of it?"

SELF-TESTING DRILL NO. 11

87. Directions. Time allowed: **15 minutes.** Follow the directions given for Drills 1 to 5. Rate yourself. **Seventh-grade pupils should make their written work easy to read.**

1. *Subtract:*

1374.411
485.410

2. *Add:*

.522
9.666
.864
5.777
3.196

3. *Subtract:*

1181577
852986

4. $14 - 6\frac{1}{6} = ?$

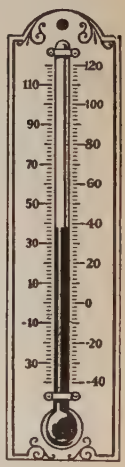
5. *Multiply:*

586.79
.45

6. $465\overline{)264120}$

7. $5.69\overline{)4500.79}$

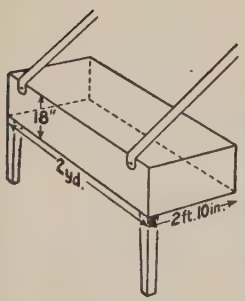
8. $3\frac{5}{12} \div 1\frac{3}{8} = ?$



9. What temperature does the thermometer show?

10. How much interest does $450 earn in 3 yr. 1 mo. @ 4%?

11. 1 mi. − 2482 ft. = ?



12. 144 is what per cent of 96?

13. Divide 9 sq. ft. 120 sq. in. by 8.

14. The picture at the left shows a locker in the captain's cabin on a "freighter." Find the volume of the locker in cubic feet.

Go on to the next page.

15. Mr. White's share of the profits of a sale was \$2763.75. This was $83\frac{1}{3}\%$ of the total profits. What were the total profits?

STANDARDS

Number correct	0–1	2	3	4	5	6	*	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Careful pupils will make sure to understand and correct every mistake they have made. The *Self-Help Index*, page 471, will guide you in needed review work.

THE FORMULA

88. Five Facts about the Formula. (Class Work.) The area of a figure like a rectangle may be stated in two ways:

(1) In words, thus: "The area of a rectangle equals the product of the altitude and the base."

(2) By a **formula,** thus: $A = a \times b$.

The formula may be regarded as an abbreviation of the full statement in words, or a sort of "shorthand." The formula $A = a \times b$ can be made even shorter by leaving out the times sign. It would then be written $A = ab$. ab means a **times** b. xy means x **times** y, etc.

In the formula $A = ab$, a and b are the **factors** of A.

Following are five facts about the formula which you should learn at once. If you master the formula, it will be one of your most helpful tools in arithmetic.

1. The letters in a formula represent numbers, and they can be replaced by such numbers. Thus, if a has the value 5 and b has the value 9, the A in the formula $A = ab$ must equal 5×9, or 45. We can replace the A by 45, and the

statement, $A = ab$, is still true, thus: $45 = 5 \times 9$. All of the following statements are also true:

$45 = a \times 9$ $\quad$ $45 = 5 \times b$ $\quad$ $A = 5 \times b$ $\quad$ $A = a \times 9$

2. Every formula contains an equality sign $(=)$. The equality or "equals" sign means that the number at the left of the equality sign is equal to the number at the right of this sign. (Remember that "number" as applied to a formula means either a number or a letter standing for a number.)

The formula $A = ab$ means that A equals the product of a and b. You should note that A does not equal a or b alone, but A does equal the product of its factors, a and b. $A = ab$ is called an **equation.** An **equation** is merely a statement that one number equals another. A is spoken of as the "left **side** of the equation" and ab as the "right **side** of the equation." You can think of the $=$ as a sign that separates every equation into two sides. It tells you that the result or number on one side is equal to the result or number on the other side.

3. The two sides of the equation may be exchanged without destroying their equality. The phrase "without destroying their equality" means without making the left and right sides of the formula unequal. That is, if $A = ab$, then $ab = A$. This of course seems foolishly simple, but keeping the fact in mind often helps you find a simpler method of working with a formula. Thus, $7 \times 8 = 56$ and $56 = 7 \times 8$; $9 \times a = 9a$ and $9a = 9 \times a$.

4. The order of the individual factors does not change the value of their product. Thus, $A = ab$ and $A = ba$. Similarly, in the formula for the area of a triangle, $A = \frac{1}{2} \times a \times b$, we can also say: $A = a \times \frac{1}{2} \times b$ and $A = a \times b \times \frac{1}{2}$.

Prove Fact 4 for the formula $A = \frac{1}{2} \times a \times b$ or $A = \frac{1}{2}ab$ by having A, a, and b equal various numbers. Thus, letting $A = 48$, $a = 8$, and $b = 12$:

(1) $48 = \frac{1}{2} \times 8 \times 12$ (2) $48 = 8 \times \frac{1}{2} \times 12$ (3) $48 = 8 \times 12 \times \frac{1}{2}$

5. Both sides of the equation in a formula may be multiplied or divided by the same number (or a letter standing for a number) **without destroying the equality.** Thus, if $A = ab$, then $2A = 2ab$, $3A = 3ab$, $4A = 4ab$, etc. You can easily see that this is true, for if $24 = 8 \times 3$, then $2 \times 24 = 2 \times (8 \times 3)$ and $3 \times 24 = 3 \times (8 \times 3)$.

Likewise, if $A = ab$, $A \div x = ab \div x$ and $A \div r = ab \div r$. Letting numbers take the place of letters in the formula makes this very clear. Thus, if $24 = 8 \times 3$, then $24 \div 3 = (8 \times 3) \div 3$, and also $24 \div 4 = (8 \times 3) \div 4$.

You have already learned that formulas may be shortened by leaving out the times sign. They may also be shortened by omitting the division sign. When this is done, we write the number to be divided above the divisor; thus, $a \div x$ is written $\frac{a}{x}$, $ab \div x$ is written $\frac{ab}{x}$. These would be read "a over x" and "ab over x."

If $24 = 8 \times 3$, then $\frac{24}{3} = \frac{8 \times 3}{3}$ (divided by the same number). Work $\frac{24}{3} = \frac{8 \times 3}{3}$ to see if dividing both sides of $24 = 8 \times 3$ by the same value changes the equality.

These facts about formulas seem absurdly simple and even foolish when you use numbers for letters, but they are very important facts when you are dealing with letters only.

At the top of the next page are the five facts to be remembered about formulas.

1. The letters in a formula represent numbers.
2. Every formula contains an equality (=) sign.
3. The two sides of the equation can be exchanged without destroying the equality.
4. The order of the factors (numbers and letters) does not change the value of their product.
5. Both sides of the equation may be multiplied or divided by the same number without destroying the equality.

89. Practice with the Formula. (WRITTEN WORK.) In doing the work called for below, read again the five facts about the formula, given above.

1. Write the formula $A=\frac{1}{2}\times a\times b$ in a still simpler form. For what is this the formula?

2. Solve for A in the formula $A=ab$, for each of the following sets of values. The first one is already done as a sample. "Solve for A" means to find what number A equals by replacing a and b by the numbers which they represent.

I. $a=10,\ b=6$

SOLUTION:
$A=a\times b=10\times 6=60$

II. $a=9,\ b=8$

III. $a=6,\ b=\frac{1}{2}$

IV. $a=.5,\ b=125$

3. (*a*) What is the left side of the equation in the formula $p=2l+2w$?

(*b*) What is the right side of the equation?

(*c*) Does this formula mean that p is equal to $2l$? To $2w$? To what is p equal?

(*d*) The formula $p=2l+2w$ has something to do with rectangles and parallelograms. See if you can find out what p, l, and w represent. State the formula in words.

90. Using the Facts about Formulas. (STUDY PERIOD.) The facts you have learned about equations in Article 88 are very useful in making new formulas from formulas that you already know. Thus, you know the formula for the area of a parallelogram, $A = ab$. This formula tells how to find the area. From it you can make a formula that will tell you how to find the base of a parallelogram. The work below explains how this is done.

We start with the formula for the area of a parallelogram (Step 1). Then we change the formula to read $ab = A$ (Step 2), because we want to find what b equals. We know that we can make this change (see fact 3, page 147).

Now, what we want on the left side of the equation is b alone, instead of ab. So we must remove the a. We can do this by dividing ab by a. But since we are dividing ab by a, we must also divide A by a (see fact 5, page 147). The formula then reads as shown in Step 3.

STEP 1.	$A = ab$
STEP 2.	$ab = A$
STEP 3.	$\frac{ab}{a} = \frac{A}{a}$
STEP 4.	$\frac{\not{a}b}{\not{a}} = \frac{A}{a}$
STEP 5.	$b = \frac{A}{a}$

We are now ready to cancel in order to remove the a from the left side. We do this (Step 4), and find our formula for b. The final formula for the base of a parallelogram is $b = \frac{A}{a}$ (Step 5).

This formula tells us that the base of a parallelogram equals the area divided by the altitude.

Now try to work out a new formula for yourself. This will help you to understand formulas more clearly. See if you can get the formula for the altitude of a parallelogram. Use the same steps shown in the explanation above. With what formula will you start for Step 1? How will

you change it in Step 2? What do you want to get rid of in Step 3? Can you do this by dividing? What do you cancel in Step 4? If you have worked out all your steps correctly, you find that the formula for the altitude of a parallelogram is $a=\frac{A}{b}$.

There is another way to find that $a=\frac{A}{b}$. You know that $A=ab$, or $A=a\times b$. a and b are factors and A is their product. Do you remember the rule for finding a factor (page 74)? It is this: "Product divided by one factor gives other factor." So $A\div b$ gives a, or $a=A\div b$. $A\div b$ is the same as $\frac{A}{b}$. Hence, $a=\frac{A}{b}$.

91. Practice in Finding New Formulas. (Class Work.) Study Articles 88 and 90 again if you find these problems difficult.

1. A little later on in the seventh grade you will study circles. Then you will use a certain formula $d=2r$. From this formula work out another one that will show what r equals.

2. The formula for the base of a rectangle is $b=\frac{A}{a}$. From this formula work out another one showing what A equals.

3. The formula for the volume of a rectangular solid is $V=lwh$. From this formula get a formula for l. For w. For h.

4. The area of a triangle is given by the formula $A=\frac{1}{2}ab$. Starting with this formula, find the formula for the base of a triangle. (Hint: What is the formula when both sides of $A=\frac{1}{2}ab$ are multiplied by 2? Is this a good way to start getting the formula you want?)

PROBLEM SCALE NO. VI

92. Problems Using Data in Tables. (WRITTEN WORK.) You can earn 70 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III.

Use the following table in working the problems below.

The suits which men and boys wear vary in quality and hence in cost. This table reports certain interesting facts about men's and boys' suits.

QUALITY OF SUITS SOLD BY 128 STORES IN 1927

QUALITY AS JUDGED BY COST IN DOLLARS	QUANTITY	PER CENT OF ALL SUITS
Under $20	16,516	10.1
$20 to $29.99	32,977	20.3
30 to 39.99	48,996	30.1
40 to 49.99	39,307	24.2
50 to 59.99	17,335	10.6
60 to 79.99	7,117	4.4
80 and over	427	.3

1. What per cent of all suits sold by the 128 stores is accounted for in this table? (6)

2. How many suits were sold in all? (5)

3. How many more suits costing under $40 were sold than suits costing $40 or over? (7)

4. If the 1928 sales of the 128 stores reported in the table above were 112% of the 1927 sales, how many suits did these stores sell in 1928? (8)

5. Find the total cost of the suits sold in these stores which cost from $20 to $29.99 apiece. The average cost of these suits was $25. (8)

Sole leather is the kind of leather used in making the soles or bottoms of shoes. Study the following table and use the facts in working the next problems.

CHIEF SOLE LEATHERS PRODUCED IN THE U. S. IN 1919

Kinds	Pieces of Leather Produced	Wholesale Cost Per Pound
Oak	10,086,228	49.1¢
Union	73,141,834	45.0¢
Hemlock	1,731,545	41.9¢

6. Find the total number of pieces of leather produced in 1919 in the United States. (4)

7. How much more Union leather was produced than Oak and Hemlock together? (6)

8. About what per cent of all sole leather produced in 1919 was Oak? (7)

9. What per cent of the cost of Hemlock per pound is the cost of Oak? (9)

10. Estimating the average weight of a piece of leather as 5 lb., figure the total cost of the sole leather listed in the table above. (10)

STANDARDS

Superior work:	21–70 points	Slow work:	5–7 points
Good work:	14–20 points	Inferior work:	0–4 points
Average work:	8–13 points		

MEASUREMENTS OF AREA

93. The Area of the Rectangle. (Class Work.) A rectangle is a surface bounded by two pairs of parallel lines, and its angles are all right angles. Its opposite sides are equal. The length and width (or **base** and **altitude**) are called the **dimensions** of the rectangle. We usually think of the longest side as the base of a rectangle and the shortest side as the altitude. However, any side may be the base.

Study this rectangle (Fig. 1).

The **perimeter** is the **sum** of all the **sides.**

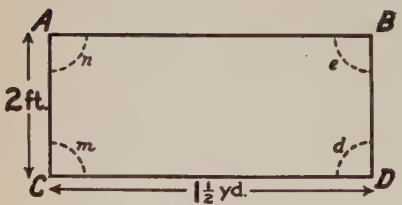


Fig. 1

The perimeter of the rectangle shown in Fig. 1 is 13 **ft.**

Lines AB and CD are parallel and equal.

Lines AC and BD are parallel and equal.

Angles m, n, d, and e are right angles.

Line AC is **perpendicular** to line CD. *Perpendicular* means that AC and CD form a right angle (square corner). Can you find three other pairs of lines perpendicular to each other? The two edges which form a corner of a rectangular desk make a right angle and are said to be perpendicular to each other. The walls of your schoolroom are perpendicular to the floor.

The altitude of a rectangle is the **perpendicular distance** from the base to the side opposite the base. In a rectangle the altitude is always the length of the side that meets or joins the base.

Common units of measure of areas are the square inch, the square foot, the square yard, the square rod, the acre, and the square mile. Area is **always** expressed in **square units.** Remember this when writing answers which are areas.

The rule for finding the area of a rectangle is usually written as the **formula** $A = ab$, where A = area in square units, a = the altitude, and b = the base. In using this formula, a and b must be expressed in the same linear units.

The area of the rectangle in Figure 1 would **not** be found by multiplying 2 by $1\frac{1}{2}$; these dimensions are **not** expressed in the same unit. Its area in sq. ft. would be found thus: $2 \times 4\frac{1}{2}$. (Why $4\frac{1}{2}$?) Its area in sq. yd. would be found thus: $1\frac{1}{2} \times \frac{2}{3}$. (Why $\frac{2}{3}$?)

Remember the formula for the area of a rectangle: $A = a \times b$ or $A = ab$.

QUESTIONS

1. What are the lengths of lines AB and BD in Fig. 1?
2. What lines are perpendicular to each other in Fig. 1?
3. What kind of angle is angle d?

Note: Angles are described in two ways.

(1) By use of a single letter, as for $\angle m$ or $\angle d$ in Fig. 1.

(2) By the use of the three letters which indicate the two lines forming the angle, as for $\angle ACD$ in Fig. 1. $\angle ACD$ can also be read or written $\angle DCA$. Can you name $\angle d$ in two other ways? The letter at the point (vertex) of the angle is always the middle one of the three letters.

4. What are six units of measure of **area?**
5. What is the area of Fig. 1?
6. Name two pairs of parallel lines in Fig. 1.

7. What are the areas of the six rectangles whose dimensions are given below? Use the formula $A = ab$. Express the dimensions in the same unit.

Put one dimension in the place of a and the other in the place of b. Remember that ab is the product of a and b.

(*a*) 4 in. by 9 in.
(*b*) 2 ft. 3 in. by 4 ft.
(*c*) 1 yd. 1 ft. by $4\frac{1}{2}$ yd.
(*d*) $5\frac{1}{2}$ yd. by 5 ft.
(*e*) 6 mi. by $\frac{2}{3}$ mi.
(*f*) 3 yd. 1 ft. by 4 yd. 2 ft.

94. The Area of the Parallelogram. (Class Work.) The parallelogram is a figure bounded by two pairs of parallel lines. Its opposite sides are equal. Its angles are not right angles (except when the figure is also a rectangle).

Study this parallelogram (*Fig. 2*).

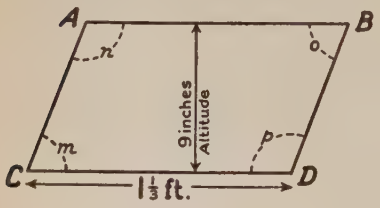


Fig. 2

Lines CA and BD are parallel lines and are equal.

Lines CD and AB are parallel lines and are equal.

$\angle m$ is an **acute** angle (less than a rt. $\angle$).

$\angle n$ is an **obtuse** angle (more than a rt. $\angle$).

The area of a parallelogram is found by multiplying its base by its altitude, expressed in the same linear units.

The altitude is the **shortest** distance between the base and the side opposite the base. An altitude can also be thought of as the length of a line drawn at right angles from the base to the side opposite, or **perpendicular** to them. Thus, if the base of Fig. 2 is CD, its altitude is 9 in.

The base of a parallelogram may be any one of its four sides.

The formula for finding the area of a parallelogram is $A = a \times b$ or $A = ab$, where A = area in square units, a and b equal an **altitude** and a **base** expressed in the same linear units. (a and b are **not** sides. They are a base and an altitude.)

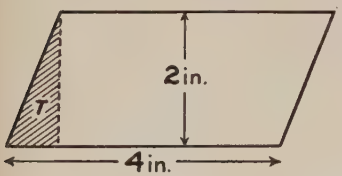
The area in square units of Fig. 2 is **not** given by the product of $9 \times 1\frac{1}{3}$, because these dimensions are **not** in the same linear units. The area of Fig. 2 is 9×16 or 144 **sq. in.**, or $\frac{3}{4} \times 1\frac{1}{3}$ or **1 sq. ft.**

Remember this formula for the area of a parallelogram: $A = a \times b$ or $A = ab$.

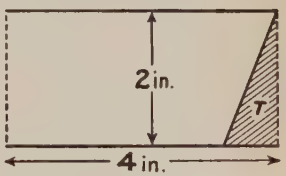
All rectangles are parallelograms; so the formulas for their areas are the same.

QUESTIONS

1. What two pairs of parallel lines are there in Fig. 2?
2. Angle o (Fig. 2) is what kind of angle?
3. Angle p (Fig. 2) is what kind of angle?



(A)



(B)

4. Are the areas of parallelogram A and rectangle B equal?

5. Do you see that by taking triangle T from the left of parallelogram A and placing it at the right of the parallelogram, we get rectangle B?

6. What are the areas of the six parallelograms whose dimensions are given below? Use the formula $A = ab$.

	Base	Altitude		Base	Altitude
(*a*)	6 in.	3 in.	(*d*)	2 yd.	$1\frac{1}{2}$ ft.
(*b*)	3 yd. 1 ft.	1 yd. 2 ft.	(*e*)	$\frac{4}{5}$ rd.	20 rd.
(*c*)	6 ft.	2 ft. 8 in.	(*f*)	3 mi.	0.5 mi.

95. The Area of the Right Triangle. (Class Work.) A right triangle (rt. △) is a figure bounded by three straight lines; one of its angles is a **right angle.**

Study the right triangle (Fig. 3) below.

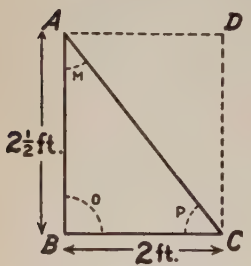


Fig. 3

Do you see that rt. $\triangle ABC$ is $\frac{1}{2}$ of the rectangle $ABCD$?

The area in square units of a rt. △ is equal to one-half the product of its base and its altitude, or one-half the product of its two **shorter** sides.

The formula is this: $A = \frac{1}{2} \times a \times b$, or $A = \frac{1}{2}ab$, where A = area in square units, and a and b equal the two shorter sides expressed in the same linear units. This formula may also be written as $A = \frac{ab}{2}$ $(ab \div 2)$.

Remember this formula for the area of a right triangle: $A = \frac{1}{2}ab$ or $A = \frac{ab}{2}$.

QUESTIONS

1. What kind of angle is $\angle o$?
2. What kind of angles are m and p?

3. In Fig. 3 what line is perpendicular to line BC?

4. What are the areas of the rt. $\triangle$ whose dimensions are given below? Use the formula $A = \frac{1}{2}ab$.

(*a*) $b = 4$ ft., $a = 2$ ft. (*d*) $a = 4.5$ ft., $b = 3.7$ ft.
(*b*) $b = 3$ ft., $a = 11$ in. (*e*) $a = 2$ yd., $b = 5$ ft.
(*c*) $b = 2\frac{1}{4}$ yd., $a = 1\frac{1}{2}$ yd. (*f*) $a = 4$ ft. 6 in., $b = 2\frac{3}{4}$ ft.

96. The Area of Any Triangle. (Class Work.) Most triangles are not right triangles. A triangle which does not have a right angle is called an **oblique** triangle. To find the area of an oblique triangle, we must know a base and an altitude. (**Oblique** is a new word to be remembered).

Study Figure 4.

Note that $\triangle ABC$ is **one-half** of the parallelogram $ABCD$.

The area of a parallelogram $= ab$ (altitude times base).

The area of a triangle which is one-half of a parallelogram is $\frac{1}{2}ab$.

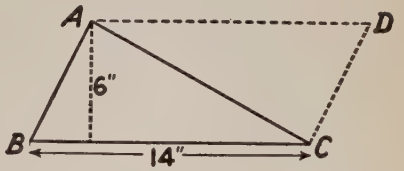


Fig. 4

Any triangle can be thought of as one-half of some parallelogram. Thus:

The formula for finding the area of a triangle is $A = \frac{1}{2}ab$ or $A = \frac{1}{2} \times a \times b$, or $A = \frac{ab}{2}$, where A is the area and a and b are the length of the altitude and base of the triangle.

Remember this formula for the area of any triangle: $A = \frac{1}{2}ab$ or $A = \frac{ab}{2}$

QUESTIONS

1. The area of Fig. 4 on page 157 is 42 sq. in. By using the formula, do you get 42 sq. in. for its area?

2. What are the areas of the triangles whose dimensions are given below? Use the formula $A = \frac{1}{2}ab$.

(*a*) $a = 3$ ft., $b = 2$ ft. (*c*) $a = 2.75$ ft., $b = 1.5$ ft.

(*b*) $a = 4\frac{1}{3}$ yd., $b = 7$ ft. (*d*) $a = 2$ ft. 6 in., $b = 1$ ft. 10 in.

3. Do the three △ below have equal bases?

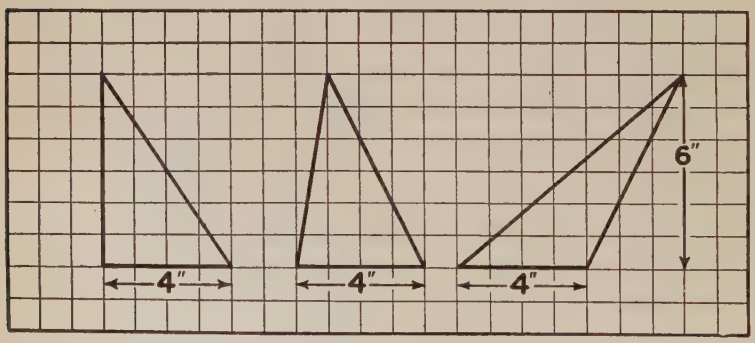


FIG. 5

4. Do the three △ in Figure 5 have equal altitudes?

5. Should they have equal areas?

6. Count the squares in their areas to see if all areas really are equal (as nearly as you can estimate).

7. What should be the area of each?

8. Find by using the formula, the areas of triangles of the dimensions given below:

(*a*) altitude, 16 in., base, 25 in
(*b*) altitude, $2\frac{1}{4}$ ft., base, 60 in.
(*c*) altitude, $3\frac{2}{5}$ ft., base, $6\frac{2}{3}$ yd.
(*d*) altitude, 8 rd., base, 20 rd.
(*e*) altitude, 4 yd. 2 ft., base, 6 yd. 1 ft.

97. Rapid Drill. (ORAL WORK.) This oral practice will help you in the Self-Testing Drills. In the division examples give the correct first quotient figure only.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	*Add:* 7, 2, 4, 1	145 ×6	$7 \div \frac{2}{3} =$ $7 \times$	$49\overline{)441}$	$\frac{1}{2}$ of 1% = what decimal?	154 −58
2.	179 −89	21 yd. = ...ft.	101 −78	*Add:* 1, 3, 9, 4	106 −68	$58\overline{)359}$
3.	*Explain:* Scale 1″:50′	93 −68	309 ×8	$36\overline{)255}$	427 ×7	*Add:* 8, 2, 2, 6
4.	958 −700	*Add:* 9, 6, 1, 7	*Reduce:* $\frac{17}{3}$	180 −97	$94\overline{)282}$	106 −51
5.	132 −39	$\frac{7}{12} = \frac{?}{36}$	*Add:* 7, 1, 5, 9	562 ×9	$42\overline{)361}$	.62 = ...%
6.	359 ×3	147 −58	$78\overline{)405}$	$6\frac{3}{4} = \frac{?}{4}$	*Add:* 9, 2, 2, 3	602 −25

+ − × ÷ SELF-TESTING DRILL NO. 12 + − × ÷

98. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Are you carefully correcting each mistake that you make in the Self-Testing Drills?**

1. Margaret said the quotient of 282,150 divided by 594 is 475. If this is correct, write "Correct" as your answer. If 475 is incorrect, give the correct answer.

2. *Subtract:*

112284.64
12364.67

3. *Add:*

6901
9688
800
8026
870
5517

4. $487.388 \div 83.6 = ?$

5. $4\frac{1}{8} \times 4\frac{2}{3} = ?$

6. *What is the total amount?*

2 qt. 1 pt.
3 qt.
5 qt. $1\frac{1}{2}$ pt.
4 qt. $\frac{1}{2}$ pt.
1 qt. 1 pt.

7. *Multiply:*

2379
9146

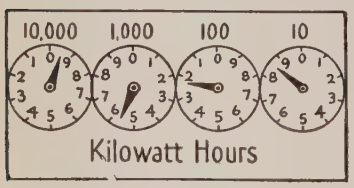
8.

$3\frac{1}{4}$
6
$2\frac{1}{3}$
$3\frac{3}{8}$

9. Is $\frac{4}{5}$ of \$645 about: \$2400, \$500, or \$120?

10. \$3.75 is 1% of what amount?

11. 7200 is 100% of a number. What per cent of the number is 936?

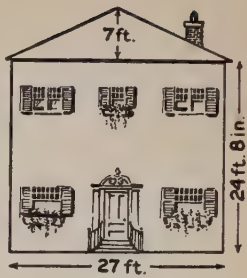


12. *Multiply:*

6 pk. 2 qt.
9

13. The electric meter shows how many kilowatt-hours?

14. Find the number of square feet in the area of the end of the house shown in the drawing.



15. I loaned some money to one of my friends at $5\frac{1}{2}\%$. I receive $454.30 interest each year. How much money did I loan?

STANDARDS

Number correct.....	0–1	2	3	*	4	5	6	7	8	9–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

THE CIRCLE

99. Facts about the Circle. (Class Work.) There are several facts about the circle which almost everyone knows. These are:

1. The bounding line of the circle is called the **circumference.** Line A to D to E to B to A in Fig. 1 is the circumference of the circle. A circumference is really a perimeter. Why?

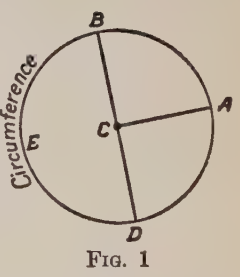


Fig. 1

2. A line from the center of the circle to the circumference is called a **radius.** Line CA is a radius. We can draw more than one **radius** in a circle. CB is a radius; so also is CD. The plural of radius is **radii.**

3. Line BD in Fig. 1 is called a **diameter.** It is a straight line touching the circumference at two points and passing through the center C. In the same circle one diameter equals two radii or one radius equals one-half a diameter.

Study the circle (Fig. 1) and estimate an answer to this question: "How many diameters equal the length of the circumference?" Any one can see that the length of the circumference equals at least one diameter. Would it equal about 2 or 3 diameters?

The relation between the diameter and the circumference of any circle can be thought of as the number of diameters it takes to make a circumference, or the number a diameter must be multiplied by to get the circumference.

If the diameter of Fig. 1 is 6 in., would the circumference be equal to about: 1×6 in., 2×6 in., or 3×6 in.?

Study the measurements below, and see if you can find the number by which to multiply the diameter of a circle in order to find the circumference. These measurements were made very accurately and are correct to the nearest one-hundredth. Carry answers to at least two decimal places.

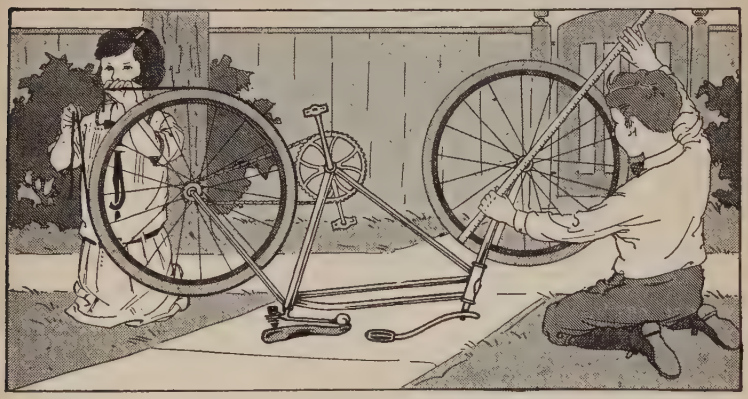
Will dividing the circumference by the diameter give you the number you are asked to find?

Object	Circumference	Diameter	Number to Multiply by
Ink bottle.....	5.50 in.	1.75 in.	3.14+
Round tin box....	6.13 in.	1.95 in.	
Fence Post.....	26.52 in.	8.44 in.	
Globe map.....	56.74 in.	18.06 in.	
Barrel-head.....	71.47 in.	22.75 in.	
Iron ring.....	17.81 in.	5.67 in.	

It has been found that if you multiply the diameter of a circle by 3.14 or by $3\frac{1}{7}$, you will get a rather accurate measure of the circumference. Also, if you divide the circumference by 3.14 or $3\frac{1}{7}$, you will get a good approximate measure of the diameter. The relation or, as is said

in arithmetic, **the ratio of a circumference of a circle to its diameter is about $3\frac{1}{7}$ or 3.14+** (a little more than 3.14)

➟It is good fun to find the ratio of the circumference to the diameter of a circle by making actual measurements.



Every seventh-grade pupil should measure the circumference and diameter of at least three circles and compute the ratios. Be careful to measure as exactly as possible.

The ratio $3\frac{1}{7}$ (approximately) or 3.14+ is called **pi**, represented by the Greek letter π. (Pronounced *pie.*) The value of pi is often given to four decimal places; thus, 3.1416.

Pi or $\pi = 3\frac{1}{7}$ or 3.1416 or 3.14+

100. Formulas to Use with the Circle. (Class Work.) We have learned in Article 99 that when we know the diameter we can find the circumference by multiplying the diameter by 3.14. We call this number pi and write it π. So we now have a formula or rule for finding the circumference: $c = \pi d$ (circumference = 3.14 × diameter).

There is also another formula for finding the circumference. It is $c=2\pi r$. Although the formula, $c=\pi d$ may seem somewhat simpler to you than the one, $c=2\pi r$, the latter will prove more useful in actual work with circles. You must understand, however, how $c=2\pi r$ is obtained from the formula $c=\pi d$. Study the following three statements to see how it is done:

(1) $d=2r$ (The diameter equals 2 times the radius.)

(2) $c=\pi\times d$ (The circumference = pi times the diameter.)

(3) $c=\pi\times 2r=2\pi r$ (The circumference equals 2 times pi times the radius.)

Notice that the d in formula 2 was replaced by $2r$ to obtain formula 3. We can do this because a diameter equals 2 times the radius.

Formula 3 can be simplified as follows:

$$c=2\pi r=2\times 3.14\times r=6.28r.$$

Remember these formulas for the circumference of a circle:

$c=\pi d$ or $3\frac{1}{7}d$ or $3.14d$ $\quad$ $c=2\pi r$ or $6\frac{2}{7}r$ or $6.28r$

Sometimes we may know that a circle has a circumference of 32 feet and want to find the diameter or the radius. So we must have two more formulas or rules.

In Article 90 you learned how to make formulas from formulas that you already know. Let us therefore start with the formula $c=\pi d$ and make a formula for finding the diameter of a circle. First look back and study the Steps on page 148.

It is d that we want to find; so we change the sides of the equation to make it read $\pi d=c$. Then, because we want to

find d alone, we must get rid of π. We do this by dividing both sides of the equation by π $\left(\frac{\pi d}{\pi} = \frac{c}{\pi}\right)$. Now we can get rid of π by canceling $\left(\frac{\cancel{\pi} d}{\cancel{\pi}} = \frac{c}{\pi}\right)$, and our formula for finding the diameter reads $d = \frac{c}{\pi}$.

Starting again with the formula $c = \pi d$, let us now find a formula for getting the radius. Let us replace d with $2r$, since we want a formula with r in it. The formula then reads $c = \pi 2r$, or $c = 2\pi r$. Now we follow the same steps we have used before to find new formulas. $2\pi r = c$ (step 2). $\frac{\cancel{2}\cancel{\pi} r}{\cancel{2}\cancel{\pi}} = \frac{c}{2\pi}$ (steps 3 and 4). $r = \frac{c}{2\pi}$ (step 5).

Remember these formulas for the diameter and the radius of a circle: $d = \frac{c}{\pi}$. $r = \frac{c}{2\pi}$.

101. Problems about Circles. (Written Work.) Here are some problems in finding circumferences and diameters.

1. Find the circumferences of the circles with the diameters given below. Give answers to two decimal places.

(*a*) 8 ft. (*b*) 6 yd. 2 ft. (*c*) $4\frac{1}{2}$ ft.

2. Find the circumferences (to two decimal places) of circles with the radii given below.

(*a*) 9 in. (*b*) 2 ft. 4 in. (*c*) $2\frac{1}{2}$ yd. (*d*) 5 ft. 6 in. (*e*) 13 ft.

3. Using π as 3.14, find to two decimal places the diameters of the circles whose circumferences are given below.

(*a*) 1884 ft. (*b*) 12.5 ft. (*c*) $\frac{1}{2}$ mi.

4. Using π as 3.14, find to two decimal places the radii of the circles whose circumferences are given below.

(*a*) 236.5 yd. (*b*) $9\frac{1}{4}$ in. (*c*) $11\frac{3}{4}$ ft.

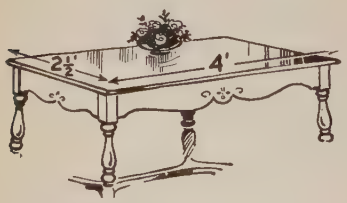
5. The distance around a circular pond is $1\frac{1}{2}$ mi. About what distance is it straight across?

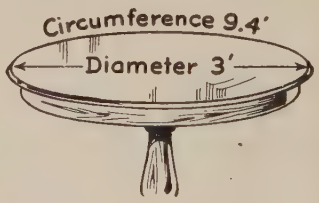
6. A circular running-track is to be constructed on a square piece of land. The land is 145 yards wide. About what length of track can be built?

7. A circular saw has a radius of $2\frac{3}{4}$ inches. On the circumference of the saw there are 4 teeth to the inch. How many teeth are there on the saw? (Disregard decimals in in the answer.)

8. A certain radio coil is $1\frac{1}{2}$ inches in diameter. There are to be 60 turns of wire on it. About how many feet of wire will be needed for the coil?

102. The Area of the Circle. (Class Work.) Two seventh-grade pupils wanted to compare the areas of the two dining-room tables shown below. They could find the area of the rectangular table, but they did not know how to compute the area of the





circular table. After they had finished studying this article, they computed the approximate area of the circular table without any trouble. You can do it also.

A simple way of discovering how the area of a circle is found is to cut the circle into twelve equal parts as shown in Fig. 1.

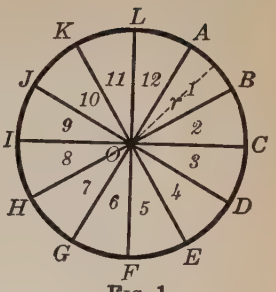


Fig. 1

If Fig. 1 were cut apart along each radius shown (at *AO*, *BO*, *CO*, etc.), it could be laid out in the form shown in Fig. 2 below.

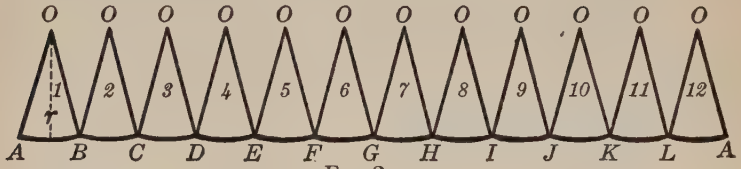


Fig. 2

Notice the following facts about Fig. 2:

Figure 2 is made up of 12 exactly equal parts. Each one of these parts is about equal to a triangle. Since they look so much like triangles, it seems reasonable to measure their areas by using the same sort of formula that is used to measure the areas of triangles: $A = \frac{1}{2} \times a \times b$. When you study geometry in high school, you will learn that this method measures the area of parts like these in Fig. 2 correctly.

The area of the part *AOB* is $\frac{1}{2}$ times *AB* times *r*. The total area of the circle is made up of 12 such parts, or $12 \times \frac{1}{2} \times AB \times r$. The formula is $A = 12 \times \frac{1}{2} \times AB \times r$, or we can write it in different order thus, $A = \frac{1}{2} \times r \times 12 \times AB$.

Notice that $12 \times AB$ is the same as the circumference of the circle, or $c = 12AB$. We can shorten the formula therefore to $A = \frac{1}{2} \times r \times c$, or $A = \frac{1}{2}rc$, or $A = \frac{rc}{2}$ or $rc \div 2$.

Remember this formula for the area of a circle:

$$A = \tfrac{1}{2}rc, \text{ or } A = \frac{rc}{2}$$

If you understand this article thus far, you can go a step further. In Art. 100 you learned that $c = 2\pi r$. (Circumference equals 2 times pi times radius.)

Now see if you can understand the following statements:

(1) $A = \dfrac{r \times c}{2}$ and (2) $c = 2\pi r$. Now in (1) we put $2\pi r$ in place of c and find that (3) $A = \dfrac{r \times 2\pi r}{2}$. The 2's cancel and $r \times r = r^2$, therefore (4) $A = \pi r^2$.

r^2 is read "r square" or "the square of r." The following facts will tell you what the "square" of a number or letter is:

(1) 6^2 (6 square) $= 6 \times 6 = 36$
(2) 10^2 (10 square) $= 10 \times 10 = 100$
(3) 5^2 (5 square) $= 5 \times 5 = 25$

In the same way, $r^2 = r \times r$, or r multiplied by itself. If $A = \pi r^2$, does A equal $\pi \times r \times r$?

Remember this formula for the area of a circle: $A = \pi r^2$

You have learned that there are two formulas for finding the area of a circle: $A = \dfrac{rc}{2}$ and $A = \pi r^2$. To use the first formula you must know both the circumference and the radius. Which one of the formulas could we use when it is impossible or inconvenient to find both the circumference and the radius?

103. Problems. (WRITTEN WORK.) Here are some problems in finding areas. Use $\pi = 3.14$.

1. Find the approximate areas of the circles whose radii or diameters are given below:

(*a*) Radius = 9 in.
(*b*) Radius = 3 ft.
(*c*) Radius = 14 yd.
(*d*) Diameter = 26 ft.
(*e*) Diameter = 3 mi.
(*f*) Diameter = 15 yd.
(*g*) Radius = 5′9″
(*h*) Diameter = 4 yd. 2 ft.

2. Which of the two dining-room tables shown on page 166 has the larger area?

3. The diameter of a drum-head is 2.5′. How many square inches of skin are in the surfaces of the two heads?

4. The circumference of a circular window is 58.12″. What is its radius? (Use $r = \frac{c}{2\pi}$) What is its area?

5. The diameter of a circular water tank is exactly 9 ft. The pressure of the water on the bottom of the tank is 1.3 lb. per sq. in. What is the total water pressure on the bottom of the tank?

6. In the center of a war memorial building is a circular room with a diameter of 50 ft. How much paint will be needed to cover the ceiling of this room if one gallon of paint will cover 400 sq. ft.?

7. The wind is blowing squarely against a circular sign-board, whose diameter is 32.75 ft. The pressure exerted on the sign-board by the wind is 25.5 lb. per sq. ft. Find the total wind pressure against the board.

SELF-TESTING DRILL NO. 13

104. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **You should make a special study of the examples in which you make mistakes.**

1. How much is 18 times $2\frac{2}{9}$?

2. Two-thirds plus three-fourths plus one-sixth equals what?

3. *Find the average:*

354
425
332
13
147
654
525

4. Find the difference of 8,931,072 and 3,246,919.

5. *Find the sum:*

6 yd. 2 ft.
2 yd.
1 ft.
1 yd. 1 ft.
5 yd. 2 ft.

6. *Subtract:*

1551211
898752.3

7. *Multiply:*

5068
9684

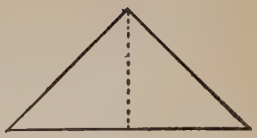
8. $21.7\overline{)776.86}$

9. 84 is what per cent of 1680?

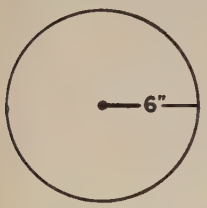
10. A 2% discount on a bill came to $19.00. How much was the bill before it was discounted?

11. $405 \div \frac{1}{5}$ is about: 2000, 810, 81.

12. The base of the triangle shown at the right is 15 yd. Its altitude is 12 yd. Find the area.



13. Express in miles and feet: 3×2346 ft.



14. Which of the following formulas would you use in finding the circumference of the circle?

(*a*) $c = \pi r$ (*b*) $c = \frac{1}{2}\pi r$ (*c*) $c = 2\pi r$

15. Fire destroyed 84% of the goods in a dry-goods store. The loss amounted to \$8198.24. What was the value of the goods before the fire?

STANDARDS

Number correct	0–1	2–3	4	5	*	6	7	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

The *Self-Help Index* beginning on page 471 will direct you to material that will help you.

PROBLEM SOLVING

105. Studying One's Own Mistakes in Problem Solving. (Class Work.) A seventh-grade class in arithmetic in a town in West Virginia voted to adopt the following six rules for use in all their problem solving during the year.

I. Read each problem at least twice carefully.
II. Get clearly in mind what is asked for.
III. Find out what facts are given in the problem.
IV. Decide what processes must be used, and in what order they should be carried out.
V. Before working the problem, estimate a reasonable answer.
VI. Check all work (at least where the numbers are large).

After these six rules were agreed upon, their teacher gave them some problems to work. Some of the pupils, however, were careless and made mistakes. Five of the problems which were worked incorrectly by certain pupils are given below. Study the problems and the solutions and find out which of the six rules were not used by the pupils who made the mistakes. The first one is discussed already as a sample of what you are to do.

Problem 1. A coal dealer sold 2400 lb. of coal to Mr. Spear for \$18.30. How much did Mr. Spear pay per ton for the coal?

As solved by Mary Roe

2400 lb. $= \frac{2400}{2000}$, or $\frac{6}{5}$ T.

$\$18.30 \times \frac{6}{5} = \$19.56$, what Mr. Spear paid for the coal.

Do you see that Mary Roe made a number of errors?

(1) She did not follow Rule II; so she did not notice that the problem called for what Mr. Spear paid **per ton** for the coal. Probably she would not have made this mistake if she had done as Rule I suggests.

(2) A reasonable answer (Rule V) would be less than \$18.30 since a ton is less than 2400 lb. If Mary had estimated a reasonable answer she might have discovered that she chose the wrong processes.

(3) But she not only chose the wrong processes; she made mistakes in her figuring. She did not check her work (Rule VI). If she had done so, she might have found the mistake she made in cancellation.

(4) The correct solution is as follows: 2400 lb. $= \frac{2400}{2000}$, or $\frac{6}{5}$ tons. If $\frac{6}{5}$ tons cost \$18.30, $\frac{1}{5}$ ton costs $\frac{1}{6}$ of \$18.30, or \$3.05. One ton costs $5 \times \$3.05$, or \$15.25.

Problem 2. My father and I went on an auto trip last summer. When we had gone 108 miles, my father said, "We have gone 75% of the total distance. Can you tell me what the total distance will be?"

As solved by John Lee

The Work	Check
108	John
.75	failed
500	to
706	check.
75.60 mi. *Ans.*	

As solved by Robert Moss

The Work	Check
108	.75
.75	108
540	600
756	750
81.00 mi. *Ans.*	81.00

Robert's check showed that he had made no mistake in his figuring. Does this prove that he had solved the problem correctly? (Is 81 mi. a reasonable answer? Would not the total distance be more than 75% of the total distance?)

Problem 3. The Boy Scouts went on an all-day hike. They left camp at 8:00 in the morning and walked until 3:30 in the afternoon, stopping 90 min. for lunch and to rest. They walked $12\frac{1}{2}$ mi. in all. What was their average rate of walking per hour, deducting the lunch period?

As solved by Ralph Birch

8 o'clock till 12 o'clock is 4 hours.

12 o'clock till 3.30 is $3\frac{1}{2}$ hours.

4 hr. and $3\frac{1}{2}$ hr. are $7\frac{1}{2}$ hours.

$1.66\frac{2}{3}$ mi. per hr. *Ans.*

$$
\begin{array}{r}
7.5\overline{)12.5} \\
75 \\
\hline
500 \\
450 \\
\hline
500 \\
450 \\
\hline
50
\end{array}
$$

Check

$$
\begin{array}{r}
1.66 \\
7.5 \\
\hline
830 \\
1162 \\
\hline
12.450 \\
+.05 \\
\hline
12.500
\end{array}
$$

Problem 4. How much is left from a man's wages for 4 weeks of 24 working days if he earns \$5.50 a day, and he has to pay \$12.50 a week for board and room?

As solved by Alice Norton

The Work				*Check*
\$5.50	\$12.50	\$132.00		24
24	4	\$ 50.00		5.50
2200	\$50.00	\$ 72.00	*Ans.*	1200
1100				120
\$132.00				132.00

Problem 5. In 1000 lb. of roasted peanuts, 326 lb. is waste and the rest is food. What per cent of roasted peanuts is food?

As solved by Joe York: $\frac{326}{1000} = .326$ or 3.26% *Ans.*

PROBLEM SCALE NO. VII

106. Problems Which Contain "Hard Spots." (WRITTEN WORK.) You can earn 60 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scale I, Art. 20. Here is a good chance to use the rules for problem solving about which you studied in Art. 105.

1. A radio set and cabinet sell for \$275. A 12% discount is allowed for cash payment. How much would be saved by paying cash for the set and cabinet? (5)

2. Four boys who went on a Saturday hike carried packs on their backs. They stopped at a country store and while resting, weighed their packs. Timothy's pack weighed $12\frac{1}{2}$ lb., Sidney's weighed 17 lb. 8 oz., Walter's,

14 lb. 9 oz., and Jerry's, 16 lb. 4 oz. How much weight had the four boys been carrying on their backs? (5)

3. The science class wanted to find how many times the diameter is equal to the circumference of a round copper plate. They measured the diameter and found that it was exactly 8 in. They measured the circumference and found it was 25.13 in. The circumference of the copper plate is how many times as large as the diameter? (Carry answer to three decimal places.) (6)

4. What was the cost of the clothing a soldier took with him overseas during the World War? Use the following table of standard equipment: (6)

1 raincoat—$5.33
2 suits of underwear @ $3.15 each
1 winter cap—$1.25
2 pr. puttees @ $2.95
1 wool coat—$9.79
1 overseas cap—$1.00
1 belt—$.30
1 set identification tags—$.04
2 pr. field shoes @ $7.50 per pair
5 pr. breeches @ $2.55 each
5 pr. socks @ $.60 per pair
1 overcoat—$12.17
1 pr. wool gloves—$.80
2 flannel shirts @ $4.08 each
1 set collar badges—$.37

5. During one summer William sold Magic Silver Polish. He received as his pay $37\frac{1}{2}\%$ of his total sales. His accounts show that he sold 87 large cans at $1.75 each, 154 medium-sized cans at 98¢, and 247 small cans at 49¢. Find his earnings on these sales. (7)

6. The science class found that the circumference of a circular rod was 5.884 in. and the diameter was 1.875 in. How many times as large as the diameter is the circumference of the circular rod? (Carry answer to three decimal places.) (7)

7. The price of lots in Woodlawn will be increased by what per cent on May 1, according to the signboard at the right? (7)

8. Use the formula $c = d \times 3.14$, where c equals the circumference and d equals the diameter, to figure the circumference of a circle whose diameter is 5.675 in. (8)

9. The class measured the diameter and the circumference of an iron ring. They learned that the diameter was 5.675 in., and the circumference was 17.828 in. What was the relation between the diameter and the circumference of the ring? (Carry answer to 3 decimal places.) (9)

STANDARDS

Superior work:	18–60 points	Slow work:	6–7 points
Good work:	13–17 points	Inferior work:	0–5 points
Average work:	8–12 points		

NOTE TO PUPILS

Perhaps you have wondered why some problems are given more points than others. For example, Problem 1 counts 5 points, but Problem 9 counts 9 points. The points given to problems depend on the number of pupils who worked them correctly when they were being tried out before being printed in this book. Problem 1 does not get as many points as Problem 9 because more pupils worked it correctly. This shows that it was easier than Problem 9. In other words, the harder the problem the more points you earn for a correct solution. Read Problems 1 and 9 again and see if you can tell why Problem 9 is harder for most pupils than Problem 1.

There is little reason why you cannot make ratings of "good" and even "superior" this year on the Problem Scales if you will make the study of "how to solve problems" an object of serious work. What articles in this book give good hints on how to solve problems? Should not these articles be carefully studied?

107. Rapid Drill. (ORAL WORK.) A few minutes of study should make it possible for you to give all these answers correctly. In quotients give first figure only.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$\begin{array}{r} 65 \\ -47 \\ \hline \end{array}$	$\frac{3}{4}+\frac{3}{10}$	$69\overline{)185}$	$12+6+9$	$\frac{7}{8}-\frac{1}{2}$	$14+5+8$
2.	$A=\pi r^2$ is formula for what?	$\begin{array}{r} 903 \\ \times 5 \\ \hline \end{array}$	Does $19.8\times.01=1.98$?	1 ft. 2 in. $\times 6$	$69\overline{)471}$	$\begin{array}{r} 94 \\ -36 \\ \hline \end{array}$
3.	\$18.25 = 10% of \$.....	$14\overline{)5600}$	$\begin{array}{r} 31 \\ -16 \\ \hline \end{array}$	$\frac{3}{4}\div\frac{1}{5}$	$\begin{array}{r} 162 \\ -58 \\ \hline \end{array}$	$8.888\div .8=?$
4.	*Reduce:* 9 gal. 9 qt.	$6+8+7$	$27+3+7$	$\frac{4}{15}\times\frac{12}{8}$	$\frac{1}{6}$ = what decimal?	$69\overline{)554}$
5.	$\begin{array}{r} 49 \\ \times 8 \\ \hline \end{array}$	π = what number?	1.1 = what per cent?	Explain "discount."	$27+7+6$	25 = what % of 125?
6.	$24\overline{)68}$	$\begin{array}{r} 76 \\ \times 7 \\ \hline \end{array}$	Explain perpendicular	$29+5+8$	$\frac{2}{3}N=66$; $N=?$	$\begin{array}{r} 68 \\ \times 7 \\ \hline \end{array}$

SELF-TESTING DRILL NO. 14

108. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Is the line on your Progress Chart steadily going up?**

1. *Subtract:*
973.3175
255.0647

2. *Add:* $4\frac{3}{4}$, $\frac{2}{5}$, 6, and $2\frac{9}{10}$.

3. *Subtract:*
1512562
637938

4. $\frac{4}{15}$ of $4\frac{1}{6}$ = ?

5. $69.3\overline{)18.5724}$

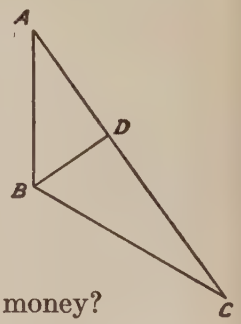
6. Would a 3% discount on a sale of \$1900 be \$570, \$57 or \$5.70?

7. Multiply 61.72 by 8307.

8. What will it cost to borrow \$4500 at 4% interest for 2 yr. 8 mo.?

9. *Find the sum:*
2 gal. 3 qt.
4 gal. 1 qt.
6 gal.
3 gal. 2 qt.
8 gal. 3 qt.

10. *Find the average:*
5897
563
878
8795
7866
4298



11. \$67.24 is 10% of what amount of money?

12. 25.2 is what per cent of 84?

13. *Multiply:*
2 yd. 1 ft. 7 in.
8

14. What is the area of the triangle above? (AD = 2 yd. 1 ft.; DC = 3 yd. 2 ft.; BD = 1 yd. 2 ft.)

15. Find the cash on hand for Oct. 1, 1927, in the cash account below.

1927			Receipts		Expenditures	
Sept.	1	Cash from August, 1927	815	85		
	2	Received from H. A. Forbes on Acc't	113	15		
	7	Check to Auto Supply Co.			24	38
	8	Life Insurance Premium			98	68
	20	Dividends on Telephone stock	76	16		
	24	3d payment on lot			500	00
	29	Salary for September	275			
Oct.	1	Cash on hand				

STANDARDS

Number correct	0	1–2	3	4	*	5	6	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

THE USE OF COMMISSION

109. Commission as a Form of Pay. (Silent Reading.) Many men and women work for wages or salaries. They are paid a stated sum each week or month for their services. Another form of pay is **commission.** Thus, a salesman is given a certain per cent of his sales as pay. If a salesman were working on a 10% commission and sold $100 worth of goods, he would receive 10% of $100, or $10, as his commission (pay), for selling the goods.

You will understand the meaning of commission by reading the conversations given on the next page.

I. JOHN: Does your father work on a salary or commission?

GEORGE: He works on a commission. He likes this, because the harder he works the more he earns.

II. MRS. DEWEY (to real estate agent): Mr. Jones, I want you to sell my house for me.

MR. KING (the agent): I shall be glad to do this for you, Mrs. Dewey, and I shall get the best possible price for you.

MRS. DEWEY: How much will you charge for your services?

MR. KING: All realtors (real-estate agents) in this city work on a commission. We get 5% or $50 for the first $1000 and $2\frac{1}{2}\%$ for all amounts above that. Suppose I sell your house for $5500. I'll charge 5% of $1000 and $2\frac{1}{2}\%$ of $4500.

MRS. DEWEY: I've always wondered what "agent's commission" meant and I am glad you have explained it to me.

III. MARY: My father is a doctor. What does your father do?

ESTHER: My father is a commission merchant.

MARY: What do you mean by commission merchant?

ESTHER: Often people who manufacture things do not sell them directly to stores. In different parts of the country they have men, called commission merchants, who attend to these sales for them. The commission merchant is not paid a salary, but gets a percentage of all sales.

My father explained it to me like this: The Widen and Cass Factory asked him to sell four carloads of furniture at a commission of 7%. He said that he would get 7% of the amount of money the four carloads of furniture were sold for as commission for his services as salesman.

➡Commission is a form of pay for helping someone else sell goods. A per cent of the money paid for the goods goes to the one who helped sell as his **pay for his services.**

The per cent paid as a commission varies greatly. Some kinds of goods are sold on as low a commission as 2%, while others are sold for as high as a 50% commission. Five

automobile dealers in one town were asked what commission they received for selling new cars. These commissions were 18%, 19%, 21%, 24%, and 30%. The commission varies somewhat according to the price of the car, its popularity, and the ease or difficulty of making a sale.

Sometimes an agent gets a regular salary in addition to his commission. But in such cases the commission is smaller, of course, than if the agent were getting only a commission.

110. Finding Commissions. (WRITTEN WORK.) The problems below are about commission. You will see that they are solved by using what you already know about percentage. Before working the problems, study the three samples.

SAMPLE I. During the summer vacation Henry sold a patented ironing pad. He was paid a 40% commission on his sales. One week he sold $26.00 worth of pads. What was his commission?

THINK: I must find 40% of $26.00

THE WORK:

$$\begin{array}{r} \$26 \\ .40 \\ \hline \$10.40 \end{array}$$

$10.40 amt. earned (commission) *Ans.*

SAMPLE II. Mr. Smith received $350 for selling an automobile priced at $1750. His commission was reckoned at what per cent?

THINK: $350 is what per cent of $1750?

THE WORK: $\frac{350}{1750}$ or $1750\overline{)350.00}$ (.20)

.20 = 20% commission *Ans.*

Notice that finding commission is merely finding the per cent in a percentage problem.

SAMPLE III. An agent working on a 4% commission gets a commission of $34 on a sale. What was the amount of the sale?

THINK: I must find the total amount (100%) when I know that $34 = 4% of it.

THE WORK:

$$.04\overline{)34.00} \quad \$850.$$

$850. amt. of sale. *Ans.*

Notice that this is merely finding 100% of a number from a given per cent of it. It is what we have learned as "the third use" of percentage.

Every problem about commission can be worked by one of the three uses of percentage. Before starting to work any problem about commission, decide which one of the three uses of percentage is the correct one for solving that problem.

PROBLEMS

All seventh-grade pupils should be able to work the problems below.

1. Mrs. Jones gave an order amounting to $24.00 to an agent selling household brushes. She asked him how he was paid for his work. He told her that he "worked on a $33\frac{1}{3}$% commission." How much did he make on his sale to Mrs. Jones?

2. Frank sold small fire-extinguishers to people living in cottages at a lake resort. One summer he sold $1250 worth of extinguishers. His commission was 45%. How much did Frank earn that summer?

3. A realtor sold a house. The facts of the sale are these: *Sales price*, $17,200; *commission*, 5% on first thousand, $2\frac{1}{2}$% on the remainder. What was the realtor's commission on this sale?

4. The clerks in the Washburn Department Store get a regular salary and a commission of 2% on all sales made above a certain amount. Miss Long gets a salary of $15 a week and a 2% commission **on all sales above $400 a week.** One week Miss Long sold $650 worth of goods. What were her **total** earnings that week?

5. Here is a letter from a commission merchant to a farmer. Find what should be on each blank.

M. K. THOMAS
Commission Merchant
Maryville, Vt.

June 6, 1928

Dear Mr. Jones:

Below is a statement of sales of the 40 crates of asparagus which I sold for you last week.

Sold 40 crates @ 3.25 ....?....
My commission @ 8% ....?....

Enclosed find my check for ..... due you on this sale.

Yours truly,
M. K. Thomas

6. On the next page is a letter from the Boyles Auction Co. to Mrs. L. T. Wicks for whom the company sold some goods at auction. Find what should be on each blank.

Dear Mrs. Wicks:

Enclosed find our check for.....due you on the following sales:

Sundry garden tools	$18.25
Sundry kitchen goods	9.50
4 rockers	51.00
One bedroom set	26.50
Dining table and chairs	17.75
Total	.....
Our commission @ 2%	.....
Amt. Due	.....

7. Find what should be on each blank. (Optional Work.)

	Amount of Sales	*Commission in Money*	*Commission in Per Cent*
(*a*)	$764.		30%
(*b*)	$87.	$34.80	
(*c*)		$15.60	20%
(*d*)	$1360		22%
(*e*)	$4500		5% on first 1000 $2\frac{1}{2}$% on the rest
(*f*)	$736.	$110.40	
(*g*)		$34.50	6%
(*h*)	$45.		$12\frac{1}{2}$%

CRITICAL STUDY OF SOME COMPUTATIONS

111. Finding and Correcting Errors. (WRITTEN WORK.) One of the most important uses of arithmetic is that of finding and correcting errors. The wise pupil always looks over his written work to find and correct all errors.

Men and women "hunt for errors" in bills and statements before paying them. The way to reduce your own errors is to train yourself to be self-critical.

In the work in this article, **there are some errors.** Find and correct them.

EXERCISE

1.

$$\begin{array}{r} 648 \\ 39 \\ 456 \\ 480 \\ 10 \\ 293 \\ \hline 1942 \end{array}$$

2. *Subtract:*

$$\begin{array}{r} 80736 \\ 31989 \\ \hline 59745 \end{array}$$

3. *Multiply:*

$$\begin{array}{r} 905 \\ 409 \\ \hline 8145 \\ 3820 \\ \hline 46345 \end{array}$$

4.

$\frac{2}{3}+\frac{5}{6}=1\frac{3}{6}$

5.

$$89\overline{)72184}\quad 810\tfrac{4}{89}$$

$$\begin{array}{r} 712 \\ \hline 98 \\ 98 \\ \hline 04 \end{array}$$

6. *Subtract:*

$$\begin{array}{r} 6 \\ 3\frac{2}{5} \\ \hline 3\frac{2}{5} \end{array}$$

7.

$\frac{4}{5}\times\frac{4}{9}=\frac{8}{45}$

8.

$\frac{2}{3}\div 6=\frac{2}{18}$

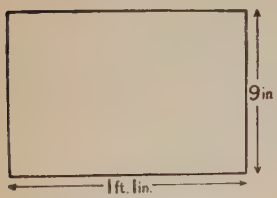
9.

14% of 84 = 1176

10. 114 is what % of 310? 1.5% *Ans.*

11. If \$12.25 = 35% of N, N = what? \$350 *Ans.*

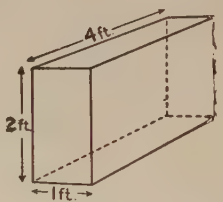
12. *Find the area of the rectangle below:*



99 inches *Ans.*

13. π or pi equals about 3.24

14. *Find the volume of the solid below:*



8 sq. ft. *Ans.*

SELF-TESTING DRILL NO. 15

112. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Needless mistakes are sometimes made because of careless writing of figures.**

1. *Subtract:*

17863183
9376429

2. *Subtract:*

119.6228
56.7684

3. $4\frac{1}{3}+2\frac{7}{12}+1\frac{5}{6}=?$

4.

7389
644
9588
977
898
8656

5. $8\frac{1}{6}\times 9=?$

6. *Multiply:*

3579
709

7. *Divide:*

55603.2 by 76.8

8.

2 T. 1500 lb.
5 T. 1225 lb.
6 T. 1000 lb.
3 T. 500 lb.
2 T. 1600 lb.

9. *Multiply:*

2 qt. 1 pt.
8

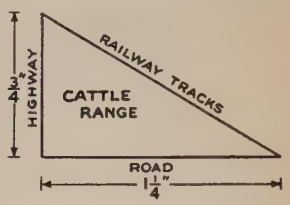
10. If \$65.24 equals 7% of a number, 100% of the number equals what?

11. Is $358.25 \div 356$ about: 100, 10, or 1?

12. Which of these formulas is the correct one for finding the area of a triangle? (*a*) $\frac{1}{2}A = ab$; (*b*) $A = ab$; (*c*) $A = \frac{1}{2}ab$.

13. If a 15% commission comes to \$993.75, what is the amount on which the commission is paid?

14. The cattle range shown at the right is drawn to a scale of 1 in. : 4 mi. Find the area of the range.



15. If 100% of $N = 51$, then $34 =$ what % of N?

STANDARDS

Number correct	0–1	2–3	4	5	*	6	7	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

SUMMARY

113. General Review. (Class Work.) Study each item below in order to be ready to give the correct answer to each statement. The article numbers following certain of the statements will tell you where you can look for review if you are not certain of the answers.

➡ ALL seventh-grade pupils should be able to answer correctly at least 25 of the 27 items after having had a few minutes for study.

1. We can measure not only distances but also ..... and ..... (Art. 78.)

2. Common units of measure of distance are ....., ..., etc. (Art. 79.)

3. The reason why we need several units of measure of length is ..... (Art. 79.)

4. A distance stated in inches can be changed to feet by ..... the number of inches by .....

5. A distance given in feet can be changed to yards by ..... the number of feet by .....

6. We sometimes measure small distances inaccurately because ..... (Art. 82.)

7. Good cautions to use in measuring are ..... (Art. 82.)

8. Scale drawing means..... (Art. 84.)

9. A scale $1'' : 10'$ means that .....

10. If two or more numbers are multiplied, the numbers are said to be ..... of the product. (Art. 46.)

11. An abbreviation of a full statement of a rule, such as $A = ab$ or $A = \pi r^2$, is called a ..... (Art. 88.)

12. What are the five important facts about formulas? (Art. 88.)

13. Go to the board and illustrate the meaning of the words: *rectangle, dimensions, perimeter, perpendicular lines, parallel lines, a parallelogram, a right angle.* (Arts. 93-95.)

14. What are common units of measure of area? (Art. 93.)

15. What is the formula for finding the area of a rectangle? (Art. 93.)

16. Explain the formula for finding the area of a parallelogram. $A = ab$. (Art. 94.)

17. Write the formula for finding the area of any kind of triangle. (Art. 96.)

18. Be ready to draw a circle and explain these words: *circumference, radius, diameter.* (Art. 99.)

19. The letter π or pi stands for approximately what number? (Art. 99.)

20. Explain this formula. $c = \pi d$. (Art. 100.)

21. Explain the formulas $A = \frac{rc}{2}$, $A = \pi r^2$. (Art. 102.)

22. Tell clearly just how you would use the 10 formulas listed below in finding important facts about these figures. Use the blackboard to illustrate your explanation if you wish to do so.

FIG. A FIG. B FIG. C FIG. D FIG. E

(1) $A = \frac{rc}{2}$

(2) $A = \frac{1}{2}ab$

(3) $A = ab$

(4) $p = l + l + w + w$

(5) $c = \pi d$

(6) $r = \frac{c}{2\pi}$

(7) $c = 2\pi r$

(8) $p = 2l + 2w$

(9) $A = \pi r^2$

(10) $\frac{1}{2}c = \pi r$

23. I sold a brush at \$5.95, a floor mop at \$6.55, and a push broom at \$6.75, and received a commission of 45% on the sales. My commission on these sales was \$.....

24. A real estate salesman received a commission of \$50 for selling a city lot for \$800. What was the rate (%) of his commission?

25. A salesman received a salary of \$25 a week plus a 2% commission on the week's sales. What was the agent's total pay for a week in which the sales were \$2450?

26. Commission, \$31.20; rate of commission, 20%; amount of sales, ...

27. Amount of sales, \$368; commission, \$55.20; rate of commission, ...%.

PART TWO

CHAPTER V

TESTING ARITHMETIC ABILITY

REVIEW TESTS

114. Testing Your Ability in Arithmetic. (Class Work.) You have now completed about half of your seventh-grade work in arithmetic. It may be well, therefore, to stop new work long enough to take an inventory of your abilities in arithmetic. In this way you can locate any "weak spots" you may have in your arithmetic ability, and you can strengthen these "weak spots" by using the *Self-Help Lessons* for study and practice (pages 471-478).

Arts. 115 to 124 give a number of tests covering past work. They are like the ones you took when you began seventh-grade work. Turn back now and read Article 2 on pages 2 and 3, so that you may understand more fully how to use these tests. It would be wise for you to make a chart on which to keep your record of your test scores (see page 2).

115. A Location Test in Integers. (Written Work.) Your scores on the next test will "locate" for you any serious weaknesses in your ability to add, subtract, multiply, and divide integers (whole numbers).

If you reach the standards given in the table which follows the test, you need not spend time in reviewing integers. If you fall below the standard in any one of these

four processes (addition, subtraction, multiplication, and division), you should do all of the work in the articles suggested for that process in the right-hand column of the table on page 193. These articles are on pages 388-409.

Notice that you find your standing on each of the four processes **separately.** Thus, your answers to Examples 1, 7, 9, and 13 will determine your score in addition. If you missed more than one of these examples, you need to review addition of integers now.

Directions for the test: You will be allowed **20 minutes** to work the 16 examples. Be careful to number your papers correctly. Make your figures easy to read. Wait for your teacher to say "Go!" before beginning to work.

After your teacher says "Stop!", she will read the answers. As your teacher reads the correct answers, mark each correct example with a "C." You can then count up your score (the number correct). The standards on page 193 will show you whether you need review.

LOCATION TEST NO. I

1.

7678
7792
345
4082
5167

2.

Multiply:
7893
97

3.

Subtract:
12734
9358

4.

$38\overline{)34428}$

5.

Subtract:
82825
27467

6.

Multiply:
846
806

7.

932
463
9589
4246
2390

8.

$39\overline{)277186}$

9.	10.	11.	12.
388	*Subtract:*	*Multiply:*	*Subtract:*
9259	84790	5046	144916
598	54803	597	59746
9680			
2477			

13.	14.	15.	16.
470	706)3037212	*Subtract:*	*Multiply:*
378		169130	5791
466		94286	436
418			
559			

STANDARDS FOR LOCATION TEST NO. I
GUIDE TO REVIEW WORK

PROCESS	EXAMPLES	POSSIBLE SCORE	STANDARDS (Number you should get right)	ARTICLES FOR STUDY (The articles listed below are good ones for study and practice if you fall below standard in any process.)
ADDITION	1, 7, 9, 13	4	3 or 4	Arts. 213-217
SUBTRACTION	3, 5, 10, 12, 15	5	4 or 5	Arts. 218-222
MULTIPLICATION	2, 6, 11, 16	4	3 or 4	Arts. 223-228
DIVISION	4, 8, 14	3	2 or 3	Arts. 229-235

116. A Location Test in Fractions and Mixed Numbers. (WRITTEN WORK.) On the next page you will find Location Test No. II. It is like the one you have just taken except that it will "locate" your ability in fractions and

mixed numbers. Most people think it is harder to work with fractions than with whole numbers; but if you understand **how** to do the work, it is not difficult.

Directions for the test: You will be allowed **20 minutes** to work the 16 examples. Number your papers correctly and work the examples in order. Wait for your teacher to say "Go!" before starting to work.

After your teacher says "Stop!", she will read the answers. Count up your score (the number of examples you worked correctly); then look in the table of standards and see what processes, if any, you need to review.

LOCATION TEST NO. II

1.

$7\frac{5}{12}$
4
$6\frac{5}{6}$
$12\frac{1}{2}$

2.

Subtract:
$17\frac{1}{5}$
$7\frac{3}{5}$

3.

$\frac{3}{4} \times \frac{5}{6}$

4.

$\frac{2}{3} \div \frac{1}{5}$

5.

$8 \div 2\frac{2}{5}$

6.

$\frac{2}{3}$ of $7\frac{1}{2}$

7.

$4\frac{1}{4}$
$8\frac{5}{6}$
$\frac{1}{2}$
$10\frac{2}{3}$
2

8.

$14 - 6\frac{1}{3}$

9.

Subtract:
$15\frac{1}{4}$
$8\frac{2}{3}$

10.

$25\frac{3}{4}$
15
$7\frac{1}{6}$
$4\frac{1}{8}$

11.

$5\frac{1}{4} \div 1\frac{7}{9}$

12.

$2\frac{2}{3} \times 2\frac{5}{6}$

13.

$1\frac{9}{16} \div 1\frac{1}{4}$

14.

Subtract:
$13\frac{1}{4}$
$6\frac{5}{6}$

15.

$11\frac{1}{3}$
$4\frac{2}{5}$
42
9
$13\frac{3}{4}$

16.

$6\frac{1}{8} \times \frac{5}{7}$

STANDARDS FOR LOCATION TEST NO. II
GUIDE TO REVIEW WORK

PROCESS	EXAMPLES	POSSIBLE SCORE	STANDARDS (Number you should get right)	ARTICLES FOR STUDY AND PRACTICE (If you need review)
ADDITION	1, 7, 10, 15	4	3 or 4	Arts. 239-247
SUBTRACTION	2, 8, 9, 14	4	3 or 4	Arts. 248-252
MULTIPLICATION	3, 6, 12, 16	4	3 or 4	Arts. 253-262
DIVISION	4, 5, 11, 13	4	3 or 4	Arts. 263-268

117. A Location Test in Decimal Numbers. (WRITTEN WORK.) Below is a test on decimals similar to the Location Tests you have already taken on integers and fractions. Use the standards on page 196 to "locate" your ability and to see whether you need review of any of the four processes with decimals.

The table also shows which articles to use for study and practice if you fail to reach the standards.

Directions for the test: You will be allowed **20 minutes** to work the 15 examples. Number your paper correctly and work the examples in order. Wait for the teacher to give the signal to begin.

LOCATION TEST NO. III

1. *Copy and find the sum:* 9.41, 4.05, 6.28, and 5.39.
2. *Copy and add:* .96, .78, .42, .54, and 35.
3. *Find the difference of* 13.713 and 8.287.

Turn over the page and finish the test.

4. *Subtract:* 836.98 from 1839.23.

5. *Find the product:*

$$\begin{array}{r} 58.02 \\ \underline{3.9} \end{array}$$

6. $35\overline{)141.75}$

7. $3.28\overline{)88.56}$

8. *Multiply:*

$$\begin{array}{r} 374 \\ \underline{8.9} \end{array}$$

9. $3.9\overline{)8.856}$

10. *Multiply:*

$$\begin{array}{r} 69.4 \\ \underline{7.06} \end{array}$$

11. $.48\overline{)18.912}$

12. *Subtract:*

$$\begin{array}{r} 54.547 \\ \underline{19.068} \end{array}$$

13. *Multiply:*

$$\begin{array}{r} 51.96 \\ \underline{8.19} \end{array}$$

14. *Subtract:*

$$\begin{array}{r} 1709.12 \\ \underline{922.45} \end{array}$$

15. $.72\overline{)2750.4}$

STANDARDS FOR LOCATION TEST NO. III
GUIDE TO REVIEW WORK

PROCESS	EXAMPLES	POSSIBLE SCORE	STANDARDS (Number you should get right)	ARTICLES FOR STUDY AND PRACTICE (If review is needed)
ADDITION	1, 2	2	2	Art. 270
SUBTRACTION	3, 4, 12, 14	4	3 or 4	Art. 270
MULTIPLICATION	5, 8, 10, 13	4	3 or 4	Art. 271
DIVISION	6, 7, 9, 11, 15	5	4 or 5	Art. 272

If you do the study and practice that your test scores show you need, you will make a better record in your arithmetic work during the rest of Grade Seven.

118. A Location Test in Percentage. (WRITTEN WORK.) Location Test No. IV covers most of the facts of percentage. The test is divided into five separate parts. Some of the parts are again divided into smaller groups of examples. In such cases there are separate sets of directions for each group of examples. It will be necessary for you to read each set of directions carefully or you will make errors.

Directions for the test: You will be allowed **30 minutes to do all five parts** of the test. This time should be divided accurately as follows:

Part 1	10 minutes	*Part 4*	5 minutes
Part 2	4 minutes	*Part 5*	8 minutes
Part 3	3 minutes		

Wait for the teacher to give the signal to start work on each new part of the test.

LOCATION TEST NO. IV

Part 1—Equivalent Fraction, Decimal, and Per Cent Relations. (Time: 10 minutes.)

Directions: Change the fractions below to per cents. In each statement N stands for a number or an amount. Study the sample. Number the examples carefully and write only the answer.

SAMPLE: $\frac{1}{2}$ of N = ... of N. ANS. 50%

1. $\frac{1}{4}$ of N = ...% of N.
2. $\frac{2}{3}$ of N = ...% of N.
3. $\frac{7}{10}$ of N = ...% of N.
4. $\frac{3}{5}$ of N = ...% of N.
5. $\frac{1}{8}$ of N = ...% of N.
6. $\frac{8}{100}$ of N = ...% of N.
7. $1\frac{1}{4}$ or $\frac{5}{4}$ of N = ...% of N.
8. $\frac{1}{5}$ of N = ...% of N.
9. $1\frac{1}{2}$ or $\frac{3}{2}$ of N = ...% of N.
10. $\frac{4}{4}$ of N = ...% of N.

Turn over the page and finish Part 1.

Directions: Change the per cents below to fractions. Notice the sample. Number your answers carefully and write only the fraction that should be on the blank. Fractions should be in lowest terms.

SAMPLE: 50% of N = ... of N. ANS. $\frac{1}{2}$

11. $33\frac{1}{3}\%$ of N = ... of N.
12. 125% of N = ... of N.
13. $12\frac{1}{2}\%$ of N = ... of N.
14. 40% of N = ... of N.
15. 5% of N = ... of N.
16. 32% of N = ... of N.
17. $66\frac{2}{3}\%$ of N = ... of N.
18. 30% of N = ... of N.
19. 75% of N = ... of N.
20. $37\frac{1}{2}\%$ of N = ... of N.

Directions: Write on your paper the per cents below as decimals. Notice the sample. Number answers carefully.

SAMPLE: 18% of N = ... of N. ANS. .18

21. 50% of N = ... of N.
22. 3% of N = ... of N.
23. 60% of N = ... of N.
24. $5\frac{1}{2}\%$ of N = ... of N.
25. 108% of N = ... of N.
26. 2% of N = ... of N.
27. 125% of N = ... of N.
28. 4% of N = ... of N.
29. $66\frac{2}{3}\%$ of N = ... of N.
30. $5\frac{1}{4}\%$ of N = ... of N.

Directions: Write the decimals below as per cents. Notice the sample.

SAMPLE: .42 of N = ... of N. ANS. 42%

31. .35 of N = ...% of N.
32. .3 of N = ...% of N.
33. .08 of N = ...% of N.
34. 1.24 of N = ...% of N.
35. .6 of N = ...% of N.
36. .01 of N = ...% of N.
37. 1.52 of N = ...% of N.
38. .0425 of N = ...% of N.

Part 2—Expressing Areas in Per Cents.

Directions: Write as your answer the per cent of the area that is shaded. The first figure (Figure A) is already answered as a sample. **Time: 4 minutes.**

SAMPLE: Fig. A. 50 %

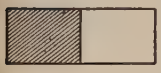


Fig. A

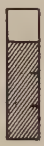


Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

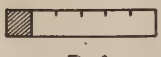


Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Directions: If the figures below were increased as shown by the dotted lines, what per cent of the first area would the **total** area be? The lines above the figures will help you estimate the areas. The first figure is a sample. Study this before you begin to work.

SAMPLE: Fig. B. Total area = 150% of first area.

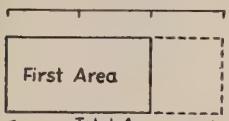


Fig. B

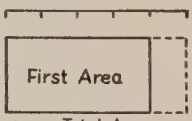


Fig. 11

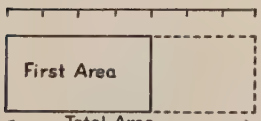


Fig. 12

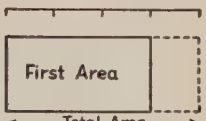


Fig. 13

Part 3—Choosing Correct Solutions in Percentage.

Directions: For each example below, five ways of solving are given. **Do not work** the examples, but simply **write the letter of the solution you would use,** as shown in the sample. Number examples carefully. **Time: 3 minutes.**

SAMPLE: 5 is what % of 22? ANS. (*d*)

(*a*) $.05\times22$ (*b*) $.05\overline{)22}$ (*c*) $22\overline{).05}$ (*d*) $\frac{5}{22}$ or $22\overline{)5.00}$ (*e*) $\frac{1}{5}\times22$

1. Find 7% of 361.

(*a*) $361\overline{)7}$ (*b*) $.07\overline{)361}$ (*c*) $361\times.07$ (*d*) $\frac{1}{7}\times361$ (*e*) 361×7

2. 34 is what per cent of 171?

(*a*) $171-34$ (*b*) $.171\overline{)34}$ (*c*) $34\overline{)171}$ (*d*) $171\times.34$ (*e*) $171\overline{)34}$

3. If 65 is 4% of N, 100% of N equals what?

(*a*) $.04\overline{)65}$ (*b*) $65\times.04$ (*c*) $65\overline{)4}$ (*d*) 65×100 (*e*) $.04\overline{)100}$

4. When 82% of N is 2890, 100% of N is what?

(*a*) $100\overline{)82.00}$ (*b*) 2890×100 (*c*) $2890\overline{)82.00}$ (*d*) $.82\overline{)2890}$ (*e*) $2890\times.82$

5. 84 is what per cent of 5672?

(*a*) $5672\times.84$ (*b*) $5672\overline{)84}$ (*c*) $84\overline{)5672}$ (*d*) $5672\overline{).84}$ (*e*) $\frac{1}{84}$ of 5672

6. Find 135% of 6000.

(*a*) $6000-1350$ (*b*) $1.35\overline{)6000}$ (*c*) 6000×1.35 (*d*) $6000\overline{)135.00}$ (*e*) $6000\times.135$

7. 422 is what % of 3480?

(*a*) $3480\overline{)422}$ (*b*) $\begin{array}{r} 3480 \\ \times .422 \\ \hline \end{array}$ (*c*) $\begin{array}{r} 3480 \\ \times 422 \\ \hline \end{array}$ (*d*) $.3480\overline{)422.00}$ (*e*) $422\overline{)3480}$

8. Find 28% of 4365.

(*a*) $\frac{1}{28}\times 4365$ (*b*) $\begin{array}{r} 4365 \\ \times 28 \\ \hline \end{array}$ (*c*) $.28\overline{)4365}$ (*d*) $4365\overline{)28.00}$ (*e*) $\begin{array}{r} 4365 \\ \times .28 \\ \hline \end{array}$

9. 100% of N is what, if 535 is 32% of N?

(*a*) $\frac{1}{32}\times 535$ (*b*) $.32\overline{)535}$ (*c*) $\begin{array}{r} 535 \\ \times .32 \\ \hline \end{array}$ (*d*) $.32\overline{).535}$ (*e*) $\begin{array}{r} 535 \\ \times 100 \\ \hline \end{array}$

10. When $25 = 4\frac{1}{2}\%$ of N, 100% of N equals what?

(*a*) $\begin{array}{r} 25 \\ \times 100 \\ \hline \end{array}$ (*b*) $\begin{array}{r} 25 \\ \times .45 \\ \hline \end{array}$ (*c*) $45\overline{)25}$ (*d*) $.045\overline{)25}$ (*e*) $\begin{array}{r} 25 \\ \times .045 \\ \hline \end{array}$

11. Find $4\frac{1}{4}\%$ of 6730.

(*a*) $\begin{array}{r} 6730 \\ \times .0425 \\ \hline \end{array}$ (*b*) $.0425\overline{)6730}$ (*c*) $\begin{array}{r} 6730 \\ \times .425 \\ \hline \end{array}$ (*d*) $.425\overline{)6730}$ (*e*) $\begin{array}{r} 6730 \\ \times 4\frac{1}{4} \\ \hline \end{array}$

Part 4—Easy Work in Percentage.

Directions: Write as your answer the number which should be on each dotted line. **Time: 5 minutes.**

1. 3 = ... % of 6.

2. 25% of 16 = ...

3. If 21 is 75% of N, 100% of N is .....

4. If 36 is 300% of N, 100% of N = ...

Turn over the page and finish Part 4.

5. $4 = \_\_\_\_\%$ of 12.
6. If 20 is 5% of N, 100% of $N = \_\_\_\_$.
7. $8 = \_\_\_\_\%$ of 800.
8. If 10 is $33\frac{1}{3}\%$ of N, 100% of $N = \_\_\_\_$.
9. $40 = \_\_\_\_\%$ of 20.
10. $66\frac{2}{3}\%$ of $30 = \_\_\_\_$
11. 3% of $600 = \_\_\_\_$.
12. $4 = \_\_\_\_\%$ of 32.
13. $37\frac{1}{2}\%$ of $36 = \_\_\_\_$.
14. If 30 is $62\frac{1}{2}\%$ of N, 100% of $N = \_\_\_\_$.
15. 400% of 6 is____.

Part 5—Harder Work in Percentage.

Directions: Find the answers to the examples below. Write your answers so that they will be easy to read. **Time: 8 minutes.**

1. Find 4% of 650.
2. 32 is what per cent of 400?
3. If 45 is 125% of N, what is 100% of N?
4. 4032 is what per cent of 3600?
5. Find 18% of 562.
6. If 75.32 is 14% of N, what is 100% of N?
7. 75.4 is what per cent of 2816?
8. Find 72% of 415.
9. If $19\frac{1}{2}\%$ of N equals 210, what is 100% of N? (Carry answer to two decimal places.)
10. 244 is what per cent of 1325?

STANDARDS FOR LOCATION TEST NO. IV
GUIDE TO REVIEW WORK

PART	POSSIBLE SCORE	STANDARDS (Number you should get right)	ARTICLES FOR STUDY (If you need review)
1	38	30	Art. 287
2	13	10	Art. 287
3	11	7	Art. 21, 291
4	15	10	Arts. 21, 287
5	10	7	Arts. 22, 288-292

TABLES OF MEASURES

119. Test on Knowledge of the Tables of Measure. (WRITTEN WORK.) In each of the following, select the one of the five possible answers which makes a true statement. Write the one you select as your answer. **Time allowed: 8 minutes.**

SAMPLE:

5 qt. are 20 pt. 10 pt. 40 pt. 15 pt. $2\frac{1}{2}$ pt. ANS. 10 pt

1. 30 gal. are 60 qt. 90 qt. 120 qt. 150 qt. 180 qt

2. 3600 yd. are about $\frac{2}{5}$ mi. $\frac{1}{2}$ mi. $\frac{2}{3}$ mi. $1\frac{1}{3}$ mi. 2 mi.

3. $3\frac{1}{2}$ ft. are 42 sq. in. 37 in. 42 in. 46 in. $1\frac{1}{2}$ yd

4. 40 T. are 800 lb. 4000 lb. 8000 lb. 40,000 lb 80,000 lb.

5. 4 bu. are 16 pk. 150 pt. 130 qt. 16 qt. 32 qt,

Go on to the next page.

6. 3 sq. ft. are 36 sq. in. 360 sq. in. $\frac{1}{3}$ sq. yd. 432 sq. in. 4320 sq. in.

7. 320 A. equal $\frac{1}{8}$ sq. mi. $\frac{1}{4}$ sq. mi. $\frac{1}{2}$ sq. mi. 1 sq. mi. 2 sq. mi.

8. 9 pk. are 72 qt. 54 qt. 45 qt. 36 qt. 18 qt.

9. 60 oz. are $2\frac{1}{2}$ lb. $3\frac{3}{4}$ lb. 4 lb. $4\frac{1}{2}$ lb. 6 lb.

10. 1 cu. ft. equals 27 cu. in. 144 cu. in. $\frac{1}{3}$ cu. yd. 1728 cu. in. 12 cu. in.

Write as your answers for Exs. 11-20 the numbers which should stand on the dotted lines. Be sure to number each answer carefully.

11. 14 pt. = ... qt. ..... pt.

12. 284 in. = ... ft. ..... in.

13. 40 pk. = ... bu. ..... pk

14. 4 ft. 6 in. = ... in.

15. 3 T. 1500 lb. = ... lb.

16. 20 ft. = ... yd. ..... ft.

17. 205 min. = ... hr. ..... min.

18. 25 qt. = ... gal. ..... qt.

19. 3 lb. 28 oz. = 4 lb. ..... oz.

20. 3 yd. 8 ft. = 5 yd. ..... ft.

If you did not get at least 15 of these examples correct, study the Tables of Measure on page 470 and the Self-Help Lessons in Arts. 274-275.

DENOMINATE NUMBERS

120. Test of Your Ability to Use Denominate Numbers. (WRITTEN WORK.) The examples in this test will show you your "weak spots" in denominate numbers, if you have any.

After you have finished the examples, look over your answers a second time to make sure that they are left in proper form. **Time allowed: 20 minutes.**

1. *Add:*

2 yd. 1 ft. 8 in.
2 ft. 5 in.
7 yd. 2 in.
10 in.

2. *Subtract:*

7 hr. 40 min.
2 hr. 25 min.

3.

3 gal. 1 qt.
×8

4.

4)9 hr.

5. *Subtract:*

5 gal. 2 qt.
2 gal. 3 qt.

6.

3)7 bu. 5 pk.

7. What is the best form of 78 inches?

8. What is the best form of 6 lb. 40 oz.?

9. Give the best form of 3 pk. 14 qt. 8 pt.

10. What is the best form of 4 hr. 100 min?

11.

$5\overline{)6\frac{1}{2}}$ lb.

12. *Add:*

5 sq. ft. 82 sq. in.
3 sq. ft. 105 sq. in.

13. *Add (change ans. to gal. qt. pt.):*

3 qt. 1 pt.
5 qt.
3 qt. 1 pt.
2 qt. 1 pt.

Go on to the next page.

14. *Subtract:*	15.	16. *Add:*
3 bu. 2 pk.	5 sq. ft. 132 sq. in.	3 lb. 2 oz.
1 bu. 3 pk. 5 qt.	×4	1 lb. 11 oz.
		5 lb.
		3 lb. 4 oz.
		6 oz.

If you did not solve correctly at least two examples in each process, use the following as a guide for study:

Process	*Examples*	*Articles for Study*
Correct Form for Ans.	7 to 10	275
Addition	1, 12, 13, and 16	276, 277
Subtraction	2, 5, and 14	278
Multiplication	3 and 15	279
Division	4, 6, and 11	280

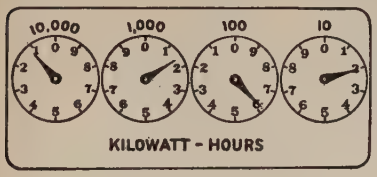
EVERYDAY USES OF ARITHMETIC

121. Test of Your Ability to Find References. (Class Work.) You may think of this article as a test of two things. First, your work upon it will show how well you can find references and topics in this book. Second, in answering the questions correctly you will show that you understand several important uses of arithmetic.

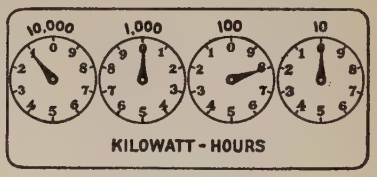
Study the questions below without help from anyone before your teacher begins to call for the answers. In getting ready to answer the questions think of good ways of giving answers as well as the facts to use in the answers.

1. On what page of the Self-Help Index is a reference to electric meters printed?

2. How much electricity was used during April according to the meters shown below?



READING ON MAY 1



READING ON APRIL 1

3. Where is there a statement in this book which tells you that gas is measured by the cubic foot?

4. What article describes the records which are kept of a checking account in a bank?

5. The item $115.00 appears on stub No. 9 on page 84. Why is this item on stub 9?

6. Is the check shown on page 85 indorsed? If so, who indorsed it?

7. On page 93 find the amount $291.45. How was this amount gotten?

8. What four rules do some savings banks follow ir computing interest on savings accounts? Defend your answer by a reference in this book.

9. What kinds of expenditures may well be put under the item of Shelter in a good budget? Defend your answer by a reference in this book.

10. If a man has an income of $2500 a year, about what per cent of his income should he plan to save? Defend your answer by a reference in this book.

11. If $A = ab$, then $A \div 2 = ab \div 2$ and $2A = 2ab$. What facts about the formula have been used here? What are three other facts about the formula?

12. What does the next sentence mean? Mr. Fellows paid a commission to the Progressive Real Estate Company for selling his house for him.

Defend your answer by exact references in this book.

13. What amounts should appear in place of the two question marks in the statement on page 183?

14. What is a factor? On what page does the book tell what a factor is?

MENSURATION

122. Critical Thinking in Mensuration. (CLASS WORK.) Think of this test as a measure of your ability to think, using what you have learned about mensuration.

You will notice that some of the statements are **correct** and some are **incorrect.** One important use of arithmetic in life is the finding or recognizing of mistakes. You may also think of your work on this test as a measure of your ability to find errors and correct them.

Study each statement below. If it is correct, be ready to explain why it is correct. If it is wrong, be ready to tell why it is wrong and what the correct statement is.

1. In seventh-grade arithmetic only two kinds of measurements are studied. These are measurements of length and measurements of volume. (78)

2. In everyday life the word capacity is always used to mean the actual volume. Thus the capacity of a freight car always means its volume in cubic feet, cubic yards, or some other cubic measure. (78)

3. A measure of length expressed in one unit can be "translated" or changed to another unit of measure. Thus: 4 miles could be expressed in other units. (80)

4. After all, the real reason for having different units of measure of length is simply to make hard work for pupils studying arithmetic. (79)

5. In drawing to scale there is only one thing to keep in mind. It is this: Choose a scale which will make the drawing of reasonable size on your paper. (84)

6. $1'':8'$ means that one inch on a drawing represents eight feet on the object itself. (84)

7. A parallelogram and a rectangle which have the same base and the same altitude also have the same area. (96)

8. An altitude is always perpendicular to the base. (94)

9. If a means altitude, and b means base, and A means area, then the formula for finding the area of a rectangle is $A = ab$. (93)

10. The formula $b = \frac{A}{a}$ means that the base equals the area divided by the altitude. This formula would be used to find the base of a rectangle when its area and altitude are known. (90)

11. In using the formulas in 9 and 10 above it does not matter whether a and b are expressed in the same units or not. (93)

12. In the formulas in 9 and 10 above, A is always expressed in square units. (93)

13. The formula for finding the area of any triangle is $A = \frac{1}{2}ab$. In Figure 1, b equals 10 in. and a equals 7 in. (96)

FIG. 1

14. The areas of the triangles in Fig. 2 are equal to each other. (96)

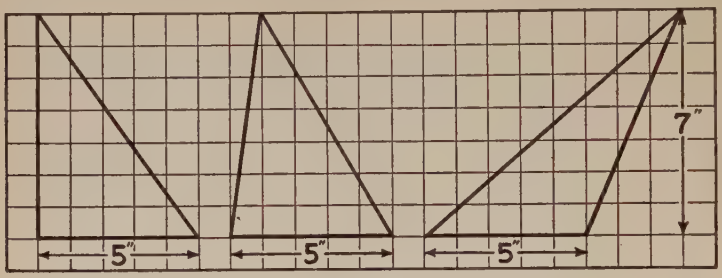


FIG. 2

15. The circumference of any circle is always less than its radius. (99)

16. The circumference of a circle is always exactly three times as great as the diameter. (99)

17. $A = \pi r^2$ is a formula for finding the area of a circle. r means radius and 1.34 can be used for π. (99, 102)

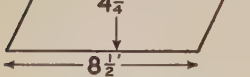


FIG. 3

FIG. 4

FIG. 5

18. Area of Figure 3 = $34\frac{7}{8}$ sq. ft. (94)

19. Area of Figure 4 = 68 in. (96)

20. Area of Figure 5 = 22 sq. ft. (very close.) (102)

The article reference following each statement shows you where you can find Self-Help work for things you did not know.

CHECKING

123. Informal Test on Checking. (WRITTEN WORK.) The examples below have been worked, but some of them were solved incorrectly. **Check** to find which printed answers are wrong, and write the correct answers.

If you cannot check all the examples in **15 minutes,** you need further practice on checking.

1.
456
892
30
109
75
689
2251

2.
9007
×86
54642
72056
775202

3. *Subtract:*
435.87
149.563
286.303

4.
.216
.005).01080

5.
47.6
2.68
19.3
.45
70.03

6.
$19\frac{3}{8} \rightarrow \frac{9}{24}$
$3\frac{2}{3} \rightarrow \frac{16}{24}$
$41\frac{1}{4} \rightarrow \frac{6}{24}$
$7\frac{5}{6} \rightarrow \frac{20}{24}$
$72\frac{6}{24}$ $\frac{51}{24}$

7.
278500.
.34)94790.00

8. *Subtract:*
4 yd. 1 ft. 5 in.
1 yd. 2 ft. 4 in.
2 yd. 0 ft. 1 in.

9. $\frac{1}{10} \div 2\frac{1}{2} = \frac{1}{4}$

10.
.948
×7.2
1896
6636
6.8256

On pages 47-49, 56-57 and 410-411, you will find lessons that will help you in checking.

ESTIMATING

124. Informal Test on Estimating. (WRITTEN WORK.) Seventh-grade pupils should be able to estimate "in their heads" about what a reasonable answer would be to an example.

Each of the 15 examples below has four answers given. In each case one of the four is approximately correct; that is, one of them is a good estimate of the answer to the example. **Without actually working the examples,** select the best estimate. Number your papers carefully.

Suggested working time: 15 minutes.

1. $356530 \div 89 =$ 400 500 4000 5000
2. $8098 \times 99 =$ 80,000 700,000 800,000 8,000,000
3. $37999 \div 481 =$ 1100 800 80 8
4. $78 \times 19 =$ 780 1400 1480 1600
5. $3.167 \times .532 =$ 1.5 15 150 1500
6. $1.684 \div 421 =$ .004 .04 .4 4
7. $2300 \div .392 =$ 6 60 600 6000
8. $2\frac{1}{2} \div 7 =$ 17 7 2 $\frac{1}{2}$
9. $4\frac{1}{9} \times 8\frac{1}{8} =$ 30 33 60 240
10. 8% of 792 = 56 63 560 6300
11. 312 is what % of 601? $\frac{1}{2}$% 50% 2% 200%
12. If 58 is 41% of N, 100% of $N =$ 24 80 140 580
13. $201\frac{1}{2}$% of 80 = 16 160 400 16,000
14. 911 is what % of 307? 3 $33\frac{1}{3}$ $133\frac{1}{3}$ 300

15. Find the interest on \$500 @ 4% for 2 yr. \$4000 \$400 \$40 \$20

If you had trouble with this work, you can use Art. 237, pages 412-413, for study and practice.

If you have taken the tests in Chapter V honestly and have faithfully done all needed extra study and practice, you can go on with confidence in your ability to do good work in arithmetic during the remainder of this year.

CHAPTER VI

THE MEASUREMENT OF VOLUMES

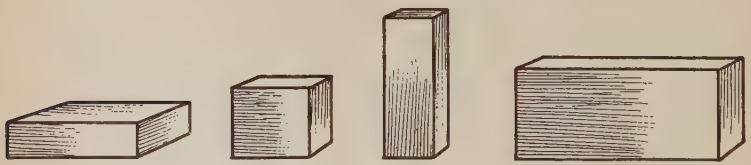
KINDS OF SOLIDS

125. Names of Important Types of Solids. (Class Work.) Perhaps you are used to thinking that a "solid" is something that is hard all through, like a block of wood or a stone. But in arithmetic the word "solid" has a special meaning.

Anything that has length, width, and height (or depth) is called a **solid.** A brick is a solid and so also is any given amount of air or water. We call a barrel or a pail a solid, too. The number of cubic units of space a solid occupies is called its **volume.** We have to use arithmetic with solids very often in everyday life. Turn back to page 131 and read what is said there of volumes.

Solids are of different shapes, and they are named from their shapes. The surfaces of a solid are called its **faces.**

RECTANGULAR SOLIDS



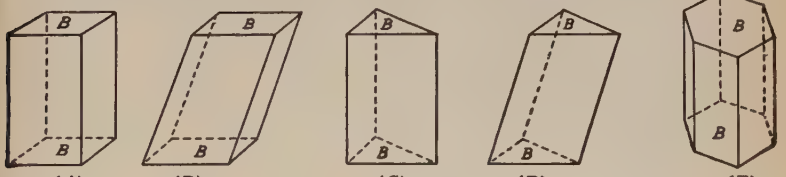
The figures above are called **rectangular solids.** Rectangular solids may be of any size and of various shapes. They may be tall or short, long or narrow, thick or thin; but the faces of a rectangular solid must all be rectangles. The rectangular solid is also called a **rectangular prism.**

A **prism** may be described as a solid figure, in which

(1) two faces are equal and parallel, and

(2) all other faces are parallelograms.

Any two equal and parallel faces are called the **bases.** (The bases are labeled *B* in Figs. A to E below.)



The bases may have three, four, or more sides. Fig. C and Fig. D above have triangular bases, and hence are called **triangular prisms.** The bases of Fig. A and Fig. B have four sides. The bases of Fig. E have six sides.

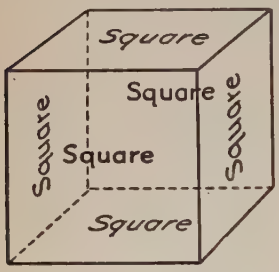
There are special names for prisms having various numbers of sides in their bases, but these names are not important for you to learn at this time. Fig. A is important because it shows a form of prism very common in everyday life. It is called a **rectangular prism,** because all of its faces (including the two bases) are rectangles.

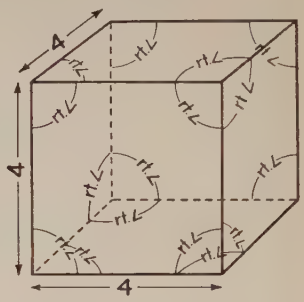
When all the faces (including the bases) of a prism are rectangles, it is called a rectangular prism.

QUESTIONS

1. Why is Fig. B above not a rectangular prism?
2. Give two reasons why Fig. D is not a rectangular prism.
3. Why is Fig. E above not a rectangular prism?
4. Is a chalk box a rectangular prism? Is a brick?
5. Name some objects which are rectangular prisms.

The **cube** is a special type of rectangular solid. Its six faces are all squares (a special type of rectangle). You





can think of a cube as a rectangular solid whose twelve edges are all equal in length and whose angles are all right angles. The cube is also called a rectangular prism.

SELF-TESTING DRILL NO. 16

126. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*
   4978579
   540630

2. $12 \times 2\frac{1}{12} = ?$

3. *Subtract:*
   9170.414
   7987.818

4. *Add:*
   $2\frac{3}{4}$
   $4\frac{7}{8}$
   $3\frac{6}{16}$

5. Jack said the right quotient of $274{,}396 \div 758$ is 372. If you do not agree with him, give the quotient you think correct. If you agree, write "Correct" as your answer.

6. *Multiply:*
   3954
   7806

7. $.612\overline{)53488.8}$

8. Is $6.66 \times \frac{2}{3}$ about: 38, 13.0, or 4.5?

9. *What is the total distance?*

6 mi. 2780 ft.
2 mi. 4196 ft.
3 mi. 3900 ft.
1 mi. 3854 ft.
4 mi. 1110 ft.

10. *Find the average:*

95981
10897
28102
56718

11. Interest at 6% on $835 for 3 yr. 8 mo. will amount to what?

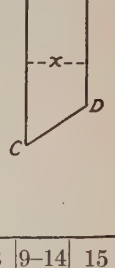
12. *Multiply:*

4 sq. ft. 100 sq. in.
6

13. 260 is what per cent of 208?

14. Find the area.

$CA = 4$ yd. 1 ft.
$AB = 2$ yd.
$x = 1$ yd. 2 ft.



15. $6781.25 was earned in one year by a business concern. This was an interest rate of $7\frac{3}{4}\%$ on the money invested. How much was invested?

STANDARDS

Number correct	0–1	2	3	4	*	5	6	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

A USEFUL PROBLEM-SOLVING HABIT

127. Estimating Answers. (Oral Work.) Before you work a problem, you should stop and think what answer would be reasonable. Then compare the answer you get with your estimate. This habit often prevents foolish mistakes. The exercise on page 218 is to help you form the habit of estimating reasonable answers before you begin work.

Do not work the examples, but tell which of the answers on the right is the best estimate for each example on the left. The estimates are not in order.

The best estimate for Example 1 is (*g*). In this case

you might reason thus: 61 is about 60. 994 is about 1000. The product must be about 60×1000, or 60,000.

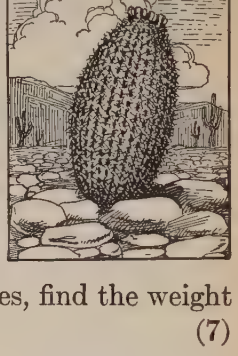
Examples	Estimated Answers
1. 61×994	(*a*) 3200
2. $8.47\overline{)7.80087}$	(*b*) 940
3. $87246 - 34182$	(*c*) 9.4
4. 94 is 10% of what number?	(*d*) 94
5. Dimensions, $6'' \times 8'' \times 10''$; volume = ?	(*e*) .94
6. $8.47\overline{)78.0087}$	(*f*) .094
7. 493×406	(*g*) 60,000
8. $0.94 \div 10$	(*h*) 53,000
9. 9% of 7324	(*i*) $16\frac{2}{3}\%$
10. Dimensions, $16' \times 30'$; area = ?	(*j*) 200,000
11. $4.28 + 9.90 + 15.5 + 4.8 =$	(*k*) 0
12. 84 is what % of 500?	(*l*) 480 cu. in.
13. 5.73×4.62	(*m*) 480 sq. ft.
14. $8.47\overline{)780.087}$	(*n*) 700
15. $428 + 990 + 1550 + 48$	(*o*) 32.00
16. 94×0	(*p*) 25

PROBLEM SCALE NO. VIII

128. Problems Using Science Facts. (Written Work.) You can earn 70 points by working every problem correctly. Find your score and rate yourself. **Before working an example, try estimating an answer.**

1. Very young grass on the desert often contains 80% of water. As the blades of grass become older and tougher,

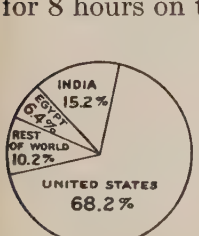
the amount of water shrinks to 50% of the total weight. How much water is there in 100 lb. of old desert grass? (5)



2. Animals which live in the desert often get water, not from the rivers and streams, but from the water in desert plants. One of the most valuable of the desert plants that contain water is the Viznaga. A man found one Viznaga which he estimated had 200 pounds of water in its fruit. He also estimated that 90% of the weight of the fruit came from the water in it. Using these estimates, find the weight of the fruit of this plant. (7)

3. A man studying plant life on the desert found a single pad, or joint, of cactus 9 by 12 inches. It weighed 22 ounces when cut from the plant. He split it open and dried it thoroughly. It then weighed 17 oz. Its loss in weight came from losing water through evaporation. What per cent of its weight did it lose through evaporation? (7)

4. A section foreman estimates that a man working on the railroad in the desert during hot weather needs 4 gallons of water for every ten hours of work. How much water will be needed for a crew of 15 men who are to work for 8 hours on the desert during the summer? (7)



5. The graph shows the distribution of cotton production during a certain year. The total production for the whole world for that year was 19,595,000 bales. Find the amount of cotton produced in the United States for that year. (7)

Have you ever wondered how often certain easy words are printed in textbooks that you use? In a set of textbooks that one pupil uses the word "the" comes 7267 times, the word "you" comes 1385 times, the word "will" comes 591 times, and the word "very" comes 366 times. Use these facts for Problems 6 to 10.

6. How many times as often does the word "the" come as the word "you"? (Answer to one decimal place.) (6)

7. For every time the word "very" comes, how many times does the word "will" come? (To one decimal place.) (7)

8. How many times as frequent is the word "you" as the word "very"? (Answer to one decimal place.) (7)

9. The frequency with which the word "very" comes is what per cent of the frequency of the word "will"? (8)

10. The frequency with which the word "you" comes in these textbooks is what per cent of frequency of the word "the"? (9)

STANDARDS

Superior work:	21–70 points	Poor work:	6–8 points
Good work:	13–20 points	Inferior work:	0–5 points
Average work:	9–12 points		

THE MEASUREMENT OF VOLUMES

129. Volume of Rectangular Solids. (Class Work.) When a builder is figuring the cost of a concrete basement for a house, he has to find the number of cubic feet or yards of concrete he will use. When the owner of the home orders his coal for the first winter, he has to know about how much coal his bin will hold. To do this he has to find the space in the bin (or the volume). You already know that gas is measured by the cubic foot. So you see that there are many reasons why we should know how to find volumes.

Rectangular solids are the kind with which we will most commonly have to work. The formula for finding the volume of a rectangular solid is $V = l \times w \times h$, or $V = lwh$, where V = volume (in cubic units), l = length of base (in linear units), w = width of base (in linear units), h = height of the solid (in linear units).

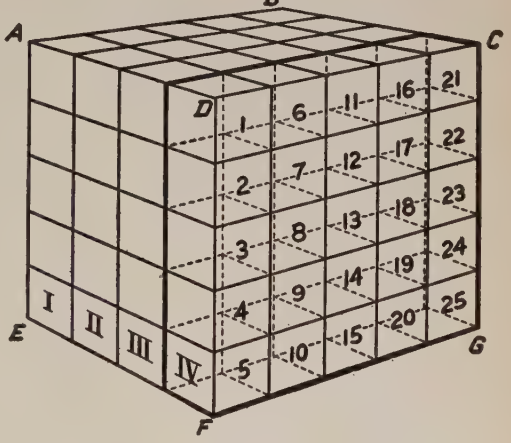


FIG. 1

First of all, let us study Fig. 1 to see if we can really count the number of cubic units there are in its volume. Let a small cube, like number 5, for example, 1 unit long, 1 unit wide, and 1 unit high, be the cubic unit of measure. Try to count the number of cubes or blocks in Figure 1.

Now let us compute the volume by using the formula.

Let line FG be the length of the base. There are 5 linear units in line FG.

Let line FE be the width of the base. There are 4 linear units in line FE.

Let line DF be the height of the solid. There are 5 linear units in line DF.

Formula: $V = lwh$.

For Fig. 1, $V = 5 \times 4 \times 5$ cubic units $= 100$ cubic units.

Does the volume as given by the formula agree with the volume you got by studying the figure itself?

There is another formula which we can use to find the volume of a rectangular solid. You can easily understand it by thinking a moment about the formula you already know, $V = lwh$.

In the formula $V = lwh$, you know that the l and the w stand for the length and the width of the base of the solid. You also know that the length of the base times the width of the base will give you the area of the base. So, in $V = lwh$ we can put a capital B in place of the lw. This B stands for the area of the base, and the new formula reads $V = Bh$.

This new formula will give you not only the volume of a rectangular prism, but of any prism.

Remember these formulas for the volume (in cubic units) of a rectangular prism:

$V = lwh$. $V = Bh$, where B stands for the area of the base.

Use the formula $V = lwh$ in computing the volume for Fig. 2; study the figure to see if the formula gives the correct answer. Find the volumes of Fig. 3 and Fig. 4.

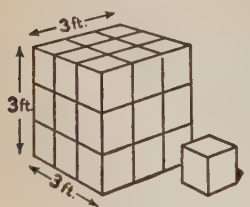


Fig. 2

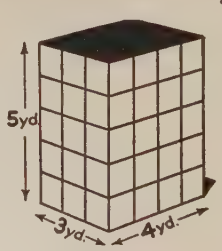


Fig. 3

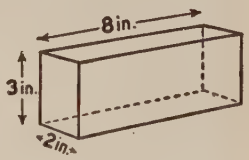


Fig. 4

Cautions: There are three cautions to keep in mind in working with rectangular solids. These are:

1. Use cubic measure in expressing volumes. Why?

2. All dimensions must be in the same unit before multiplying to get the volume. Why?

3. When a dimension contains a fraction, do not get worried. You know how to multiply by a fraction. Thus: $l=4\frac{1}{2}$, $w=3$, $h=2\frac{1}{3}$. $V=4\frac{1}{2}\times3\times2\frac{1}{3}=\frac{9}{2}\times\frac{3}{1}\times\frac{7}{3}=\frac{63}{2}=31\frac{1}{2}$ (cubic units).

If you knew the volume and only two dimensions of a rectangular solid, could you find the other dimension? Read carefully what is said below.

Suppose you were building a bin in which to put sand for your little brother's sand-pile. You have a space 6 ft. by 4 ft. in the yard for the bin. Your father has ordered a load of sand, and he tells you that it will contain about 2 cubic yards of sand (54 cu. ft.) Could you find how high to build the bin in order to hold the sand?

Think for a minute of what you learned in Art. 46 about finding a missing factor: Product of two factors divided by one factor gives the other factor. But in this problem we have a product of three factors, 54. Two of its factors are 6 and 4. We want to find the third factor. $6\times4\times?=54$. We can multiply the two factors, 6 and 4, and then use their product in place of these two factors. That is, $24\times?=54$.

Now you can solve the problem, since this example is just like other examples you worked in Art. 46 to find the missing factor. How high would you make the bin in order to hold the sand?

130. Practice in Calculating Volumes. (WRITTEN WORK.) Find the volumes or the missing dimensions of the rectangular solids whose dimensions are as given at the top of page 224 and work the problems.

	Length of Base	Width of Base	Height	Volume
1.	$2\frac{1}{2}$ in.	$3\frac{1}{4}$ in.	5 in.	----------
2.	4.5 ft.	6 ft.	3.5 ft.	----------
3.	7 yd.	13 yd.	5 yd. 2 ft.	----------
4.	3 ft.	9 ft.	----------	54 cu. ft.
5.	2 yd.	----------	10 yd.	40 cu. yd.
6.	----------	6 in.	$7\frac{1}{2}$ in.	$112\frac{1}{2}$ cu in.
7.	Area of base = 14 sq. in.			84 cu. in.

8. A concrete block 6 ft. by $4\frac{1}{2}$ ft. by 3 ft. was needed as a base for a statue. How many cubic yards of concrete were needed?

9. Wood for fuel is often measured by the "cord." A cord equals 128 cu. ft. At $12.00 a cord what will be the cost of a straight pile of wood 16 ft. by 8 ft. by 5 ft.?

10. A farmer planned a wheat bin to be 12 ft. long and 6 ft. wide. He wanted it to hold 500 bushels of wheat. How deep did he have to make it? A bushel of wheat takes up $1\frac{1}{4}$ cu. ft. of space. (Hint: How many cubic feet would there be in the 500 bu.? Can you find a missing factor?)

11. In figuring the cost of a house, a builder had to find how much concrete he would need for the walls of the basement. The outside dimensions of the basement were 24 ft. by 31 ft. The walls were 18 inches thick and 9 ft. high. How many cubic yards of concrete would he need?

12. One of New York City's water reservoirs is 2400 ft. long, 1500 ft. wide and the same depth throughout. It holds 435,000,000 gal. of water. How deep is it? Use $7\frac{1}{2}$ gal. to the cubic foot. (Hints: This problem is much like Problem 10. Start by changing the gallons to cubic feet.)

131. Rapid Drill. (ORAL WORK.) In division examples give only the first quotient figure. You should be able to give the answers quickly after a few minutes of study.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$53\overline{)477}$	*Add:* 9, 3, 2, 1	24 qt. = ... pk.	165 -79	250% = what decimal?	592 $\times 9$
2.	134 -56	$37\overline{)222}$	*Add:* 8, 5, 4, 8	385 $\times 4$	684 $\times 6$	$5\frac{2}{3} = \frac{?}{3}$
3.	$1.26 \times N$ = ...% of N	100 -45	561 $\times 1$	*Add:* 5, 6, 6, 5	$69\overline{)1794}$	$\frac{2}{3} = \frac{?}{15}$
4.	129 -42	*Reduce:* $\frac{15}{6}$ =	752 $\times 3$	$70\overline{)5180}$	*Add:* 8, 8, 6, 3	93 -87
5.	$\frac{7}{8}$ of N = ... % of N	192 $\times 8$	$48\overline{)4992}$	141 -59	$c = \pi d$ means?	*Add:* 5, 3, 3, 9
6.	492 $\times 5$	$1 \div 2\frac{1}{3}$ = $1 \times$	142 -96	*Add:* 5, 9, 2, 7	113 -31	$95\overline{)4275}$
7.	*Add:* 6, 8, 3, 4	$20\overline{)1900}$	$1\frac{1}{2} + \frac{2}{3}$	1 cu. yd. = ... cu. ft.	.6+1.5+ .8	126 $\times 7$
8.	$37\overline{)263}$	Meaning of $V = lwh$?	146 -72	350% of \$3	132 -37	3 wk. 3 da $\div 4$

SELF-TESTING DRILL NO. 17

132. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*

$$\begin{array}{r} 1171513 \\ 834578 \\ \hline \end{array}$$

2.

$$\begin{array}{r} 102.1 \\ 29.2 \\ 50.4 \\ 18.9 \\ 403.3 \\ 12.1 \\ 233.9 \\ \hline \end{array}$$

3. $\frac{5}{6} \div 2\frac{3}{4} = ?$

4. $682\overline{)471262}$

5. *Multiply:*

$$\begin{array}{r} 40.68 \\ 70.3 \\ \hline \end{array}$$

6. *Subtract:*

$$\begin{array}{r} 34\frac{1}{6} \\ 17\frac{2}{3} \\ \hline \end{array}$$

FIRST NATIONAL BANK

DEPOSITED BY H. T. Jones

ADDRESS 3 Walker Terrace

IOWA CITY, IOWA, 4-5 1928

Currency	125	
Silver	10	63
Gold		
CHECKS AS FOLLOWS.		
Merchants Bank	95	32
Bank of Riverside	10	16
Cotton Growers Bank	139	48
Total $	391	59

7. Figure a 20% commission on a sale of $872.

8. Mr. Jones presented a deposit slip like the one at the left to the cashier of his bank. The cashier added the amount and found it to be incorrect. What is the correct amount?

9. How much heavier is 3 lb. 4 oz. than 2 lb. 11 oz.?

10. *Multiply:*

$$\begin{array}{r} 9735 \\ 3285 \\ \hline \end{array}$$

11. 6 sq. yd. 7 sq. ft. $\div 7 = ?$

12. The length, width, and height of a rectangular bin are $5\frac{1}{2}$ ft., 8 ft., and 14 ft. What is its volume?

13. 3000 is 125% of what number?

14. How much money must I have earning 8% interest to pay me $20 a year?

15.* (*a*) $\frac{1}{8}$ equals what decimal? (*b*) $1.12\frac{1}{2}$ equals what mixed number? (*c*) $12\frac{1}{2}\%$ equals what decimal? (*d*) 16 yd. = how many feet?

*To count Example 15 correct, each part must be correct.

STANDARDS

Number correct.....	0–1	2–3	4	5	*	6	7	8	9–10	11–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

You should take time to **correct any mistakes** and use the *Self-Help Index*, page 471, to find articles you may need to study.

REVIEW OF FUNDAMENTALS

133. Drill for Mastery. (Oral Work.) Every seventh-grade pupil should be able to give correct answers to the work below after a short study period. Be sure you understand the directions that go with each set of examples.

1. What common denominators would you use in each of the following examples?

(*a*) $\frac{}{2}+\frac{}{3}+\frac{}{4}$
(*b*) $\frac{}{5}+\frac{}{2}+\frac{}{4}$
(*c*) $\frac{}{3}+\frac{}{8}+\frac{}{12}$
(*d*) $\frac{}{3}+\frac{}{4}+\frac{}{6}$
(*e*) $\frac{}{10}+\frac{}{6}+\frac{}{15}$
(*f*) $\frac{}{6}+\frac{}{9}+\frac{}{2}$
(*g*) $\frac{}{8}+\frac{}{4}+\frac{}{12}$
(*h*) $\frac{}{6}+\frac{}{3}+\frac{}{8}$

2. Tell how each division would be changed to work the example.

(*a*) $N \div \frac{1}{2} = N \times$ .....
(*b*) $N \div \frac{4}{5} = N$ .....
(*c*) $N \div 1\frac{1}{3} = N$ .....
(*d*) $N \div 3\frac{1}{4} = N$ .....
(*e*) $N \div \frac{6}{7} = N$ .....
(*f*) $N \div 2\frac{2}{5} = N$ .....
(*g*) $N \div 6\frac{1}{2} = N$ .....
(*h*) $N \div 7\frac{5}{6} = N$ .....

3. The figures in the quotients below are correct, but the decimal points have been omitted. Read the correct quotients, supplying zeros where needed.

(*a*) $41\overline{)251.74}$ 614

(*b*) $4.1\overline{)2517.4}$ 614

(*c*) $.41\overline{)25174}$ 614

(*d*) $.041\overline{)25174}$ 614

(*e*) $.041\overline{)251.74}$ 614

(*f*) $.41\overline{)2.5174}$ 614

(*g*) $4.1\overline{).25174}$ 614

(*h*) $.41\overline{).25174}$ 614

(*i*) $4.1\overline{)25.174}$ 614

(*j*) $.041\overline{)2.5174}$ 614

4. Make up questions in percentage which would be answered by working each of the examples below:

(*a*) $.06 \times \$476$

(*b*) $\frac{31}{421}$ or $421\overline{)31}$

(*c*) $.08\overline{)316}$

(*d*) $1.5\overline{)590}$

(*e*) $\frac{817}{2434}$ or $2434\overline{)817}$

(*f*) $.22\overline{)9146}$

(*g*) 2.5×931

(*h*) $\frac{652}{148}$ or $148\overline{)652}$

5. Tell which example in each pair below will give the larger answer.

	(*A*)		(*B*)
I.	967×34	or	967×43
II.	$47\overline{)19326}$	or	$32\overline{)12672}$
III.	6% of 985	or	5% of 1142
IV.	$2\frac{1}{2} \times 3\frac{1}{3}$	or	$3\frac{1}{4} \times 2\frac{2}{5}$
V.	$4 \div \frac{2}{5}$	or	$4 \div \frac{3}{5}$

6. On the left are questions. On the right are examples. Name the letter of the example which, when solved, will answer each question.

I. What is 29% of 49?	(*a*) $.49\overline{)29.}$
II. 29 is what per cent of 49?	(*b*) $\frac{4\,9}{2\,9}$ or $29\overline{)49.}$
III. If 49 is 29% of N, $N=$?	(*c*) $49\times.29$
IV. 49 is what per cent of 29?	(*d*) $\frac{2\,9}{4\,9}$ or $49\overline{)29.}$
V. If 29 is 49% of N, $N=$?	(*e*) $.29\overline{)49.}$
VI. What is 49% of 29?	(*f*) $29\times.49$

134. A Matching Exercise in Mensuration. (Written Work.) On the left below are certain phrases, on the right are formulas or other facts. On your paper write the number of each phrase and beside each number write the letter of the formula or other fact which matches it.

1. Area of rectangle	(*a*) $V=lwh$
2. Units of measure of length	(*b*) 3.14 or $3\frac{1}{7}$ or $\frac{2\,2}{7}$
3. Units of volume of a rectangular solid	(*c*) sq. in., sq. ft., sq. yd.
4. Area of a circle	(*d*) $A=ab$
5. Units of measure of area (how much surface?)	(*e*) in., ft., yd., mi.
6. Scale drawing	(*f*) $A=\pi r^2$
7. Formula for volume of rectangular prism	(*g*) $A=\frac{1}{2}ab$
8. Area of any triangle	(*h*) cu. in., cu. ft., cu. yd.
9. π or pi	(*i*) $1''$: 4 mi.

STUDYING CLASS RECORDS

135. The Standing of Classes on Problem Scales. (WRITTEN WORK.) Suppose 4 pupils in one class rated superior in problem solving, and in another class 6 pupils rated superior. How would you find which class had the larger percentage of superior problem-solvers?

If both classes were of the same size, the class with 6 pupils rating superior would stand the higher. But if one class had 20 pupils and the other class had 36 pupils, we would have to find what per cent of each class rated superior and compare the per cents.

Here is a table showing the records of certain classes in three schools. Below it are some questions for you to answer.

TABLE SHOWING THE RECORDS OF SCHOOLS *A*, *B*, AND *C*

SCHOOL	GRADE:	IV	V	VI	VII
A	No. of pupils	30	32	27	25
	No. rating superior	5	4	9	5
B	No. of pupils	40	45	42	36
	No. rating superior	5	5	7	9
C	No. of pupils	45	40	39	35
	No. rating superior	3	2	3	5

QUESTIONS

1. What per cent in School *A*, Grade IV, rated superior?
2. What per cent in School *B*, Grade VI, rated superior?
3. Which fifth grade had the highest per cent of superior pupils?
4. Which seventh grade rated the lowest?

5. To which class, of all in the table, would you give first prize for having the largest per cent of superior pupils?

6. To which class should second prize be given?

7. After you have worked Problem Scale No. IX, find the percentage of your class rating superior on Scale IX.

PROBLEM SCALE NO. IX

136. Problems Using Troublesome Computation. (WRITTEN WORK.) You can earn 60 points by working every problem correctly. **Careful reading of problems is the first step toward getting correct answers.**

1. Mrs. Chase bought $\frac{3}{8}$ of a yard of velvet advertised to sell at $4.80 a yard. She gave the saleslady a five-dollar bill in payment for the purchase. How much change should she have received? (3)

2. A box of apples weighed 45 lb. 4 oz. The empty box weighed 1 lb. 11 oz. What did the apples alone weigh? (4)

3. Frank went to his grandmother's for the Christmas vacation. He was on the train exactly 5 hours. The times that the train stopped amounted to $1\frac{5}{6}$ hr. How long was the train actually running while Frank was on it? (5)

4. Ruth is planning a garden which is rectangular in form and has these dimensions: 46 ft. and 58 ft. She wants to draw a plan of the ground so that she can figure out on paper just how large to make plots for different vegetables and flowers. Her drawing is to have a scale of 1″ to 10′. What will be the length and width of her drawing of the garden? (5)

5. Mary takes care of Mrs. Williams's baby in the afternoons for 25¢ an hour. Mrs. Williams figures the time that Mary works very exactly. Last week she took

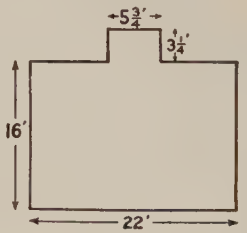
care of the baby for $\frac{3}{4}$ of an hour on Monday, $1\frac{5}{12}$ hours on Tuesday, $2\frac{1}{6}$ hours on Tuesday, $2\frac{1}{6}$ hours on Thursday, and $1\frac{1}{3}$ hours on Friday. How much money should Mrs. Williams pay Mary for the week's work? (6)

6. I stopped at the Travelers' Garage to have a leak in the radiator of my automobile fixed. The mechanic started work at 11:50 and finished at 12:40. The labor cost of this repair was one dollar. What is the labor charge at the garage per hour? (6)

7. Last Thanksgiving Julia's father bought a $15\frac{1}{4}$-lb. turkey for $7.93. Julia lives in Michigan. Her cousin who lives in Texas wrote her that they bought a $16\frac{3}{8}$-lb. turkey for $3.99. How much less per pound did turkey cost in Texas than in Michigan? (6)

8. From a barrel of maple syrup holding 39 gal. 3 qt., Mr. Logan drew off enough syrup to fill 4 cans, each holding 3 qt. 1 pt. He sent these cans as presents to his four boys. How much syrup had Mr. Logan left in the barrel? (6)

9. A schoolhouse of 50 years ago such as many of our fathers and mothers attended had a floor plan like that of the drawing. Find the floor area of this plan. (9)



10. The Acme Wholesale Grocery allows discounts of 10%, 8%, 2%. Find the saving to be made by taking advantage of all three discounts on a bill of $149.50. (10)

STANDARDS

Superior work:	28–60 points	Slow work:	8–13 points
Good work:	20–27 points	Inferior work:	0–7 points
Average work:	14–19 points		

137. Rapid Drill. (ORAL WORK.) Can you work these examples without using your pencil? It is worth while to be able to work simple arithmetic examples "in your head."

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	7.5 ×8	23)71	250% = what decimal?	*Add:* 9, 9, 7, 5	21 da. = ...wk.	97.2 −37.0
2.	89)801	168 −99	$\frac{3}{4} = \frac{?}{12}$	57 ×7	$\frac{5}{6}$ of N = what % of N?	*Add:* 5, 6, 9, 7
3.	74.5 −6.8	.9 + 1.0 +.9	1 cu. yd. −7 cu. ft. = ? cu. ft.	Find 6% of $40	67)483	94 ×.1
4.	52 ×6	57)363	$\frac{1}{9} + \frac{1}{3}$	*Add:* 10, 1, 3, 7	4 pt. + 2 qt. = ...qt.	422 −28
5.	.6 + .7 +3.5	*Best form for ans?* $\frac{18}{16}$	8.20 −5.01	7 = 10% of what number?	95 ×5	13)28
6.	310 −83	68 ×5	$\frac{1}{2} \times 1\frac{1}{2}$	78)702	*Add:* 5, 5, 10, 4	40 min. ×3 .....hr.
7.	37)257	3)2 ft.	*Add:* 9, 8, 9, 3	73 ×1000	2 $-1\frac{3}{8}$	696 −421
8.	433 −302	20 is what % of 16?	.38 ×1	Give the factors of 21.	46)320	*Add:* 5, 7, 6, 6

SELF-TESTING DRILL NO. 18

138. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*

1318508
978786

2. $2.09
7.11
3.88
2.92
7.44
9.89

3. *Subtract:*

66819.50
62983.54

4. *Multiply:*

3.867
415

5. *Divide:* 281,542 by 478.

6. Find a 9% discount on $931.00.

7. What is the difference of $56\frac{1}{12}$ and $17\frac{1}{4}$?

8. $3\frac{3}{5} \div 1\frac{1}{9} = ?$

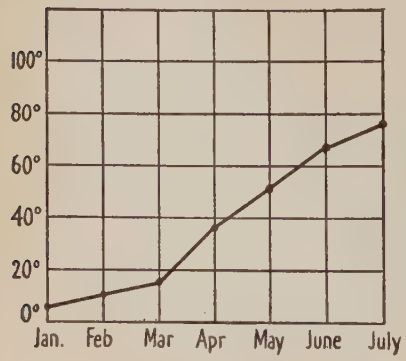
9. 75 is what per cent of 225?

10. If the work to the problem shown below is correct, write C as your answer. If not, write the correct answer.

PROBLEM: Find the volume of a rectangular solid of the dimensions 9 yd. by 9 yd. by 12 yd.

WORK: 9+9+12=30 yd.

11. Subtract 2 gal. 3 qt. 1 pt. from 4 gal.



Month	*Average Temperature*
January	6°
February	11°
March	24°
April	38°
May	52°
June	68°
July	78°

12. Which monthly temperature is plotted (marked) incorrectly according to the figures given in the table on page 234?

13.* (*a*) $4\frac{1}{2}$ lb. = ... oz. (*b*) $7\frac{1}{2}$ yd. = ... in. (*c*) 16 qt. = ... pk. (*d*) 900¢ = ... dollars.

14. 3 yd. 2 ft. 8 in. $\div 2 = ?$

15. $6\frac{1}{4}\%$ of a certain amount of money = \$525.25. What is 100% of this amount?

*To count as correct, each part of Example 13 must be correct.

STANDARDS

Number correct	0–1	2	3	4	*	5	6	7	8–9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

SUMMARY

139. General Review of Processes. (Class Work.) Practically all of the work of arithmetic calls for the use of one of the four processes: addition, subtraction, multiplication, division. But these processes are used with different kinds of numbers and for many different purposes.

Work the examples which are numbered. As you finish each example, take time to talk over the reasons for the mistakes that any of you may have made.

USES OF ADDITION

Whole Numbers

1. 427
93
816
24
718

Be sure to carry correctly.

Remember to check.

Fractions, Mixed Numbers

2. $4\frac{1}{2}$
$3\frac{1}{3}$
$\frac{3}{4}$
8

Add sum of fractions to the whole numbers.

Reduce fractions in answers.

Decimals

3.
63.7
4.07
18.09
6.06
.84

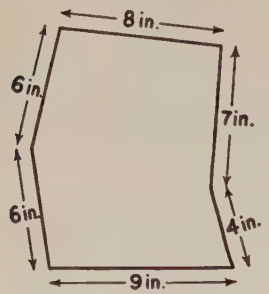
Remember to check. Zeros and gaps must be watched. Keep points in a straight line.

Denominate Numbers

4.
4 ft. 6 in.
2 ft.
3 ft. 11 in.
1 ft. 8 in.

Tables on p. 470 should be mastered. Reduce answers.

5. Find the perimeter.



Words Used in Addition

What is the **sum?**
What is the **total?**

32
41
68
92
233 ← *Sum* or *total*

USES OF SUBTRACTION

Whole Numbers

6. *Subtract:*
↓↓↓
60142
38176

Watch places like these ↓
Check by adding.

7. *Subtract:*
31824
1427

Fractions, Mixed Numbers

8. *Subtract:*
$6\frac{1}{2}$
$4\frac{2}{3}$*

*Watch when the fraction in the subtrahend is the larger.
Watch vacant places (↓)
Reduce answers.

9. *Subtract:*
8↓
$6\frac{3}{8}$

Decimals

10. *Subtract:*

214.7
61.9

Watch vacant places (↓)
Keep decimal points in a straight line.
Remember to check.

11. *Subtract:*

824.5↓
314.27

Denominate Numbers

12. *Subtract:*

4 hr. 15 sec.
2 hr. 40 sec.

Tables on p. 470 should be memorized. Be careful when a subtrahend number is the larger.

13. *Subtract:*

3 gal. 1 qt.
2 gal. 3 qt.

Questions answered by subtraction are:

How much more?
What is left?
What is the difference of?
How much greater?
How much less?
How much must be added?

Words Used in Subtraction

438←*minuend*
217←*subtrahend*
221←*difference*

USES OF MULTIPLICATION

Whole Numbers

14. *Multiply:*

647
304

Watch out for zeros.
Check by reversing.
Place figures carefully.
Remember to carry.

15. *Multiply:*

3092
408

Fractions and Mixed Numbers

16. $\frac{2}{3} \times \frac{9}{12}$

17. $4\frac{1}{2} \times \frac{2}{3}$

18. $\frac{2}{3}$ of 15

Cancel when possible. "Of" means ×. Change mixed numbers to improper fractions. Reduce fractions in answers.

Changing Form but not Value

$\frac{2}{3} = \frac{4}{6}$ $\left(\frac{2 \times 2}{3 \times 2} = \frac{4}{6}\right)$ $\qquad 2\frac{1}{2} = \frac{5}{2}$ $\left(\frac{(2 \times 2) + 1}{2}\right)$

$\frac{4}{5} = \frac{12}{15}$ $\left(\frac{4 \times 3}{5 \times 3} = \frac{12}{15}\right)$ $\qquad 7\frac{2}{3} = \frac{23}{3}$ $\left(\frac{(3 \times 7) + 2}{3}\right)$

Decimals

19. *Multiply:*

47.3

6.8

Remember to place decimal point in product. Check by reversing.

20. *Multiply:*

92.18

42.6

First Use of Percentage

21. 4% of 18

22. 125% of 83

23. 25% of 104

"Of" means $\times$. Change per cent to decimal form. Use equivalent fractions when you can.

Denominate Numbers

24. *Multiply:*

4 yd. 2 ft.

6

Remember to reduce answers. Tables on page 470 should be mastered.

25. *Multiply:*

6 bu. 2 pk.

5

Finding Areas

Dimensions must be in same unit. Areas are expressed in square units. Tables on page 470 should be mastered. Formulas for finding areas should be memorized.

26. Find the area of a rectangle whose dimensions are 11 in. by 2 ft. 3 in.

27. Find the area of Figure 1.

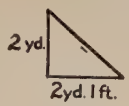


FIG. 1

28. Find the area of Figure 2.

FIG. 2

Finding Volumes

Dimensions must be in same unit. Volumes are expressed in cubic units.

Tables on p. 470 should be mastered. Formulas for finding volumes should be memorized.

29. Volume of Fig. 3 = ?

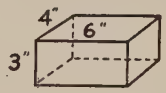


FIG. 3

Words Used in Multiplication

436 ← *multiplicand*
28 ← *multiplier*
3488
872
12208 ← *product*

USES OF DIVISION

Whole Numbers

30. $24\overline{)14428}$

31. $39\overline{)15834}$

Estimate each quotient figure first.

Watch zeros in quotient.

Remember to use the remainders in the quotient in correct form.

Express remainders in fraction form, reduced to lowest terms. or in decimal form.

Check by multiplying quotient and divisor and adding remainder.

Reducing Fractions

$\frac{10}{15} = \frac{2}{3}$ $\left(\frac{10 \div 5}{15 \div 5} = \frac{2}{3}\right)$

$\frac{17}{5} = 3\frac{2}{5}$ $\left(\begin{array}{r} 3\frac{2}{5} \\ 5\overline{)17} \end{array}\right)$

$\frac{23.8}{145} = 145\overline{)23.80}$ .16 or 16%

Fractions and Mixed Numbers

Change mixed numbers to improper fractions.
Invert divisor and multiply.
Cancel after inverting if possible.
Reduce answers to lowest terms.

32. $\frac{2}{3} \div \frac{1}{2}$

33. $2\frac{1}{2} \div 7$

34. $4\frac{2}{3} \div 2\frac{1}{6}$

Decimals

35. $24.7\overline{)2.7911}$

Shift points in dividend and divisor before starting. Place decimal point in quotient. Fill in vacant places with zeros. Check by multiplying.

36. $.38\overline{)7714}$

Second Use of Percentage

37. 17 is what % of 85?

38. What % of 284 is 63?

39. 93 is what % of 31?

40. What % of 33 is 14.3?

"What per cent of" means "what fraction of." Thus, 24% of 98 can be thought: $\frac{24}{98}$ or $98\overline{)24}$.

Change answer from decimal to per cent form.

Check by multiplying. Be sure the answer is reasonable.

Third Use of Percentage

41. If 36 is 18% of N, $N = ?$
42. N = what, when 140% of $N = 183$?
43. If 47 is 23% of N, $N = ?$
44. 139 is 17% of what number?

Hunt for 100% with the given per cent in decimal form as a divisor.

Check by multiplying. Be sure the answer is reasonable.

Denominate Numbers

45. $4\overline{)11 \text{ pk. } 3 \text{ qt.}}$

46. $8\overline{)26 \text{ hr. } 40 \text{ min.}}$

Carry the correct number. Tables on page 470 should be memorized.

Check by multiplying.

Words Used in Division

$$\begin{array}{r} 11\frac{1}{24} \\ 24\overline{)265} \\ 24 \\ \hline 25 \\ 24 \\ \hline 1 \end{array}$$

265 is the *dividend*
24 is the *divisor*
1 is the *remainder*
$11\frac{1}{24}$ is the *quotient*

CHAPTER VII

FINDING INTEREST BY THE FORMULA AND BY USE OF THE INTEREST TABLE. AMOUNT. PROMISSORY NOTES. BANK DISCOUNTS

INTEREST

140. Main Ideas of Interest. (SILENT READING.) Below are 10 questions about interest. Read them, and then study the rest of the article so that you will be able to answer them when you have finished your study.

1. Why is interest paid on loans?

2. What is the difference between just keeping money and investing it?

3. To what unit of time does the rate of interest refer?

4. Is one year the only length of time for which money can be borrowed?

5. What are some common lengths of time for loaning money?

6. When investing money, what fact beside the rate of interest is always very important?

7. Why does the rate of interest one must pay for borrowed money sometimes vary?

8. What rates of interest are usually paid by savings banks?

9. What is meant by a security for a loan?

10. Why is it often unwise for a small investor to place his money at more than 6%?

Interest is the money paid for the use of money, or the money earned by money which is loaned. We may think of money as working for its owner.

A bank will pay you interest on savings deposits under certain conditions, because it can use your money and is therefore able to pay you for its use.

A man is often willing to borrow money at interest, for example, to help buy a house. He needs a house to live in, and willingly borrows money and pays for its use (interest), just as he would pay for the use of a carpenter's time to help build the house.

A man may have saved $1000. If he puts the $1000 in a safe place, he will have his money, but most people are not satisfied with just keeping money. They want that money to make more money. The wise man therefore **invests** the $1000 so that it will earn interest for him.

An important fact about interest is that one must pay for the use of borrowed money. He must pay interest on it. If one owns money and lets someone else use it, his money earns interest for him. The **borrower** pays **interest** to the owner for the use of the **loan.**

A second important fact about interest is that it is reckoned (computed) **by the year.** That is, we reckon the amount of interest to be paid one year at a time. Thus, "4% interest" means that 4% of the amount of money loaned, or borrowed, must be paid for its use **for a year.** If the loan is for six months at 4%, the interest would be paid at the **yearly rate** of 4%, but for a half-year only. This would be the same as 2% for a year. If we borrowed money for two years at 6%, we would pay six per cent of the amount each year. For the 2 years this would be the same as 12% for one year. Many loans are made for

a year, but interest often has to be figured for other lengths of time. Business men often borrow money for 60 days (2 months), for 90 days (3 months), or for 6 months. Large loans are often made for periods longer than a year. One might borrow money to use in building a house and agree to keep the loan for 2 years, or 5 years, or even longer. It is important to remember two things about the time of interest. These are: (1) interest is reckoned by the year, or **per annum;** (2) money is often borrowed or loaned for times less than a year, or more than a year, but the interest is stated and reckoned by the year.

A third thing to keep in mind about interest is the **usual rates** of interest. **Usual rates** of interest paid by savings banks are 3%, $3\frac{1}{2}$%, or 4%. Some may think these pretty low rates, especially when they often see advertisements of investments which promise to pay 10%, or more. Of course, everyone would rather have his money earn him 10% than 3%, but the rate of interest is **only one thing** to consider in lending money. When one lends money, he must consider the **safety** of the money itself. Many investments are not good investments, because the business which borrowed the money does not prosper and cannot pay back the loan. It is better to get 3% on one's money and have it safely invested than to expect 10 or 20% and not only fail to get the interest, but also fail to get back the money which was loaned (the principal).

One must usually pay more than 3% interest on money he borrows. Usual rates of interest which must be paid for borrowed money are 6%, $6\frac{1}{2}$%, 7%, and 8%. The rate of interest depends a good deal upon the length of time of the loan. Usually the longer the loan the lower the rate of interest charged. More important, however,

than the length of time, is the **security** one can offer. You will see how the rate of interest may vary with the **security** back of the loan by reading the following dialogues:

First Dialogue: Mr. Jones (to bank official): I want to borrow $5000 for 2 years. What rate of interest will you charge me?

Bank Official: What **security** can you put back of this loan?

Mr. Jones: What do you mean by security?

Bank Official: A **security** is some property which you will agree to give us in case you do not pay back the loan. If we lend you $5000 and you agree to give us your house, which is worth more than $10,000, in case you cannot pay back the loan, then you would be giving us a good security. We would then feel so safe about the loan that we could let you have it for 6%.

But if you had nothing as a security except your own promise to pay, or if you offered as security some property which we might not be able to sell for $5000 in case you could not pay back the money, you would be offering poor security.

Mr. Jones: Well, I have a house worth more than $10,000 which I will put up as security for the $5000 loan.

Bank Official: Then we will be glad to make the loan at 6%.

Second Dialogue: Mr. Easy Promise (to Mr. Everybody): I understand that you have saved $1000.

Mr. Everybody: Yes, that is so.

Mr. Easy Promise: Well, of course you want to invest it in some business that will pay you good interest. Big business men often make 15% on their money. Why shouldn't you? You know as much about business as they do.

Mr. Everybody: I would like to make 15%, but I must have a safe investment. I cannot save very much money and cannot afford to lose any that I do save.

Mr. Easy Promise: Let me tell you confidentially that I know of an investment which is bound to pay you 15%, or more. I'll get you some stock in this investment. Just sign this order blank. (Holding out a fountain pen to Mr. Everybody.)

MR. EVERYBODY: Just a minute, please. Tell me about the success of this business. It seems to me that if the owners of this business were sure they could earn 15%, they would not come to me for money. They would keep such a good thing for themselves. What do the banks think of this company? Is any bank investing some of its money in this company?

MR. EASY PROMISE: Now don't you worry about the security. You know me. I would not offer this investment to any bank. I want you to get the advantage of it.

MR. EVERYBODY: The business men I have learned to trust tell me that small investors are risking their money too much when they try to get more than 6% interest. This 15% promise without any security makes me a little suspicious. I have been told that thousands of people lose their money by trying to get too high rates of interest. I think I'll be content with less interest and more safety. Thank you for calling. Good day.

Do you think Mr. Everybody was wise in refusing to buy stock of Mr. Easy Promise? Why?

It is always more important to invest money **safely** than to earn a high rate of interest. You will make no mistake if you follow this advice given below:

> Invest money only when officials of a good bank recommend the investment. Be very suspicious of investments which pay over 6%. Higher rates of interest very often involve too much risk. Money deposited in a good bank at 3 or $3\frac{1}{2}\%$ is better than money invested in some "wildcat" scheme and lost.

The following are important ideas about interest:

1. *Interest is money paid for the use of money.*

(*a*) If you lend or invest money, it pays you interest.

(*b*) If you borrow money, you must pay interest for its use.

2. *Interest rates are stated by the year.*

(*a*) 4% interest means 4% every year.

(*b*) Money may be loaned or borrowed for any length of time.

(*c*) 3 mo., 6 mo., 1 year, 2 years, etc., are common lengths of time for borrowing and lending money.

(*d*) In figuring interest, a month is regarded as 30 da. and a year as 360 days.

3. *Interest rates vary.*

(*a*) When borrowing money, the better the security the lower the rate of interest.

(*b*) In investing money, the greater the safety of the investment the less the interest paid.

(*c*) In investing your own money a good rule to follow is: "Be content with a low rate of interest (not over 6%), but be sure of the safety of the investment."

SELF-TESTING DRILL NO. 19

141. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Try to beat your past records.**

1. *Subtract:*	**2.** *Subtract:*	**3.** *Divide:*	**4.** *Subtract:*
9942385	$43\frac{3}{4}$	$2\frac{3}{4}$ by $1\frac{5}{6}$	97699.263
9565791	$28\frac{1}{3}$		87248.489

5.
9.78
77.97
9.68
47.90
6.81
78.70

6. $857\overline{)765301}$

7. Mary thought the volume of a rectangular solid whose dimensions were $4\frac{1}{3}$ yd., 5 yd., and 9 yd., was 185 cu. yd. Was she right or wrong?

8. Find the interest on \$9300 @ $7\frac{1}{2}$% for 2 yr.

9. What per cent of 150 is 129?

10. *Multiply:*

95.14
45.19

11. 150% of 2440 equals what number?

12. What must be added to 2 pk. 3 qt. 1 pt. to equal 3 pk.?

13.* What should be on each blank in the table below?

l	*w*	*h*	*V*
2 in.	6 in.	8 in.	(*a*).....
(*b*).....	7 ft.	5 ft.	105 cu. ft.
4 yd.	(*c*).....	9 yd.	288 cu. yd.
1 ft.	1 ft.	1 ft.	(*d*).....

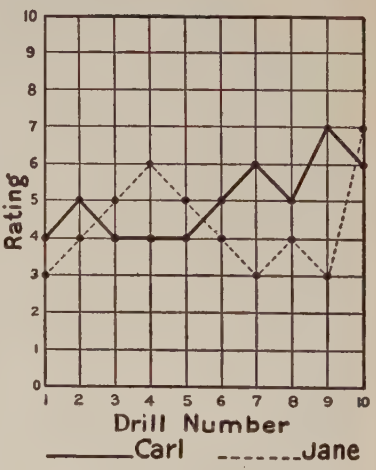
14.* Answer the following questions from the graph:

(*a*) Which pupil rated higher on Drill 8?

(*b*) Were both pupils' ratings ever equal on any drill shown?

(*c*) Which pupil did the better work on the average on Drills 1 to 10?

(*d*) Which pupil showed the more regular improvement?



15. 4 gal. 3 qt. 1 pt. ÷ 6 = ?

*To count Exs. 13 and 14 correct, each part must be correct.

STANDARDS

Number correct.....	0–1	2–3	4	*	5	6	7	8	9	10–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

142. Mixed Practice. (WRITTEN WORK.) This practice will help you in the new work to be studied in Art. 143.

I. We know that per cents may be written in decimal form. Thus 6% of N = .06 of N. $4\frac{1}{2}\%$ of N = .045 of N. $3\frac{3}{4}\%$ of N = .0375 of N. 4.4% of N = .044 of N.

What decimal should be on each blank?

1. 5% of N = ... of N
2. 45% of N = ... of N
3. 150% of N = ... of N
4. 6.25% of N = ... of N
5. $6\frac{1}{4}\%$ of N = ... of N
6. 6.4% of N = ... of N
7. 3.5% of N = ... of N
8. $3\frac{1}{2}\%$ of N = ... of N
9. 0.5% of N = ... of N
10. $\frac{1}{2}$ of 1% of N = ... of N
11. 125% of N = ... of N
12. 4.2% of N = ... of N
13. 3.7% of N = ... of N
14. 1% of N = ... of N
15. $\frac{1}{4}$ of 1% of N = ... of N
16. $\frac{3}{5}$ of 15% of N = ... of N

II. In finding interest, a month of 30 days is counted as $\frac{1}{12}$ of a year, and a year is counted as 360 days. Thus, 6 mo. = $\frac{6}{12}$ or $\frac{1}{2}$ yr. 15 mo. = $1\frac{1}{4}$ or $\frac{5}{4}$ yr. 18 da. = $\frac{18}{360}$ or $\frac{1}{20}$ of a year. 1 yr. 6 mo. = $\frac{3}{2}$ yr.

Change to fractions of an interest year.

1. 4 mo. = ... yr.
2. 8 mo. = ... yr.
3. 3 mo. = ... yr.
4. 9 mo. = ... yr.
5. 5 mo. = ... yr.
6. 30 da. = ... yr.
7. 2 yr. 6 mo. = ... yr.
8. 12 da. = ... yr.
9. 60 da. = ... yr.
10. 1 yr. 4 mo. = ... yr.
11. 90 da. = ... yr.
12. 18 mo. = ... yr.
13. 20 da. = ... yr.
14. 2 yr. 9 mo. = ... yr.

III. Whole numbers, decimal fractions, and common fractions can be multiplied. Study the next two samples.

SAMPLE A. Find the interest on \$110 at 6% for 15 months. $\$110 \times .06 \times 1\frac{1}{4} = \$110 \times .06 \times \frac{5}{4} = ?$

$$\begin{array}{r} \$110 \\ \times .06 \\ \hline 6.60 \\ \times 5 \\ \hline 4)\$33.00 \\ \hline \$8.25, \textbf{ Ans.} \end{array}$$

SAMPLE B. What is the interest on \$650 for 1 yr. 4 mo. at $6\frac{1}{2}\%$? $\$650 \times .065 \times \frac{4}{3} = ?$

$$\begin{array}{r} \$\,650 \\ \times .065 \quad (1) \\ \hline 3250 \\ 3900 \\ \hline \$42.250 \quad (2) \\ \times 4 \quad (3) \\ \hline 3)169.00 \quad (4) \\ \hline \$56.33 \end{array}$$

Find (1) in Sample B. Why is \$650 multiplied by .065?

Find (2) in Sample B. Why are there 3 decimal places in \$42.250? Can the zero in \$42.250 be dropped?

Find (3) in Sample B. Why is \$42.25 multiplied by 4?

Find (4). Why is \$169 divided by 3?

Find (5). Why is the answer \$56.33 instead of $\$56.33\frac{1}{3}$?

Find the products in the examples below. Cancel when you can. Thus, in Example 6, the 4 in $\frac{5}{4}$ can be canceled into the 2400.

1. $\$650 \times .04 \times 2$
2. $\$2000 \times .03 \times 1$
3. $\$975 \times .05 \times 3$
4. $\$64 \times .06 \times 4$
5. $\$150 \times .0425 \times 1$
6. $\$2400 \times .035 \times \frac{5}{4}$
7. $\$3500 \times .075 \times \frac{2}{3}$
8. $\$1750 \times .045 \times \frac{3}{2}$
9. $\$690 \times .07 \times \frac{10}{3}$
10. $\$840 \times .055 \times \frac{11}{4}$

COMPUTING INTEREST

143. The Interest Formula. (STUDY PERIOD.) If you understand the three uses of percentage, you already know how to do most of the arithmetic work having to do with interest. Study the three samples below. Sample 1 is finding interest (finding a per cent of a number). Sample 2 asks for the rate of interest (finding what per cent one number is of another). Sample 3 gives the interest and the rate and asks you to find the amount invested (finding the whole or 100% when a part is given).

You have already learned much about formulas. There is also a formula for finding interest, which you can always use as your rule and from which you can make other formulas for working with interest.

SAMPLE 1

Find the interest on \$600, for 2 years at 6%.

\$ 600
.06
\$36.00←1 year @ 6%.
2
\$72.00←2 years @ 6%.

SAMPLE 2

An investment of \$600 paid me \$36 interest a year. At what rate was the \$600 invested?

$\frac{36}{600}$ or $600\overline{)36.00}$ .06 = 6%
36.00

Rate per cent was 6, or the rate is 6%.

SAMPLE 3

An investment at the rate of 6% paid me \$36 per year. How much have I invested?

$.06\overline{)36.00}$ \$ 600.
36

I have \$600 invested.

Interest formula: $i = p \times r \times t$, or, more simply, $i = prt$

i means **interest** in money.

p means **principal** or the money on which the interest is reckoned.

r means the **rate** per cent of interest.

t means **time** in years.

Study the next samples:

SAMPLE 1. Find the interest on a loan of \$375 at $6\frac{1}{2}$ per cent for 3 yr. 6 mo.

Sample 1 is solved by the formula $i = prt$ as follows:

i = interest to be found.

p = \$375, the loan on which interest is figured.

r = .065, the rate per cent, $6\frac{1}{2}$, written **in decimal form.**

$t = \frac{7}{2}$ (3 yr. 6 mo. $= 3\frac{1}{2}$ yr. $= \frac{7}{2}$ yr.)

$i = \$375. \times .065 \times \frac{7}{2}$ $\quad \$375. \times .065 = \$24.375$

$\$24.375 \times 7 = \$170.625$ or \$170.63,

$\$170.63 \div 2 = \$85.31\frac{1}{2}$ or \$85.32, *Ans.*

SAMPLE 2. What is earned on an investment of \$1500 invested at 7% for 4 years?

Sample 2 is solved by the formula as follows:

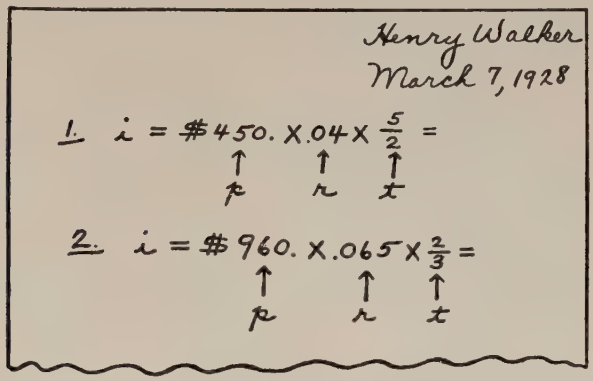
$i = \$1500 \times .07 \times 4 = \$420.00$, the interest earned.

$i = p \times r \times t$

Work Example 2 to see if \$420.00 is the correct answer.

144. Practice in Using the Formula. (WRITTEN WORK.) Do not work the examples now, but write each example so that it will show how the formula would be used. Take

time to show for each example which number is p, which is r, and which is t, as shown for the first two examples of Henry Walker's paper.



1. Find the interest on \$450 @ a rate per cent of 4 for 2 yr. 6 mo.

2. What is earned on an investment of \$960 for 8 mo. which yields (pays) $6\frac{1}{2}\%$?

3. Compute the interest on \$800 @ 7% for 3 yr. 9 mo.

4. Find the interest on \$1400 @ $4\frac{1}{2}\%$ for 1 yr. 3 mo.

5. Find the interest on \$2500 @ $5\frac{1}{2}\%$ for 30 da. (Count 30 da. as 1 mo. = $\frac{1}{12}$ yr.)

6. I borrowed \$500 @ 9% with the privilege of paying it back as soon as I could. I paid it back in 17 days. Find the interest due.

(In computing interest, 1 da. = $\frac{1}{360}$ yr. 17 da. = $\frac{17}{360}$.)

7. A clerk had to figure interest from the following facts: The time was 2 yr. 10 mo. The rate was $7\frac{1}{2}\%$. The principal was \$900.

8. I agree to pay the interest on a loan of \$1800 @ 6% every 6 mo. How much interest must I pay every 6 mo.?

9. Interest is payable quarterly (every 3 months) on a certain investment of \$1000 @ 6.4% (.064). How much interest is paid quarterly?

AN EXERCISE IN INDEPENDENT WORK

The answers to the above 9 examples are given below. These answers are not in order. Work the 9 examples and see if you can match each of the 9 answers below. Use the formula $i = prt$. (In computing interest, count .5 or more than .5 of a cent as 1 cent and drop fractions less than .5 of a cent. Thus, \$5.655 becomes \$5.66, but \$1.333 is called \$1.33.)

Answers (not in order)

\$210.	\$78.75	\$41.60	\$45.	\$16.
\$2.13	\$11.46	\$191.25	\$54.	

➡If your answers do not agree with those printed, one of two things could be done:

1. Change numbers to make answers come out right. This is a waste of time, and **cheats only yourself.**

2. Find mistakes and keep at work until your answers match those above perfectly.

AMOUNT

145. Two Meanings of the Word "Amount." (Class Work.) It often happens that a word has more than one meaning. A good example of this is the word "amount." We speak of the "amount" of candy we would like to buy, the "amount" of time it takes to walk to school, or the "amount" of money which a man owes.

In arithmetic work dealing with money, and especially in problems of interest, the word "amount" is used in two different meanings. The commoner or more general meaning of the word is already known to you. The less common or special meaning of "amount" occurs most often in working with interest.

In dealing with the interest formula, $i = p \times r \times t$, you sometimes have to find the interest and sometimes the sum of the interest and the principal. This is true because in repaying a loan the money due is the sum of the interest and the principal. It would be convenient to have a word to express "sum of the interest and principal." For such a word we use **amount**.

> **In interest problems, *amount* means the sum of the interest and the principal.**

There is no certain way of knowing which of the two meanings of "amount" is meant when the two meanings are used carelessly. For this reason you should use the word "amount" in connection with interest in its special sense of $i+p$ **(interest plus principal)**. Look at the problem below.

PROBLEM: A man borrowed \$200 for one year at 6% interest on January 1, 1927. What was the amount of the loan on January 1, 1928?

Here it is clear that the "amount" is the sum of the principal and the interest, or \$200 plus $.06 \times \$200 \times 1$, or \$212.

By careful reading you should never have any difficulty with the examples and problems of this book in spite of the fact that the word "amount" is used in both a general and a special meaning. Likewise you should not be

careless in your use of the word. One seventh-grade pupil, when asked why he was so careful to use arithmetic terms correctly, said: "I try to use words correctly so that other persons will know exactly what I am talking about." Do you think that he was right?

Watch out for the two meanings of "amount" for the next few weeks and you will never be troubled with this matter again!

Amount = principal plus interest
$a = p + prt$, or, more simply, $a = p + i$

146. Problems in Finding Interest and Amounts. (WRITTEN WORK.) In solving the problems in this article use these two formulas: $i = prt$. $a = p + i$.

➠Do not leave a problem until you answer this question: "Is my answer a **reasonable one** and does it **answer the question** which is asked?"

1. What is the **amount** to be paid on a loan of \$600 @ 6% for 2 yr. 4 mo.?

2. Mr. Wagner borrowed \$1500 at the bank for 60 days @ $7\frac{1}{2}\%$. He paid back the principal with interest as agreed. What was the amount he paid back?

3. The following is a part of a letter:

> Things have gone very well with me. I am returning your loan of \$800 which you made me 4 months ago. I am adding 7% interest. This makes the amount.....
> I want to thank you for the help.

What should be on the blank in the letter?

4. A business man needed money to buy his spring stock of goods. He borrowed from his bank $3800 for 90 da. @ 7%. He paid his loan with interest as agreed. What amount did he pay the bank?

5. Read the conversation below. How much interest will Mrs. Wellar have?

MRS. WELLAR: July 1 is the semi-annual interest day at the Home Savings Bank. I must go down and have my interest entered in my pass-book.

MR. WELLAR: How much interest will you have?

MRS. WELLAR: I have had $454 in my account since January 1. The rate is 4%. I have six months' interest due. Let's see, the interest will be.....

6. Read what Mr. and Mrs. Kent said; then answer the two questions below.

MR. KENT: Business has been good and I have some extra cash. I think I'll pay that loan Jim Bywater made me.

MRS. KENT: How much did you borrow?

MR. KENT: $650 at $7\frac{1}{2}\%$.

MRS. KENT: How long have you had the loan?

MR. KENT: For 15 months.

MRS. KENT: Let's both figure the interest so that we will be sure of the amount that we must pay.

Mr. Kent figured the interest at $56.40. Mrs. Kent figured the interest at $60.94.

(*a*) Which figured the interest correctly? (*b*) What was the amount?

7. James Wiley borrowed $900 for 9 mo. @ $7\frac{1}{2}\%$. He thought the amount to be paid back was $967.50; but when he paid back his loan, the bank said that only $950.63 was due. Was Mr. Wiley or the bank correct as to the amount?

8. "I need \$500 for 30 days," said Mr. Dyke to the bank official. "How much will the loan cost me?" The official said, "On short loans we will have to charge you 8%." How much will the \$500 for 30 days at 8% cost Mr. Dyke?

147. Rapid Drill. (ORAL WORK.) Exercises such as this one will give you practice in working without pencil. In division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	*Add:* 5, 0, 3, 3	153 −69	$\frac{1}{6}$ of N = ...% of N	89 ×7	$\times 1\frac{1}{4}$ = ×....%	89)266
2.	97 ×9	74)149	*Add:* 7, 7, 5, 6	131 −46	48 ×6	$2\frac{7}{8} = \frac{?}{8}$
3.	9 bu. = ...pk.	96 −68	76 ×5	72)284	*Add:* 2, 8, 6, 9	96 ×2
4.	.045 = ...%	*Add:* 9, 5, 4, 6	798 ×1	$\frac{5}{6} = \frac{?}{18}$	*Reduce:* $\frac{22}{8}$ =	151 −57
5.	51 −4	21)128	46 ×3	*Add:* 5, 4, 4, 9	89 ×5	$a = p + i$ means what?
6.	$37\frac{1}{2}\%$ = what decimal?	46 ×7	69 −0	63 ×8	27)95	*Add:* 7, 6, 7, 2
7.	What is a check stub for?	133 −54	Meaning of $i = prt$?	121 −85	86 ×9	$A = \pi r^2$ means what?

SELF-TESTING DRILL NO. 20

148. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. **What does the line on your Progress Chart tell about your work?**

1.	2. *Subtract:*	4.	5. *Add:*
135	832653.2	$6\frac{2}{9} \times 3\frac{3}{8} = ?$	4 pk. 7 qt.
554	129714.6		9 pk.
493			3 pk. 1 qt.
202	3. *Subtract:*		2 pk. 5 qt.
126	8125413		6 pk. 3 qt.
247	1239457		

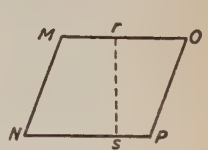
6. What is the sum of three-fourths, five-sixths, two, and one-half?

7.	8. *Multiply:*	9. *Multiply:*
$3.59\overline{)306586}$	4859	2 gal. 1 qt. 1 pt.
	378	4

10. What per cent of 1700 is 289?

11. Find the area of the figure at the right.

($MO = NP = 2\frac{1}{3}$ ft. $rs = 1\frac{5}{6}$ ft.)



12. $\frac{1}{2}\%$ of a certain amount of money equals \$1.34. What is 100% of the amount?

13. Is 964×4.99 about: 450,000, 45,000, or 4500?

14. An investment of \$175 for 2 yr. 2 mo. at 3% will earn how much money?

Go on to the next page.

15. *Write what should be on the blanks in the formula for this example. Do not solve.*

Find the interest on \$500 @ 6% for 3 years.

$i =$ ..... $\times$ ..... $\times$ .....

STANDARDS

Number correct	0–1	2–3	4	*	5	6	7	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

PROBLEM SCALE NO. X

149. Mixed Problem Practice. (WRITTEN WORK.) You can earn 65 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III. **A good question** to ask yourself in each problem is: "What does this problem ask for?"

1. The daily numbers of people who used the Municipal Bath House at the city swimming beach during one week last summer were as follows: Monday, 1308; Tuesday, 342; Wednesday, 784; Thursday, 683; Friday, 106; Saturday, 1380; Sunday, 1298. Find the total for the week. (4)

2. The yearly report of a great manufacturing company shows that the net income for 1927 was \$86,622,752. From this income \$60,305,204 was paid to those who owned stock in the company. The rest was kept in the treasury as a "balance." How much was the balance? (4)

3. Albert borrowed \$2500 from his grandmother to make a first payment on a farm. He agreed to pay 7% interest and to make semi-annual payments of the interest. How much should Albert pay his grandmother every six months? (5)

4. The Fourth Liberty Loan bonds pay $4\frac{1}{4}\%$ interest. Mary has 3 hundred-dollar bonds and 3 fifty-dollar bonds of the Fourth Liberty Loan. How much interest does she receive each year from these bonds? (6)

Use the data (the facts) below in working the next problems.

A dairyman's record for one of his cows for the month of November was as shown in the table below.

	Morning Milkings	Evening Milkings
1st week	94.4 lb.	80.8 lb.
2d "	117.2 "	96.0 "
3d "	107.6 "	99.6 "
4th "	125.4 "	94.6 "
29th and 30th days	35.6 "	32.8 "

5. What was the total weight of milk yielded for the month? (5)

6. The amount of milk from evening milkings was what per cent of the amount from morning milkings the first week? (6)

7. The yield from evening milkings was what per cent of the yield from morning milkings figured on a monthly basis? (8)

8. Judging from the yield of milk for the 29th and 30th days, how much milk would you expect the cow to give per week? (8)

9. Counting 8.6 lb. to the gallon, how many quarts of milk did the cow give during the month? (9)

10. At 9.35¢ a quart, what did this cow earn for the owner during the month? (10)

STANDARDS

Superior work:	21–65 points	Slow work:	6–10 points
Good work:	16–20 points	Inferior work:	0–5 points
Average work:	11–15 points		

THE INTEREST TABLE

150. The Interest Table. (Class Work.) Interest is computed in two ways:

(*a*) By the formula, $i = prt$.

(*b*) By means of the **interest table** or **interest book** (a collection of interest tables).

Banks and business men who have to do a great deal of computing of interest find that it saves time and increases accuracy to use an interest table.

On pages 263-265 you will find three pages from an interest table for principals of \$10, \$100, and \$1000. Notice that a full page is given to each of the principals. The time is shown in the column at the left, and the rates of interest are given across the top of the table.

The principals are usually shown dollar by dollar from \$1 to \$100; thus, for \$1, \$2, \$3, etc.; and for exact hundreds of dollars from \$100 to \$1000. From \$1000 to \$10,000 the principals are shown for exact thousands of dollars.

Study these three tables until you are sure that you understand how they have been arranged. Then study the three examples which are worked on pages 266 and 267 to make plain the use of the tables.

The body of the table gives the interest at first in cents and then in dollars and cents, as shown by the decimal point.

10 DOLLARS

TIME.	5%	6%	7%	8%	9%	10%
Days 1	0	0	0	0	0	0
" 2	0	0	0	0	1	1
" 3	0	1	1	1	1	1
" 4	1	1	1	1	1	1
" 5	1	1	1	1	1	1
" 6	1	1	1	1	2	2
" 7	1	1	1	2	2	2
" 8	1	1	2	2	2	2
" 9	1	2	2	2	2	3
" 10	1	2	2	2	3	3
" 11	2	2	2	2	3	3
" 12	2	2	2	3	3	3
" 13	2	2	3	3	3	4
" 14	2	2	3	3	4	4
" 15	2	3	3	3	4	4
" 16	2	3	3	4	4	4
" 17	2	3	3	4	4	5
" 18	3	3	4	4	5	5
" 19	3	3	4	4	5	5
" 20	3	3	4	4	5	6
" 21	3	4	4	5	5	6
" 22	3	4	4	5	6	6
" 23	3	4	4	5	6	6
" 24	3	4	5	5	6	7
" 25	3	4	5	6	6	7
" 26	4	4	5	6	7	7
" 27	4	5	5	6	7	8
" 28	4	5	5	6	7	8
" 29	4	5	6	6	7	8
Mo. 1	4	5	6	7	8	8
" 2	8	10	12	13	15	17
" 3	13	15	18	20	23	25
" 4	17	20	23	27	30	33
" 5	21	25	29	33	38	42
" 6	25	30	35	40	45	50
" 7	29	35	41	47	53	58
" 8	33	40	47	53	60	67
" 9	38	45	53	60	68	75
" 10	42	50	58	67	75	83
" 11	46	55	64	73	83	92
Yr. 1	50	60	70	80	90	1.00
" 2	1.00	1.20	1.40	1.60	1.80	2.00
" 3	1.50	1.80	2.10	2.40	2.70	3.00
" 4	2.00	2.40	2.80	3.20	3.60	4.00

100 DOLLARS

TIME.	5%	6%	7%	8%	9%	10%
Days 1	1	2	2	2	3	3
" 2	3	3	4	4	5	6
" 3	4	5	6	7	8	8
" 4	6	7	8	9	10	11
" 5	7	8	10	11	13	14
" 6	8	10	12	13	15	17
" 7	10	12	14	16	18	19
" 8	11	13	16	18	20	22
" 9	13	15	18	20	23	25
" 10	14	17	19	22	25	28
" 11	15	18	21	24	28	31
" 12	17	20	23	27	30	33
" 13	18	22	25	29	33	36
" 14	19	23	27	31	35	39
" 15	21	25	29	33	38	42
" 16	22	27	31	36	40	44
" 17	24	28	33	38	43	47
" 18	25	30	35	40	45	50
" 19	26	32	37	42	48	53
" 20	28	33	39	44	50	56
" 21	29	35	41	47	53	58
" 22	31	37	43	49	55	61
" 23	32	38	45	51	58	64
" 24	33	40	47	53	60	67
" 25	35	42	49	56	63	69
" 26	36	43	51	58	65	72
" 27	38	45	53	60	68	75
" 28	39	47	54	62	70	78
" 29	40	48	56	64	73	81
Mo. 1	42	50	58	67	75	83
" 2	83	1.00	1.17	1.33	1.50	1.67
" 3	1.25	1.50	1.75	2.00	2.25	2.50
" 4	1.67	2.00	2.33	2.67	3.00	3.33
" 5	2.08	2.50	2.92	3.33	3.75	4.17
" 6	2.50	3.00	3.50	4.00	4.50	5.00
" 7	2.92	3.50	4.08	4.67	5.25	5.83
" 8	3.33	4.00	4.67	5.33	6.00	6.67
" 9	3.75	4.50	5.25	6.00	6.75	7.50
" 10	4.17	5.00	5.83	6.67	7.50	8.33
" 11	4.58	5.50	6.42	7.33	8.25	9.17
Yr. 1	5.00	6.00	7.00	8.00	9.00	10.00
" 2	10.00	12.00	14.00	16.00	18.00	20.00
" 3	15.00	18.00	21.00	24.00	27.00	30.00
" 4	20.00	24.00	28.00	32.00	36.00	40.00

1000 DOLLARS

TIME.	5%	6%	7%	8%	9%	10%
Days 1	14	17	19	22	25	28
" 2	28	33	39	44	50	56
" 3	42	50	58	67	75	83
" 4	56	67	78	89	1.00	1.11
" 5	69	83	97	1.11	1.25	1.39
" 6	83	1.00	1.17	1.33	1.50	1.67
" 7	97	1.17	1.36	1.56	1.75	1.94
" 8	1.11	1.33	1.56	1.78	2.00	2.22
" 9	1.25	1.50	1.75	2.00	2.25	2.50
" 10	1.39	1.67	1.94	2.22	2.50	2.78
" 11	1.53	1.83	2.14	2.44	2.75	3.06
" 12	1.67	2.00	2.33	2.67	3.00	3.33
" 13	1.81	2.17	2.53	2.89	3.25	3.61
" 14	1.94	2.33	2.72	3.11	3.50	3.89
" 15	2.08	2.50	2.92	3.33	3.75	4.17
" 16	2.22	2.67	3.11	3.56	4.00	4.44
" 17	2.36	2.83	3.31	3.78	4.25	4.72
" 18	2.50	3.00	3.50	4.00	4.50	5.00
" 19	2.64	3.17	3.69	4.22	4.75	5.28
" 20	2.78	3.33	3.89	4.44	5.00	5.56
" 21	2.92	3.50	4.08	4.67	5.25	5.83
" 22	3.06	3.67	4.28	4.89	5.50	6.11
" 23	3.19	3.83	4.47	5.11	5.75	6.39
" 24	3.33	4.00	4.67	5.33	6.00	6.67
" 25	3.47	4.17	4.86	5.56	6.25	6.94
" 26	3.61	4.33	5.06	5.78	6.50	7.22
" 27	3.75	4.50	5.25	6.00	6.75	7.50
" 28	3.89	4.67	5.44	6.22	7.00	7.78
" 29	4.03	4.83	5.64	6.44	7.25	8.06
Mo. 1	4.17	5.00	5.83	6.67	7.50	8.33
" 2	8.33	10.00	11.67	13.33	15.00	16.67
" 3	12.50	15.00	17.50	20.00	22.50	25.00
" 4	16.67	20.00	23.33	26.67	30.00	33.33
" 5	20.83	25.00	29.17	33.33	37.50	41.67
" 6	25.00	30.00	35.00	40.00	45.00	50.00
" 7	29.17	35.00	40.83	46.67	52.50	58.33
" 8	33.33	40.00	46.67	53.33	60.00	66.67
" 9	37.50	45.00	52.50	60.00	67.50	75.00
" 10	41.67	50.00	58.33	66.67	75.00	83.33
" 11	45.83	55.00	64.17	73.33	82.50	91.67
Yr. 1	50.00	60.00	70.00	80.00	90.00	100.00
" 2	100.00	120.00	140.00	160.00	180.00	200.00
" 3	150.00	180.00	210.00	240.00	270.00	300.00
" 4	200.00	240.00	280.00	320.00	360.00	400.00

COMPUTING INTEREST BY THE TABLES

EXAMPLE 1. Find the interest on $100 for 1 yr. 4 mo. 15 da. at 6%.

Interest on $100 for
- 15 da. $ .25
- 4 mo. 2.00
- 1 yr. 6.00

$8.25 *Ans.*

By looking down the 6% column in the "100 dollars" table, and reading opposite in the "Days, 15" line, the $.25 was found. By going down the same column and reading opposite in the "Mos., 4" line, the $2.00 was obtained. Opposite "1 year" in the same column is the $6.00. The total interest is the sum of the three "little" examples.

Study Example 1 until you understand what was done.

EXAMPLE 2. Find the interest on $1110.00 for $2\frac{1}{2}$ yr. at 5%.

Interest on $1000 for
- 6 mo. $ 25.00
- 2 yr. 100.00

(From $1000 table)

Interest on $100 for
- 6 mo. 2.50
- 2 yr. 10.00

(From $100 table)

Interest on $10 for
- 6 mo. .25
- 2 yr. 1.00

(From $10 table)

$138.75 *Ans.*

EXAMPLE 3. Find the interest on $101 for 3 yr. 4 mo. 10 da. at 8%.

Interest on $100 for
- 10 da. $ .22
- 4 mo. 2.67
- 3 yr. 24.00

Interest on $1 for
- 10 da. .0022 ($\frac{1}{100}$ of int. from $100 table)
- 4 mo. .0267 ($\frac{1}{100}$ of int. from $100 table)
- 3 yr. .24 ($\frac{1}{100}$ of int. from $100 table)

$27.1589 = $27.16 *Ans.*

Example 3 brings out a point which you should remember about finding interest by the use of tables. Sometimes you will need to compute interest upon a principal for which no table is provided directly. Such a principal is \$10.50, or \$63.75.

In the case of \$10.50, there is a table for \$10 in an interest book, but there is no table for 50¢. Likewise you can find a page in a complete interest book giving the interest for \$63, but there is no table giving interest on 75¢.

The following rules will work for such parts of principals as are not found directly in the tables:

RULES FOR USING INTEREST TABLES

NOTE: These rules apply to the tables in an interest book. The three sample tables printed in this textbook give you the main ideas about any interest book, but they are not sufficient to solve more than a few interest problems.

1. To find the interest on 50¢, look up the interest on \$50, and divide by 100 (move the decimal point two places to the left, and insert zeros, if necessary).

2. To find the interest on 75¢, look up the interest on \$75, and divide by 100 (move the decimal point two places to the left, and insert zeros, if necessary).

3. In general, to find the interest on any number of cents, look up the interest on that number of dollars, and divide by 100.

Appoint some pupil or a committee from the class to visit a banker and ask him to lend your class his interest tables for a day or two, so that you can see how the interest book looks.

151. Practice in Using the Interest Tables. (CLASS WORK.) Have paper and pencil ready. Some of these examples you can do "in your head," but with others you will need pencil and paper.

Use the "10 Table" on page 263 for Examples 1 to 5.

1. Find the interest on \$10 at 7% for 21 days.

2. Find the interest on \$10 at 8% for 7 mo.

3. \$10 at 5% for 2 years.

4. \$10 at 6% for 5 months, 27 days. (Remember to think of 5 months, 27 days as 5 months plus 27 days.)

5. \$15 at 9% for 16 days. (Remember to think of \$15 as \$10+\$5, or \$10+$\frac{1}{2}$ of \$10.)

Use the "100 Table" on page 264 for Examples 6 to 10.

6. Find the interest on \$100 for 30 days at 7%.

7. \$100 at 5% for 8 mo. 3 da.

8. \$100 for 18 mo. at 6%.

9. \$250 at 6% for 3 mo. (Think of \$250 as \$100+\$100+$\frac{1}{2}$ of \$100.)

10. Find the amount of a \$500 loan at 10% for 11 mo. ("Amount" has a special meaning in interest problems.)

Use the "1000 Table" on page 265 for Examples 11 to 15.

11. Find the interest on \$1000 at 5% for 18 mo.

12. \$1500 at 6% for 1 yr.

13. \$2300 at 7% for 2 yr.

14. What will be the interest on \$5000 for 1 yr. 2 mo. 17 da. at 8%?

15. Find the amount of a \$2000 loan at 7% for 4 yr. 7 mo. 10 da.

152. Problems in Computing Interest by the Tables. (WRITTEN WORK.) The problems in this article can all be worked by means of the tables and rules given in Art. 150. Some of them could be solved more quickly and easily if you had the complete set of tables for all the principals.

1. Find the interest on \$100 for 6 months at 6%.

2. Find the amount due on a loan of \$1000 at 5% for 2 yr. 9 mo. (Remember the special meaning of "amount.")

3. A merchant borrowed \$500 at 8% for ninety days. What was the interest charge?

4. A loan for \$300 ran for 3 yr. 5 mo. and 11 da. What was the amount if the rate was 7%?

5. Which of the following loans earned the more interest?

LOAN A: $p = \$100$, $r = 6\%$, $t = 1$ yr. 3 mo.
LOAN B: $p = \$1000$, $r = 5\%$, $t = 1$ mo. 20 da.

6. Do you see a way of finding interest at 4% by the use of the tables in Art. 150? Write as your answer the method you would use.

7. Give two ways in which you could find by the tables in Art. 150 the interest on \$500 for 2 yr. 6 mo. at 7%.

8. The tables given in Art. 150 do not show how to compute interest when the rate is $5\frac{1}{2}\%$, $6\frac{1}{2}\%$, $7\frac{1}{2}\%$, etc. See if you can find a way to reckon the interest for 1 year on \$110.00 at $5\frac{1}{2}\%$. (HINT: Suppose you knew the interest earned at 5% and at 6%. Would that help you?)

9. By the use of the table, find the interest on \$550 for 1 yr. 3 mo. and 5 da. at 8%.

10. Compute the interest on \$1000 for 2 yr. 9 mo. 25 da. if the rate is 4%.

11. A banker loans out \$2500 at $5\frac{1}{2}\%$ for one of his customers. The interest is paid every six months.

By the use of the tables find the interest on the customer's investment every six months.

12. Some private brokers (men who lend money) charge 12% interest on short-time loans, usually 30 to 60 days. Many states allow such high rates of interest on sums as small as \$100 and less. How could you figure the interest on \$100 for 45 days at 12% by the tables in Art. 150?

SELF-TESTING DRILL NO. 21

153. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*
1126326
634529

2. John said the sum of $3086+448+7535+647+366+375$ is 12,457. Was he right? If so, write a C for your answer. If not, give the correct answer.

3. $205\overline{)63345}$

4. $874\overline{)152076}$

5. What is 5% of 846?

6.
.7399
.0865
.0988
.9597
.0769
.3496

7. How much less is 3 gal. 3 qt. than 4 gal. 1 qt.?

8. $6\frac{1}{4} \div 3 = ?$

9. $45\frac{1}{3} - 37\frac{7}{12} = ?$

10. *Multiply:*
78.96
296

11. The three dimensions of a box are 5 in., 5 in., and 15 in. What is the volume of the box?

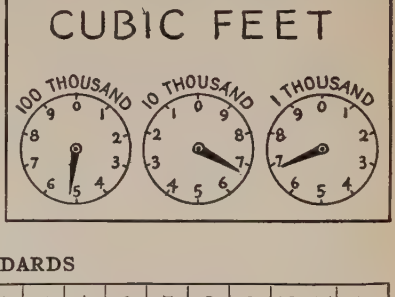
12. A discount of $12\frac{1}{2}\%$ came to \$126. Upon what amount was the discount figured?

13. When 336 = 100% of a number, 126 = what % of that number?

14.

$9\overline{)4 \text{ sq. mi. } 320 \text{ A.}}$

15. Read the gas meter shown at the right.

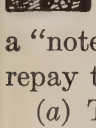


STANDARDS

Number correct	0–1	2–3	4	5	*	6	7	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

What are you doing about your mistakes in the Self-Testing Drills? Are you using the *Self-Help Index* on page 471?

PROMISSORY NOTES



154. The Main Facts about Promissory Notes. (Silent Reading.) When one person borrows money from another, the borrower should give the lender a **promissory note** (sometimes called simply a "note"). Such a promissory note is a written promise to repay the loan. There are two kinds of promissory notes:

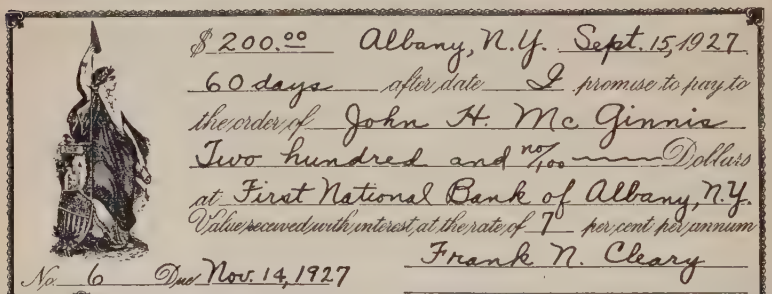
(*a*) The **demand note** is one in which the borrower agrees to pay back the money when the person who loaned it asks, or demands, that he do so. It always contains the following words, "I (or we) promise to pay upon demand," etc.

(*b*) The **time note** is one which must be paid back on a certain date. The exact length of time that the note is to "run" is stated on the note. For instance, a time note may read, "Sixty days from date, I promise to pay," etc. If the note is made on July 1, it must be paid exactly sixty

days after July 1, which is August 30. Most promissory notes are time notes.

A person who borrows money should be willing to give a written record to the lender, and the one who lends should expect such a record in order to protect himself. It is not safe to trust to memory in money affairs.

A promissory note (a time note) is shown below. Study it carefully and then look at the list of "Eight Facts about Promissory Notes" which follows the picture.


$200.00 Albany, N.Y. Sept. 15, 1927
60 days after date I promise to pay to
the order of John H. Mc Ginnis
Two hundred and no/100 Dollars
at First National Bank of Albany, N.Y.
Value received with interest at the rate of 7 per cent per annum
No. 6 Due Nov. 14, 1927
Frank N. Cleary

EIGHT FACTS ABOUT PROMISSORY NOTES

The law usually requires that eight facts be shown on a properly drawn-up promissory note. In order to help you locate these eight facts for the note shown above, certain statements are given in parentheses. These eight facts are as follows:

1. The amount of the loan (*$200*).

2. The place where the note was made (*Albany, New York*).

3. The date on which the loan and the note were made (*September 15, 1927*).

4. The time the note is to run (unless it is a demand note) (*60 days*).

5. The words "value received."

6. The place where the note is to be paid (*First National Bank of Albany, New York*).

7. The name of the person to whom (or to whose order) the payment is to be made (*John H. McGinnis*).

8. The signature of the borrower (*Frank N. Cleary*).

Printed forms of promissory notes are generally used by banks. These printed forms often contain certain additional facts such as the rate of interest, if any, the date due, the number of the note, etc.

THE VOCABULARY OF PROMISSORY NOTES

A number of new words have already been used in describing promissory notes. Some of these words are not the ones most often used by business men in talking about notes and loans.

It will be wise for you to spend a few minutes on the words defined below, for they are the ones most generally used by business men.

1. The **maker** of a note is the borrower of the money. The maker usually signs his name at the lower right-hand corner of the note.

Sometimes when a bank makes a loan, a second signature is required so that the bank will be able to collect the money from the second person in case the first one fails to pay.

2. The **payee** of the note is the one to whom (or to whose order) the money is payable. The payee is the lender. Notes usually read "to the order of ....." rather than merely "to ....." They read this way in order that the

payee may sell the note to some other person if he wishes to do so.

3. The **face value** (or **face**) of the note is the amount of money for which the note is drawn. In other words it is the sum of money (exclusive of interest, if any) which the maker must pay the payee when the note is due.

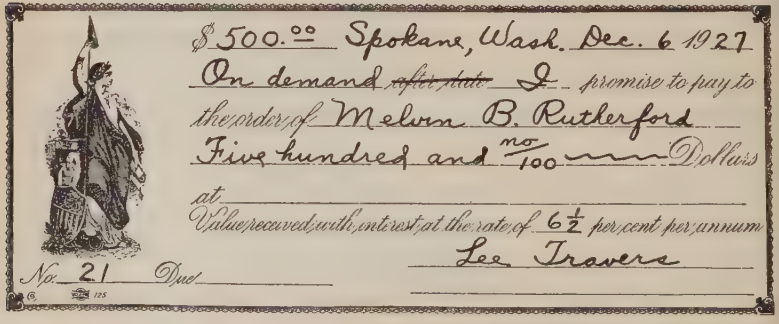
4. The **date of maturity** is the time when the loan must be paid. It is the day upon which the note is said "to fall due."

5. The **indorsement** of a note consists of the transfer of the ownership of the note to another person. The one who loaned the money and to whom the note is made out writes his name across the back of the note, just as one does in the case of indorsing a check.

6. A **time note** is one which falls due upon a certain stated date, or after a certain stated time.

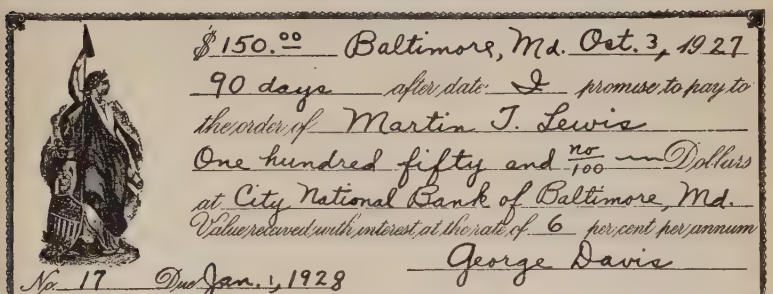
7. A **demand note** is one which the maker (borrower) must pay whenever the payee (lender) demands payment of the money.

A demand note is shown below. Study it carefully, comparing this note with the time note shown on page 272. What difference do you find?

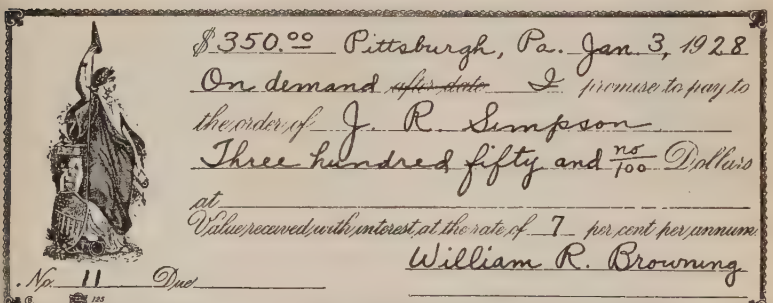

$500.00 Spokane, Wash. Dec. 6 1927
On demand ~~after date~~ I promise to pay to
the order of Melvin B. Rutherford
Five hundred and no/100 Dollars
at
Value received with interest at the rate of 6½ per cent per annum
Lee Travers
No. 21 Due

155. Practice with Promissory Notes. (WRITTEN WORK.) Be sure you understand what you have studied about promissory notes in Article 154.

1. For each of the two notes shown below, find the following facts: (1) the face value, (2) the place the note was made, (3) the date of making, (4) the time the loan is to run, (5) the payee, (6) the maker, and (7) the place of payment. (Read Art. 154 again before beginning actual work.)


$150.00 Baltimore, Md. Oct. 3, 1927
90 days after date I promise to pay to
the order of Martin T. Lewis
One hundred fifty and no/100 Dollars
at City National Bank of Baltimore, Md.
Value received with interest at the rate of 6 per cent per annum
George Davis
No. 17 Due Jan. 1, 1928

NOTE A


$350.00 Pittsburgh, Pa. Jan. 3, 1928
On demand ~~after date~~ I promise to pay to
the order of J. R. Simpson
Three hundred fifty and no/100 Dollars
at
Value received with interest at the rate of 7 per cent per annum
William R. Browning
No. 11 Due

NOTE B

2. Which note is a time note? Which a demand note?

3. What interest will be due on Note A at maturity? What is the amount due?

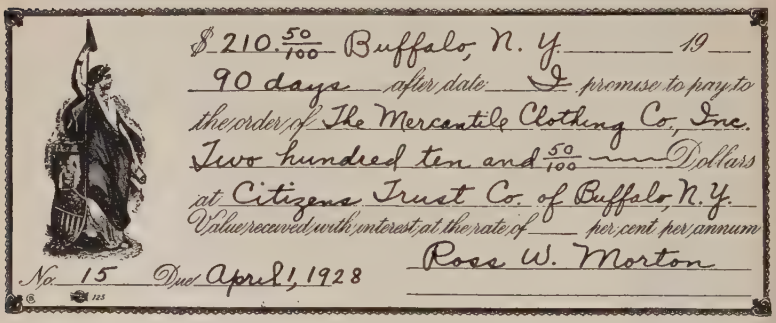
4. Obtain a blank promissory note at some bank or make one and fill it in properly, using these facts:

Loan: $125.75. Date of making: August 12, 1928.
Time note is to run: 90 days.
The payee: William W. Wilson.
The maker: Robert S. Edison.
Place of payment: Bank of Commerce, Wichita, Kansas.
Rate of interest: 7%.
Date of maturity: November 10, 1928.

5. If Frank S. Jones were willing to buy Mr. Edison's note, how could Mr. Wilson transfer the note to Mr. Jones? (See Problem 4.)

6. Make out a demand note. Make up your own facts to be entered on the note.

7. What two facts are missing on Note C below?


$210.50/100 Buffalo, N. Y. 19
90 days after date I promise to pay to
the order of The Mercantile Clothing Co, Inc.
Two hundred ten and 50/100 Dollars
at Citizens Trust Co. of Buffalo, N.Y.
Value received with interest at the rate of per cent per annum
Ross W. Morton
No 15 Due April 1, 1928

NOTE C

8. Close your books and write the list of eight facts which are required by law to appear on a promissory note.

9. Find the interest and the amount of a note for nine months at $7\frac{1}{2}\%$ interest if the face value is $400.

10. Make a list of the terms you have learned in connection with promissory notes and write a definition for each.

BANK DISCOUNTS

156. The Meaning of Bank Discount. (Class Work.) Banks often lend money on promissory notes to persons known or thought to be reliable. Very often the banks collect the interest on a note in advance. That is, the bank computes the interest which the loan will earn over the period for which the note is drawn, and then subtracts that interest from the face of the note. The borrower thus does not receive the amount for which the note was made out. He receives the **face value less the interest** he must pay for the use of the money.

This will be made clearer by an example. Mr. Walter R. Barnes wishes to borrow \$200.00 from the Bank of Commerce for a period of sixty days. The Bank of Commerce charges 7% interest on such loans. The interest on \$200 for sixty days at 7% may be found as follows:

$$i = p \times r \times t$$
$$i = \$200 \times .07 \times \tfrac{1}{6}, \text{ or } \$2.33.$$

Mr. Barnes will give the bank his note for the full \$200, but he will actually receive from the bank \$200 minus \$2.33, or \$197.67.

The \$2.33 is called **bank discount.** It is really nothing more than **interest collected in advance.** The difference (\$197.67) is often called the **proceeds** of the note.

> **Bank discount is interest collected in advance.**

A promissory note given under such conditions may not mention the rate of interest, the interest having been paid in advance. Such a note is said to be **discounted.** The

borrower receives only the face value of the note less the interest (or discount). In other words, he receives the proceeds.

Proceeds (of a note) = face value less discount (interest).

PROBLEM: Find the proceeds of a note for \$1000 for 6 months, discounted at 8%.

SOLUTION: The interest equals $\$1000 \times .08 \times \frac{1}{2}$, or \$40.
\$1000 is the face value of the note.
Proceeds equal face value less interest (discount).
Proceeds = \$1000 − \$40 = \$960.

The maker of this note would actually receive but \$960. At the end of the six months he would owe the full \$1000 (face of the note without interest). Do you see that discounting is really collecting interest in advance?

PROBLEMS FOR STUDY

Study the problems below for ten minutes so that you will be able to tell how you would work them. Remember these facts: (1) Bank discount is interest collected in advance. (2) The proceeds are face value less interest or discount.

1. Find the interest and proceeds on a note for \$500 for one year at 6% interest if the note is discounted.

2. What are the proceeds on a discounted bank loan, if the face of the note is \$500, the rate is 7%, and the time is 18 months?

3. Henry Robertson borrowed \$3000 at a bank in order to purchase a farm tractor. The bank charged 7% interest, and discounted the note. The loan was arranged

for a two-year period. What were the proceeds? (How much cash did Mr. Robertson receive?)

4. John Ellefson borrowed \$750 at the bank for a period of fifteen months at 6% interest. The bank discounted the loan.

(*a*) What was the interest on the loan?

(*b*) What were the proceeds?

(*c*) How much did Mr. Ellefson have to pay the bank when the note fell due?

157. Practice with Bank Discounts. (WRITTEN WORK.) Solve the following examples. Before beginning actual work, re-read Art. 156 and be sure you have in mind what discounts are.

Find the ***discount*** *on the following amounts for the times and interest rates stated:*

1. \$100 for 6 months at 6%.
2. \$500 for 1 year at 7%.
3. \$300 for 9 mo. at $6\frac{1}{2}$%.
4. \$2000 for 90 da. at 6%.
5. \$750 at 8% for 1 yr. 6 mo.

Find the ***proceeds*** *on the following notes:*

6. \$500 for 15 mo., discounted at 7%.
7. \$900 for 9 mo., discounted at 6%.
8. Face value of the note, \$600; interest $7\frac{1}{2}$%, and time 18 mo.
9. Banker's discount, $6\frac{1}{2}$%; time, 90 days; and face value \$2500.
10. Thirty-day note for \$10,000; discounted at 5%.

PRACTICE IN ESTIMATING

158. Estimating Reasonable Answers in Interest. (CLASS WORK.) "Wild" guesses are worth nothing as estimates. One should learn to estimate well, and the best way to learn is to practice estimating and to correct poor estimates.

There are several estimates given for each example below. In each case only one of the estimates is a good one. Can you tell which is the best estimate for each example?

1. Find the interest on \$1000 @ 7% for 1 yr. 6 mo.
   \$70 \$35 \$100 \$90
2. Find the amount of a loan of \$800 @ 6% for 2 yr.
   \$50 \$100 \$850 \$900
3. Find the interest on \$1400 for 30 da. @ 8%.
   \$9 \$50 \$110 \$160
4. Find the interest on \$490 for 8 mo. @ 7%.
   \$350 \$240 \$35 \$24
5. How much will \$2500 earn @ 3% for 4 years?
   \$75 \$100 \$150 \$300
6. What will it cost to borrow \$500 for 3 mo. @ $7\frac{1}{2}$% interest?
   \$10 \$40 \$50 \$120
7. Find the amount of a loan of \$1200 @ 9% for 6 mo.
   \$50 \$300 \$1250 \$1500
8. What is the interest on \$710 @ 6.04% for 1 year?
   \$7.10 \$42 \$420 \$4200
9. What is the amount when $p = \$500$, $r = 6\%$, $t = 3\frac{1}{2}$ yr?
   \$30 \$100 \$130 \$600
10. i = what, when $i = \$694 \times .061 \times \frac{5}{4}$?
    \$53 \$90 \$530 \$650

159. Rapid Drill. (ORAL WORK.) Spend a few minutes looking over these examples before the recitation begins. In division examples give first quotient figure only.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	72 ×7	$2\frac{1}{2}$ lb. = ...oz.	178 −79	$\frac{1}{8}$ = what %?	$27\overline{)72}$	*Add:* 7, 3, 5, 8
2.	$75\overline{)222}$	$1\frac{7}{8} = \frac{?}{8}$	.07 + .6 + 3.	$5\overline{)1}$ hr.	81 ×5	14.7 −6.9
3.	168 −69	200% of $N = 4$. $N = ?$	$83\overline{)253}$	20 ×0.8	9+5+8	$\frac{2}{3} + \frac{1}{6}$
4.	.5 + .1 + 1.8	$1\frac{1}{2} - \frac{1}{4}$	25.4 −10.4	$34\overline{)157}$	What is 25% of $20?	49 ×4
5.	941 −621	217 ×0	2 bu. = ...pk.	$6 \times \frac{1}{12}$	*Add:* 7, 7, 6, 4	$19\overline{)69}$
6.	1 T. − 1000 lb. = ?	125% = what fraction?	$28\overline{)196}$	86 ×6	895 −593	*Add:* 7, 8, 6, 6
7.	17 ×7	$65\overline{)311}$	Factors of 45 are 9 and....?	*Add:* 18, 5, 5, 7	135% = what decimal?	9.07 −8.06
8.	556 −510	6 sq. ft. ×3 ....sq. yd.	$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times$ ....?	$27\overline{)189}$	7.3 ×.06	186 −97

SELF-TESTING DRILL NO. 22

160. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Estimating reasonable answers** will help keep you from making "big" errors. **Checking** helps to avoid "little" mistakes.

1. *Subtract:*
752924
564759

2. If 781 is the right answer to the example $309276 \div 396$, put a C as your answer; if not, write the correct answer.

3. *Subtract:*
169021.7
81537.2

4. What number must be added to $28\frac{3}{4}$ to make $76\frac{4}{5}$?

5. *Add:* .897, 5.689, .456, 8.769, .578.

6. $817\overline{)804745}$

7. $4\frac{1}{6} \div \frac{5}{6} = ?$

8. *Multiply:*
3.894
3.92

9. Express $\frac{1}{5}$ of a mile in feet.

10. The volume of a rectangular tank is 126 cu. yd. It is 3 yards wide and 6 yards long. The depth is the same throughout the tank. How deep is it?

11. \$985 invested at 7 per cent for $5\frac{1}{4}$ years will earn what income?

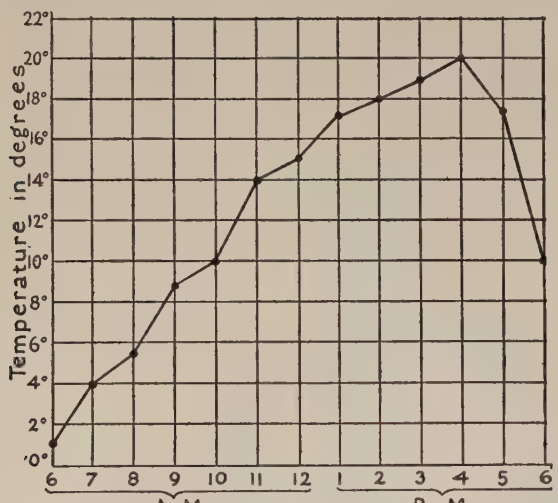
12. *Subtract:*
6 sq. ft. 74 sq. in.
3 sq. ft. 96 sq. in.

13.* *Answer the following questions by means of the graph on page 283:*

(*a*) At what hour was the temperature highest?

(*b*) What was the gain in temperature between 10 A. M. and 3 P. M.?

(*c*) What was the difference in temperature between 4 P. M. and 6 P. M.?



14. 243 is what per cent of 81?

15. A storekeeper paid $3450.30 for goods. This was 42% of what he expected to sell the goods for. For how much did he expect to sell the goods?

*To count as correct, each part of Example 13 must be correct.

STANDARDS

Number correct.....	0–2	3	4	5	6	*	7	8	9	10–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

There are eight more Self-Testing Drills for you to work before you are through with seventh-grade arithmetic. Try to make the line on your Progress Chart show steady improvement.

PROBLEM SCALE NO. XI

161. Problems Requiring Careful Copying of Numbers. (WRITTEN WORK.) You can earn 70 points by working every problem correctly. Find your score and rate yourself as directed for Problem Scales I to III.

Use this table in working the problems below:

FARM CROPS IN SEVERAL STATES

State	Total acres in harvested crops	Per cents of this acreage in: Corn	Wheat	Oats	Hay and Forage
Illinois	21,462,852	36.9	19.1	20.1	18.7
Iowa	21,609,534	41.7	6.7	25.0	21.1
Michigan	10,000,611	12.7	10.6	15.1	36.4
Minnesota	17,149,813	13.9	22.1	20.0	29.4
Montana	3,924,337	0.5	43.3	4.9	43.0
Nebraska	19,432,145	34.5	22.1	10.5	21.9
New York	8,904,678	3.6	5.1	10.5	61.8
Ohio	13,934,239	25.6	21.0	10.4	35.4
Texas	25,534,195	18.6	9.5	7.3	9.4
Wisconsin	10,265,998	11.1	5.2	22.0	44.1

1. Which state, Michigan or Wisconsin, had the more and how many more acres of land harvested? (5)

2. Which state, Minnesota or Montana, harvested the larger number of acres of wheat? (6)

3. New York had 183 milk cows for every thousand acres of harvested land. How many milk cows did the state of New York have? (Use 8904 thousand acres.) (6)

4. The table accounts for what per cent of the total acreage harvested in Illinois? (7)

5. Name three states, the sum of whose acreage about equals the total acreage for the state of Texas. (7)

6. Which state has the largest percentage of its total acreage accounted for in the table above? (7)

7. How many acres of corn were harvested in Iowa? (8)

8. How many acres of harvested land does the table fail to give account of for the state of Wisconsin? (8)

9. How many more acres of corn than wheat did Nebraska harvest? (8)

10. How many acres of hay and forage did Ohio harvest? (8)

STANDARDS

Superior work:	21–70 points	Slow work:	6–10 points
Good work:	16–20 points	Inferior work:	0–5 points
Average work:	11–15 points		

162. Quotient Drill. (Oral Work.) Spend a few minutes in studying the examples below. Be ready to tell what figure should go where the arrow points.

1. $493\overline{)17283}$ (↓)

2. $568\overline{)233912}$ (4↓)
   2272
   671

3. $698\overline{)5638}$ (↓)

4. $114\overline{)10146}$ (↓)

5. $817\overline{)3272902}$ (40↓)
   3268
   490

6. $436\overline{)13516}$ (↓)

7. $107\overline{)8453}$ (↓)

8. $397\overline{)75827}$ (1↓)
   397
   3612

9. $389\overline{)96741}$ (↓)

163. Two-Minute Talks. (Class Work.) Below is a list of words and formulas that are often used in arithmetic. Be ready to make a short talk explaining clearly what each one means.

Samples: (*a*) *Invert.*→We invert the divisor and multiply in division of fractions.

(*b*) *Quotient.*→Quotient is the name of the result in division.

1. commission
2. making change
3. reducing fractions
4. $\frac{1}{2}\ ab$
5. area
6. discount
7. cancel
8. lwh
9. promissory note
10. dimensions
11. product
12. $i = prt$
13. principal
14. budget
15. parallelogram
16. average
17. stub
18. π or pi
19. amount
20. proceeds

SUMMARY

164. A True-False Test. (Class Work.) The statements below are **not all true.** Some are **false.** Be ready to tell which are true and which are false. You will have to read each statement carefully in order to be sure that your answer is correct.

1. Everyone can easily invest his savings with great safety and earn at least 9% interest.

2. The formula $i = prt$ is used correctly below for solving this example: Find the interest on \$650 at $7\frac{1}{2}\%$ for 9 mo.

$$i = \$650 \times .075 \times 9.$$

3. The decimal form of $6\frac{1}{4}$ per cent is .0625.

4. An interest table makes it possible to obtain **amounts** directly without first obtaining the interest.

5. In the formula $a = p + i$, a means the sum of money on which the interest is reckoned.

6. $\pi = 3.14$ or $3\frac{1}{7}$.

7. The method for solving the example, "Find i when $p = \$600$, $r = 7\frac{1}{4}\%$, and $t = 8$ mo.," is $\$600 \times .0725 \times \frac{2}{3}$.

8. In computing interest 30 days is often reckoned as $\frac{1}{12}$ of a year.

9. The accounts on a check-book stub are kept by a bank clerk.

10. The example, "What per cent of 964 is 391?" would be worked as follows: $\frac{391}{964}$ or $964\overset{\bullet}{)}391.00$.

11. The formula for finding the volume of Fig. A is $V = \frac{1}{2}lwh$.

FIG. A

12. Common rates of interest paid by savings banks are 3%, $3\frac{1}{2}\%$, and 4%.

13. $17 \times 0 = 17$.

14. The result in the following division example is greater than 60. $.24\overline{)17.28}$.

15. The product of two factors is always larger than either factor.

16. The solution shown below is entirely correct for the example, "Find the interest on \$1400 @ $6\frac{1}{2}\%$ for 1 yr. 6 mo."

Solution: $i = \$1400 \times .065 \times \frac{3}{2} = \$136.50$.

17. The method shown below is correct for solving the example, "Find the interest on \$800 for 1 yr. 3 mo. @ $7\frac{1}{2}\%$."

$$\$800 \times .075 \times \tfrac{4}{5}.$$

18. According to the interest table on page 264, the interest on \$100 for 8 days @ 7% is \$1.36.

19. Amount means principal minus interest.

20. In computing interest a year is usually counted as 360 days and a month as 30 days.

21. The amount minus the principal equals the interest.

22. In figuring interest, you must remember that different months have different numbers of days.

23. A promissory note is a written promise to repay a loan.

24. All promissory notes state the rate of interest which the note bears.

25. The payee is the one who borrows money and gives his note.

26. Bank discount is interest (on a loan) which is collected in advance.

27. A demand note falls due on a stated date.

28. The "date of maturity" is the date upon which the note was drawn up.

29. Timothy Burns borrowed \$200 for six months, the bank discounting the note at 6%. He should have received \$188 in cash from the bank at the time the loan was made.

30. Time notes may be collected at any time by the payee even if the date of maturity has not arrived.

31. A promissory note differs from a check in that it cannot be indorsed and thus become the property of another person.

32. The law usually requires that the words "value received" appear on all promissory notes.

33. Promissory notes are always to be paid at the place the note was made.

34. Proceeds (of a note) equal principal plus interest.

35. When a note is discounted by a bank, the proceeds always equal the face value of the note.

CHAPTER VIII

GRAPHS. FINDING THE INTEREST RATE. PROFIT AND LOSS

GRAPHS AS PICTURES OF FACTS

165. Why We Use Graphs. (Silent Reading.) In this article are shown three methods of stating certain facts about Elizabeth Morrow, a seventh-grade girl in the Hawthorne school. Study each method carefully, so that you will be able to answer the questions at the end of this article.

Method I

Elizabeth Morrow's final grades were as follows: geography 95, history 81, physiology 85, arithmetic 98, reading 92, spelling 88, and domestic art 75.

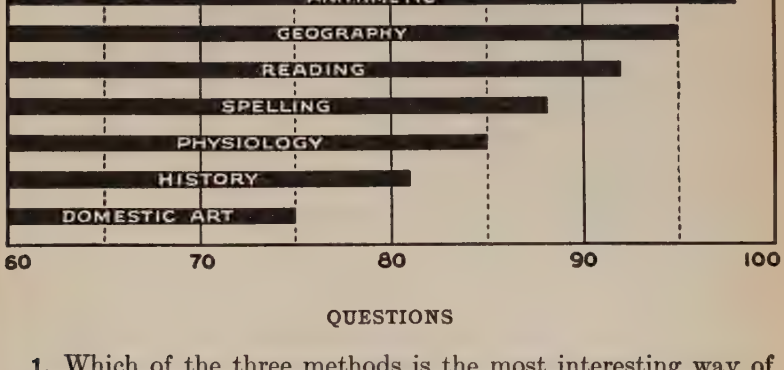
Method II

Final Grades of Elizabeth Morrow

SUBJECT	GRADE
Geography	95
History	81
Physiology	85
Arithmetic	98
Reading	92
Spelling	88
Domestic art	75

Method III

Final Grades of Elizabeth Morrow



QUESTIONS

1. Which of the three methods is the most interesting way of showing the facts about Elizabeth Morrow's grades?

2. Do all three methods tell you the same facts?

3. Which method is the easiest way of comparing the facts?

4. Which method is a "graph"? Which a "description in words"? Which a "tabulation"?

5. If you wished to show in the most interesting way the heights of the ten highest mountains in the world, which method would you use?

KINDS OF GRAPHS

166. Classification of Graphs. (Class Work.) The most common types of graphs can be classified in four groups, as follows:

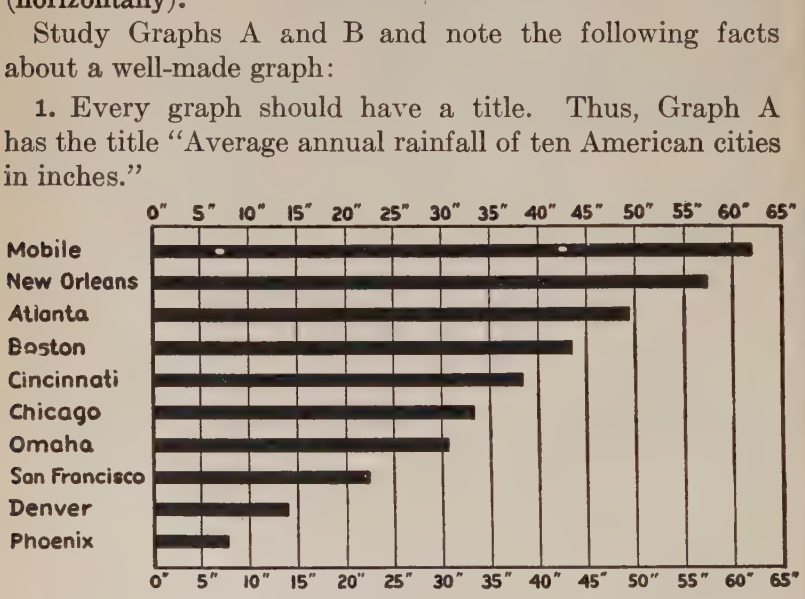
(1) The bar graph
(2) The circle graph
(3) The broken-line graph
(4) The picture graph

I. The bar graph. In the bar graph amounts are expressed by the lengths of heavy lines, or bars, drawn to a scale. The units of the scale (in., ft., dollars, yrs., per cents, etc.) are given along one or more sides of the graph. Each bar is named to show what it represents.

Sometimes the bars run up and down (**vertically**), and sometimes they run across the page from left to right (**horizontally**).

Study Graphs A and B and note the following facts about a well-made graph:

1. Every graph should have a title. Thus, Graph A has the title "Average annual rainfall of ten American cities in inches."



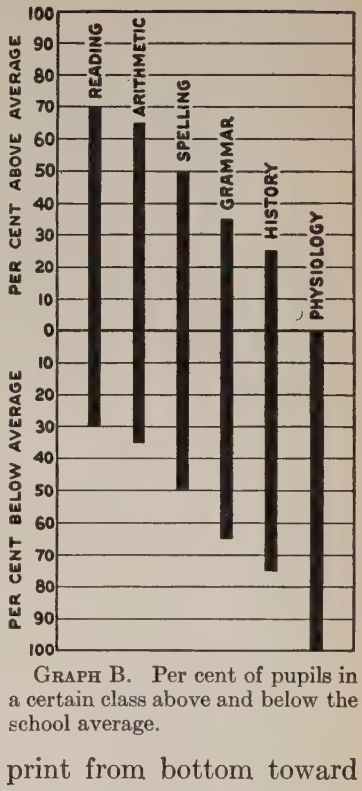
GRAPH A. Average annual rainfall of ten American cities in inches

2. When two or more graphs are given close together in a book, magazine, or newspaper, each graph is given a different letter or number to aid in referring to each one in particular. Notice that the graphs in this article are called A, B, C, etc. They might have been labeled Graphs 1, 2, 3, etc.

3. The units in which the lengths of the bars are drawn are stated along one or more sides. Thus, in Graph A the units read 0″, 5″, 10″, 15″, etc.

4. Vertical or horizontal lines are drawn at certain equal distances to guide the eye and help in estimating more exactly the amount which each bar represents.

5. Each bar is named or labeled—usually at the bottom or top in a vertical graph, like Graph B, or at the left in a horizontal graph, like Graph A.



GRAPH B. Per cent of pupils in a certain class above and below the school average.

6. Where the labels on a graph must be run up and down the page, as on Graph B, it is customary to write or print from bottom toward the top; that is, in such a way that when the drawing is looked at from the right-hand side, the writing or printing can be read easily. Graph B is made according to this rule.

7. A necessary part of any graph is a clear indication of the units of scale; that is, inches, feet, years, dollars, etc.

Now study Graphs A and B very carefully, looking to see whether they are drawn according to the 7 facts or rules we have just given. Decide in each case (*a*) the title or label, (*b*) the units of the scale, (*c*) the distance the guide

lines are apart in terms of the units of the scale, (*d*) the names of the bars, and (*e*) the amount represented by each bar. Be prepared to answer such questions as the following when your teacher calls on you.

For Questions 1 to 4 refer to Graph A.

1. What is the average annual rainfall of New Orleans?
2. What city has about twice the rainfall of Omaha?
3. Which of the cities shown has the smallest rainfall?
4. Which city has a little more than one-third as much rain as Mobile?

For Questions 5 to 8 refer to Graph B.

5. In which school subject did the pupils stand highest?
6. In which school subject were all the pupils below the average?
7. In which school subject were exactly half of the pupils above average?
8. How many more were above average in reading than in history (in per cents)?

II. The circle graph. In the circle graph, radii (plural of radius) are so drawn as to divide the area of the circle into portions which represent various amounts of the thing pictured. Very often the circle graph is used to represent per cents, fractions of a dollar, amounts of money, amounts of time, or the parts of any complete whole.

A restaurant used Graph C on page 295 to show how small a profit they make on an average meal. The restaurant receives an average of 49¢ per meal.

Graph D shows how Robert Hendricks spent the twelve hours beginning at 6 A. M. and ending 6 P. M. on Saturday, April 23, 1927.

Answer these questions about graphs C and D.

1. How much does this restaurant pay for the food served at an average meal?

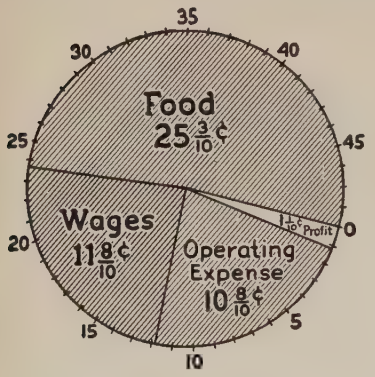
2. Which costs more, wages or operating expenses?

3. What is the average profit on a 49¢ meal?

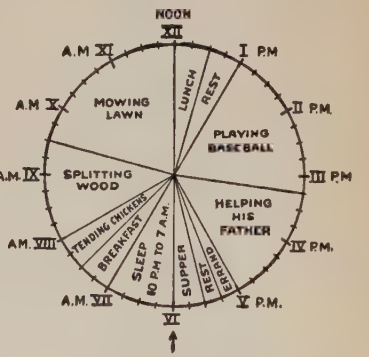
4. See if you can express the four portions of the circle in approximate per cents of the total area.

HINT 1. To make the work easier to do mentally, call the 49¢ an even 50¢. Each cent would then equal how many per cent?

HINT 2. Make the various numbers of cents ($25\frac{3}{10}$¢, $11\frac{8}{10}$¢, etc.) exact numbers of cents. Take the nearest whole cent in each case.



GRAPH C. How a restaurant spends the money it receives. The average received per meal is 49¢



GRAPH D. What Robert did from 6 A.M. to 6 P.M. one Saturday

5. What units (fractions of an hour) are marked on Graph D?

6. How did Robert spend the largest single amount of time?

7. What was the total time given to rest?

8. How much time did he spend in eating the three meals?

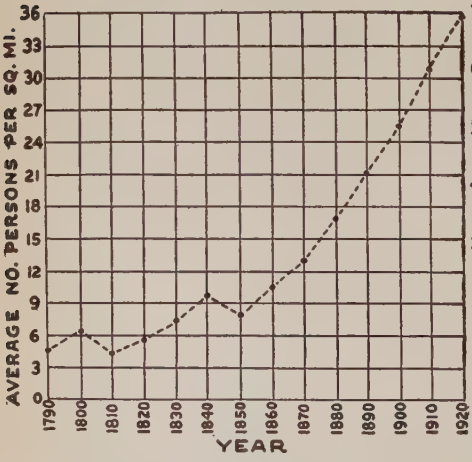
9. How long did it take him to mow the lawn?

10. How much time did Robert spend in play on this Saturday?

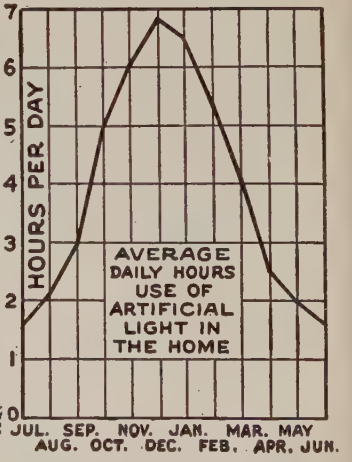
III. The broken-line graph. The broken-line graph (also called a **line graph**) is very generally used to represent facts such as increase in population by ten-year periods, hourly temperatures, changes in school ability month by month, and the like.

Graph E below shows the increase in the density of the population of the United States from 1790 to 1920 according to the last fourteen official censuses. **Density** is defined for the purposes of this graph as the average number of people per square mile of area.

Graph F appeared in a bulletin of a large electric-light company for the purpose of showing the customers why it is reasonable for the electric-light bills to run higher some months than others.



GRAPH E. Increase in density of population of United States from 1790 to 1920



GRAPH F. (Notice that the label is included in the graph itself.)

Study these two broken-line graphs and then answer the questions on the next page.

QUESTIONS

1. How many places can you find where the density of population decreased between census dates? (Graph E)

2. How many times as dense was the population in 1900 as in 1800?

3. Was the increase in density of population as rapid from 1800 to 1850 as it was from 1850 to 1900?

4. About what time did the density first reach 25 persons per square mile?

5. During what month are electric lights used the greatest number of hours per day? (Graph F)

6. How many hours of artificial light are needed per day during October?

7. What month shows almost exactly twice as many hours of artificial light as does August?

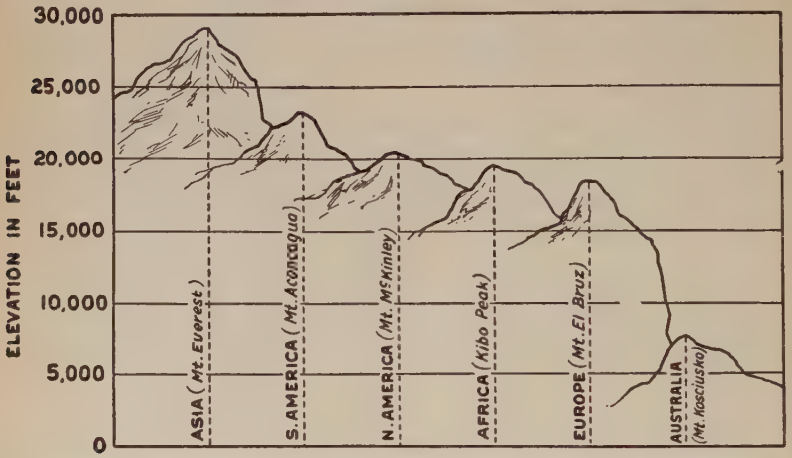
8. In what two months would the smallest amounts of electricity probably be needed for lighting?

IV. The picture graph or pictogram. Graphs G and H on page 298 show two of the wide variety of picture graphs which are found in books, magazines, and newspapers.

Graph G shows the height (elevation) of the highest mountain peak of each of the six continents of the world. Notice that there is a scale just as in the case of bar graphs, but that instead of lines or bars being used there are rough pictures of mountain peaks.

Graph H represents the sizes of the active armies of six of the large nations of the world on August 31, 1924, according to figures of the U. S. War Department. The heights of the soldiers show the relative numbers of the soldiers in the armies.

Study these two picture graphs and then answer the questions which follow the graphs.



GRAPH G. Elevation of highest mountain peak on each of six continents

QUESTIONS

1. Which is the highest mountain shown?

2. About how high is the highest mountain of North America?

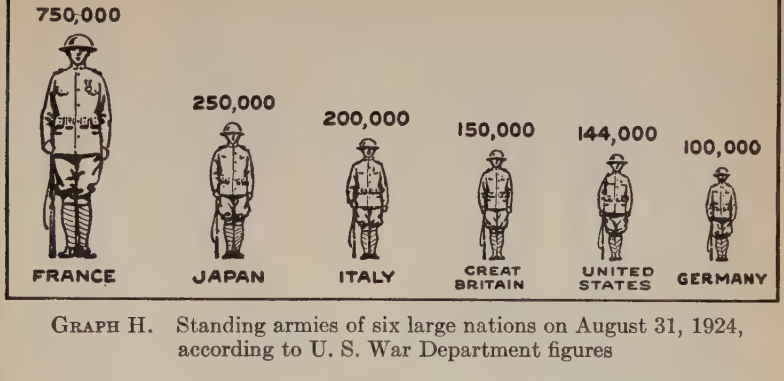
3. About how many times as high is the highest mountain in comparison with the lowest mountain represented?

4. In what way is Graph G a better one than a bar graph of the same facts?

5. If the numbers of soldiers were not given in Graph H, would it be easy to estimate closely how large these six armies are?

6. Does Question 5 suggest to you any disadvantage of such picture graphs in representing exact facts?

7. What do you think is the main value of picture graphs, the exact representation of facts or the arousing of interest on the part of the reader?



GRAPH H. Standing armies of six large nations on August 31, 1924, according to U. S. War Department figures

8. What advantages and disadvantages would a simple table of these figures about the armies of the six nations have over the picture graph shown?

9. Which allows the greater accuracy of reading, Graph G or Graph H?

10. Of the four main types of graphs which you have studied (bar, circle, broken-line, and picture) which two would you think the most useful?

167. Practice in Reading Graphs. (WRITTEN WORK.) On pages 300 and 301 are nine graphs. There are some like each of the four kinds you have just studied and perhaps one or two which are not quite like any which you have seen before.

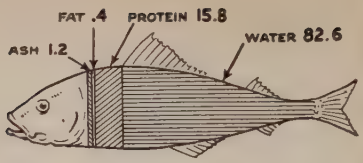
Make up at least one good question to ask the class about each of these graphs. Put your questions in writing. Be sure that you can answer all of your own questions.

Here are two questions which you can use as samples:

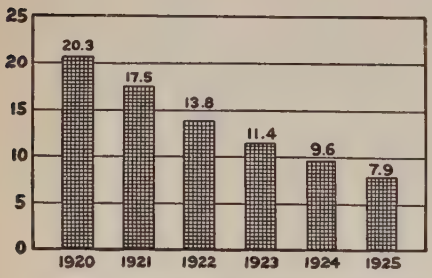
GRAPH 1. How many pupils misspelled 5 words? (*Answer:* 4.)

GRAPH 2. How many times as much water as protein does a codfish contain? (*Answer:* More than 5 times as much.)

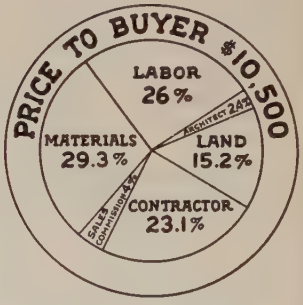
GRAPH 1. Spelling record of a class



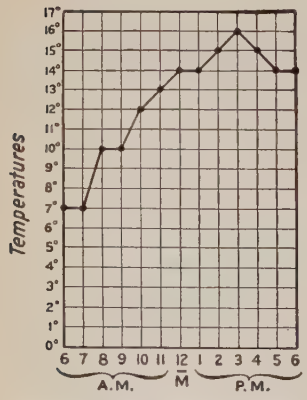
GRAPH 2. Composition of codfish in per cents



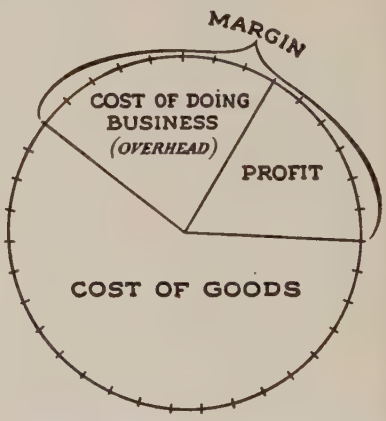
GRAPH 3. Per cent of decrease in the sale of four-cylinder automobiles from 1920 to 1925



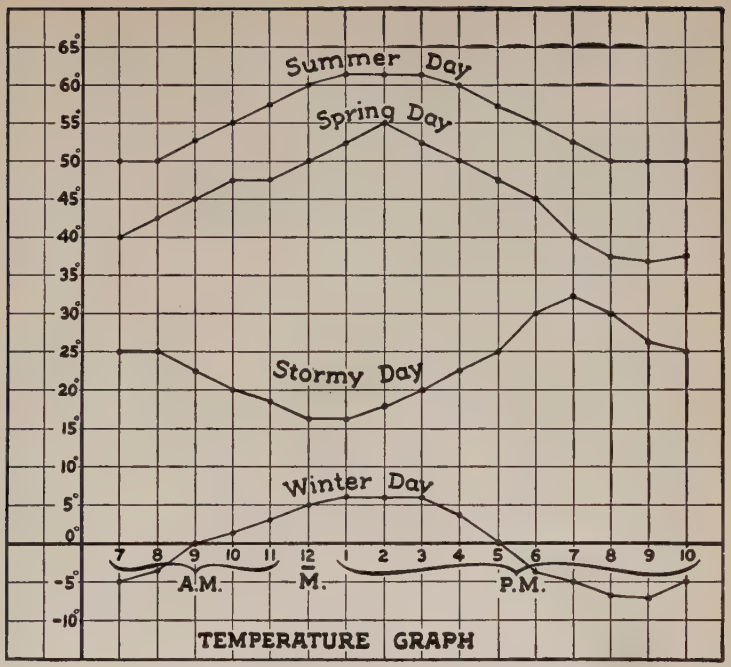
GRAPH 4. How the total cost of a certain house and lot was divided



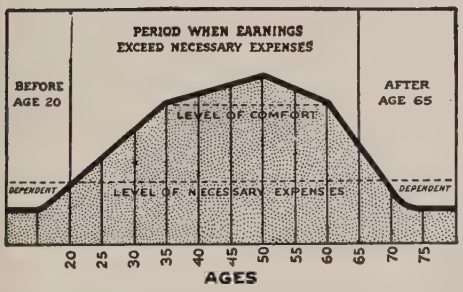
GRAPH 5. Temperatures during a certain 12 hours



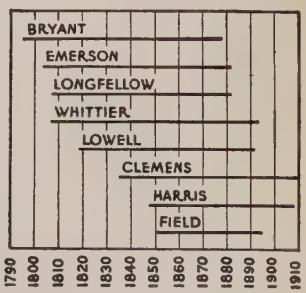
GRAPH 6. Whole circle stands for the selling price of certain goods



GRAPH 7. Hourly changes in temperature for four typical days



GRAPH 8. Earning power of the average man at various ages



GRAPH 9. Dates of certain American authors

168. Practice in Making Graphs. (WRITTEN WORK.) Below are given a number of sets of figures which will be suitable for some good graphs. Notice that this article is divided into four parts. You should make at least two graphs of each of the four kinds studied in Art. 166.

It will be wise for you to review the rules about the arrangement, choice of units, and labeling of graphs given in Art. 166.

I. BAR GRAPHS

1. Numbers of deaths by automobile accidents per 100,000 persons in ten cities of the United States for the year 1923:

Baltimore	16.5	Los Angeles	35.6
Boston	17.1	Minneapolis	14.9
Cincinnati	30.3	Nashville	20.6
Denver	15.6	New Orleans	16.6
Detroit	25.8	St. Louis	20.7

HINT: The above arrangement of the cities is alphabetical. Do you think it would be a good plan to arrange them in the order of **decreasing** per cents in making your graph?

2. Dates of the lives of some famous Americans:

Paul Revere	1735-1818	Longfellow	1807-1882
Benj. Franklin	1706-1790	Andrew Jackson	1767-1845
Abraham Lincoln	1809-1865	Washington Irving	1783-1859
Geo. Washington	1732-1799	Patrick Henry	1736-1799
James R. Lowell	1819-1891		

3. The average prices of cotton in the New York City markets for the ten-year period ending 1923, in cents per pound:

1914	*1915*	*1916*	*1917*	*1918*	*1919*	*1920*	*1921*	*1922*	*1923*
11.1	10.1	14.4	23.5	31.7	32.2	33.9	15.1	21.2	29.4

II. CIRCLE GRAPHS

1. A Nebraska farmer's investment in his 100-acre farm:

Land	50%	House	$12\frac{1}{2}$%	Stock	$12\frac{1}{2}$%
Barn	$12\frac{1}{2}$%	Machinery	$6\frac{1}{4}$%	Miscellaneous	$6\frac{1}{4}$%

HINT: It will be easier to change the per cents to fractions.

2. How Portland, Oregon, spent its dollar one year:

Education	31¢	Streets and Bridges	9¢
Interest	21¢	Sanitation	6¢
Fire Dep't	12¢	General expenses	6¢
Police Dep't	9¢	All other expenses	6¢

3. The composition of milk is as follows:

Water	87%	Fats	4%	Minerals	0.7%
Carbohydrates	5%	Protein	3.3%		

III. BROKEN-LINE GRAPHS

1. The average heights of boys and girls for different ages are (to the nearest inch):

Age	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
Boys	42	44	46	49	50	52	54	55	57	60	63	65
Girls	41	43	46	48	50	52	54	56	58	60	62	62

2. The average monthly temperatures of Philadelphia, Pennsylvania for a certain year were:

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
33°	34°	41°	52°	63°	71°	76°	75°	68°	58°	46°	36°

3. The approximate numbers of passenger automobiles in the United States from 1914 to 1923 are given below:

1914	500,000	1919	1,700,000
1915	800,000	1920	1,900,000
1916	1,500,000	1921	1,400,000
1917	1,700,000	1922	2,400,000
1918	900,000	1923	3,500,000

IV. PICTURE GRAPHS

1. Draw a picture of a loaf of white bread and show the following facts about the composition of bread:

Water	35.3%	Protein	9.2%
Carbohydrates	53.1%	Minerals	1.1%
Fats	1.3%		

2. Picture the heights of the following mountains:

Mt. McKinley	20,300 ft.	Mt. Shasta	14,380 ft.
Mt. Logan	19,000	Pike's Peak	14,108
Mt. Whitney	14,501	Mt. Mitchell	6,711
Mt. Rainier	14,408	Mt. Washington	6,293

3. Draw a picture showing the Woolworth building in New York City standing beside Pike's Peak. The Woolworth building is 792 ft. high, and Pike's Peak is 14,108 ft. high. Make your two drawings look as much like a mountain and a high building as you can.

169. Rapid Drill. (ORAL WORK.) Many seventh-grade pupils can give the correct answer to all of these examples after a short study period. See if you can do so.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	$89\overline{)722}$	*Add:* 7, 9, 8, 9	134 − 97	82 × 6	4% = what decimal?	*Add:* 7, 8, 7, 8
2.	*Reduce:* $\frac{2}{12}\frac{7}{}$	68 × 2	$73\overline{)659}$	*Add:* 8, 8, 9, 8	116 − 38	$i = prt$ means what?
3.	67 × 3	*Add:* 9, 9, 7, 9	.875 = ...%	172 − 97	$98\overline{)392}$	132 − 35

	(A)	(B)	(C)	(D)	(E)	(F)
4.	159 −64	85)286	1 gal. − 3 pt. = ___ qt. ___ pt.	$\frac{5}{8}=\frac{?}{24}$	*Add:* 8, 9, 3, 9	89 ×3
5.	$\frac{1}{3}+\frac{1}{5}$	91 ×8	37)234	236 −79	Name 4 kinds of graphs.	2 sq. yd. = ____ sq. ft.
6.	116 −67	*Add:* 8, 9, 7, 4	$87\frac{1}{2}\%$ = what fraction?	4)5 yd. 1 ft.	7×9+6	37)107
7.	743 ×.1	10 hr. = ___ min.	46)367	*Add:* 6, 5, 5, 8	$\frac{3}{4}\div 4$	Find 10% of $37.50
8.	53)159	7 = what % of 35?	7+5+.06	$2-\frac{3}{5}$	8.6 ×9	2.5−1.2

SELF-TESTING DRILL NO. 23

170. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. **Careless copying will cause mistakes.**

1. *Subtract:*
   1157500
   485739

2. *Subtract:*
   1417762
   919466

3.
   $3.46
   5.55
   13.27
   75.59
   6.76
   43.85

4. $565344 \div 936 = ?$

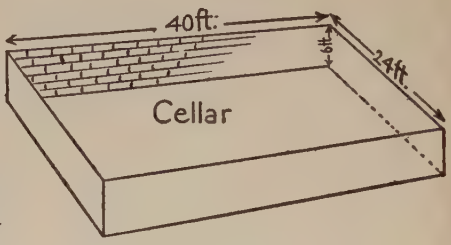
5. *Multiply:*
   6.057
   6.083

6. $2\frac{5}{6}\div 2 = ?$

7. $17\frac{1}{15}-6\frac{3}{5} = ?$

8. Find a 10% discount on $926.00.

9. How many cubic feet must be excavated for the cellar shown in the drawing at the right?



10. *Subtract:*

3 yd. 2 ft. 2 in.
2 yd. 2 ft. 8 in.

11. 1098 is 75% of what number?

12.* (*a*) 8 bu. = ... pk.

(*b*) \$7.50 = how many quarters?

(*c*) 3 sq. mi. = how many A.?

(*d*) 30 quarters are equal to how many dollars?

13.* Answer the following three questions about the formula $i = prt$.

(*a*) Is i an amount of money or a per cent?

(*b*) Is time commonly reckoned in days, months, or years?

(*c*) Is r an amount of money or a per cent?

14. 4)6 gal. 1 qt.

15. If 90 = 100% of a number, 64 equals what % of the number?

*To count Examples 12 and 13 correct, each part must be correct.

STANDARDS

Number correct	0–2	3	4	5	6	7	*	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Take time to correct any mistakes. Use the *Self-Help Index*, page 471, to find any topic you may need to study.

INTEREST IN BUSINESS

171. Applications of Interest in Business Investments. (Silent Reading.) Thus far we have considered interest as applied to savings, notes, and loans. There are a number of other uses of interest which business men must know.

Business men often wish to know the rate of interest earned by the money which they have invested in their business. Art. 172 will show you how to compute the rate of interest when the principal and the interest are known. In order to understand the explanations of Art. 172, you should know the meaning of several new business terms. Some of these will be studied more fully in eighth grade work, but you should have a general idea of the meaning of these terms now if you are to understand how business men deal with interest problems. These words are:

1. **Investment.** The word **investment** is used in several different meanings in business. It often means the amount of money (or as it is sometimes expressed, the **capital**) that a man or a company has for carrying on the business. Or, a man might say that he **invested** $3800 in a building lot. Investments are made with the idea of gain; that is, that the money invested will earn additional money, or interest. We do not always speak of the money earned by an investment as interest, but it is always possible to do so.

The money invested in business may earn profits. The profit may be regarded as interest on the money invested. A lot purchased for $3800 and sold for $4200 (after all expenses have been subtracted) gives a profit of $400. The $400 could be regarded as interest on the investment.

Some other meanings of "investment" will be shown at a later place in this article.

In dealing with problems of investment, remember that:

> **The amount invested (the capital) is the *principal*.**
> **The profit (or gain) is the *interest*.**

2. Stock. The capital of (money invested in) a large business usually represents the combined investments of many persons, since a single person seldom has money enough to conduct a very large business.

The laws of the various states allow several persons to combine their capital and form what is called a **corporation.** The combined capital is then divided into **shares of stock** usually of some stated value, like \$50, \$100, or \$1,000. If the stated (or **face**) value of each share of stock is \$100, each person receives one share of stock for each \$100 invested. The following table shows how five men (A, B, C, D, and E) formed a corporation with \$40,000 capital, and how the stock was divided among them.

Investor	*Amount of Investment*	*\$100 Shares of Stock Owned*	*Computation*
A	\$15,000	150	(\$15,000 ÷ 100 = 150)
B	\$ 3,500	35	(\$ 3,500 ÷ 100 = 35)
C	\$ 7,500	75	(\$ 7,500 ÷ 100 = 75)
D	\$ 4,000	40	(\$ 4,000 ÷ 100 = 40)
E	\$10,000	100	(\$10,000 ÷ 100 = 100)
Total	\$40,000	400	

Shares of stock are sold on the market for what they will bring, regardless of the face (or **par**) value. A stock of \$100 face or par value may sell for \$110, \$125, \$175, or even more if the corporation or company issuing the stock is

thought to be reliable and is earning good profits. Likewise stock in a poor company of doubtful earning power may sell way below par or face value.

Large corporations, like railroads, gas and electric companies, automobile manufacturers, etc., may have thousands or even hundreds of thousands of stockholders. If the corporation makes money, the profits are divided among the stockholders at some stated amount per share. Such divided profits are called **dividends.**

The amount paid for stock may be thought of as *principal.* The share of the profits may be thought of as *interest.*

3. Bonds. Sometimes businesses **borrow** money by selling (issuing) **bonds.** A bond is very much like a note; that is, it is a written or printed promise to repay the loan at a certain date. A bond pays interest, usually every six months. The buyer of a bond is not an owner of the business, as is a stockholder; he has merely loaned money to the business. The interest on bonds issued must be paid before the profits of the business are divided as dividends among the stockholders.

Bonds are sold by cities, counties, states, and governments as well as by business firms. Bonds sold by cities to provide money for paving, sewers, water supply, etc., are often called **municipal bonds.** Bonds of companies which supply gas, electricity, street car service, etc. are often called **public utility bonds.** Bonds issued by the United States Government are called **government bonds.**

Bonds are most often made out in units of $500 or $1000, and they **fall due** (**mature**) on stated dates, usually from

five to thirty years after they are issued. Bonds always state the rate of interest that is to be paid.

As in the case of stocks, bonds may sell for amounts greater or less than the face value. Although the price of a bond may vary, the amount of interest it pays is always the same. This is because the interest is always computed on the face or par value of the bond.

A $1000 bond bearing 6% interest would pay $60 interest per year (usually $30 every six months). However, if this bond could be bought for $900 ($100 less than face value), it would pay $60 interest on a $900 investment. The rate of interest would be greater than the stated rate for any bond selling below par (face value). Similarly the actual rate of interest earned is less than the stated rate for all bonds selling above par value.

Bonds bear interest at rates varying from less than 4% to 8% or more. Bonds which pay more than 6% to 7% are usually considered as not being entirely safe investments, the high interest rate showing that people are not sure of the safety of the investment.

Money paid for a bond may be considered as *principal*.

4. Yield. The gain from stocks is termed dividends. Such dividends represent profits which are distributed to the share holders. A stockholder may regard dividends received as interest on the money he has invested. Thus he might say, "I received an annual dividend of $5.75 on each $100 share of Blank stock, which is $5\frac{3}{4}\%$ interest on my investment." The dividends earned by stocks and the interest paid on bonds are often called the **yield.** Thus:

"We offer for sale $1000 public utility bonds, priced at $950, to yield about 6.3% interest."

The rate of yield (rate of interest earned) is always computed on the actual cost of the stock or bonds, not on the face or par value. This is proper, since the actual selling or cost price of the stock or bond represents the amount of the investment.

You will learn more about stocks and bonds in the eighth grade. The facts of this article will help you understand some of the uses of interest in business life.

HOW TO FIND THE INTEREST RATE

172. Finding the Rate. (Class Work.) We have already learned how to find the interest on a sum of money at a given rate and for a given length of time. We use the formula $i = prt$.

It is sometimes useful to be able to find the **rate** of interest when we know the interest, the principal, and the time. Study the next problems. We first take a problem where the interest **for one year** is given.

Problem I

ADVERTISEMENT

A SOUND INVESTMENT IN BONDS

Bond prices are low at present. We offer a safe bond in a Public Utility at $965 to yield $65 a year. Write us.

If a person were thinking of investing a thousand dollars and had confidence in the banking house whose advertisement is shown at the left, he would want to know the rate of interest which this bond at the advertised price would yield (**earn**) for him.

The advertisement does not give the rate. But we can find that for ourselves if we have a

formula or rule for finding the rate. Let us make a formula for finding the rate in this problem.

You have learned how to make new formulas from formulas that you already know. You have learned the interest formula $i = prt$. Let us use it to find a formula for r. This problem gives the interest for one year, so that if we put $t = 1$ (meaning 1 year) in the interest formula $i = prt$, the formula becomes $i = pr1$, or $i = pr$. If i is the product of the factors p and r, then i divided by p must give r, or $r = \frac{i}{p}$. You know that a product divided by one of two factors gives the other factor. For problems, then, where the interest for one year is given, we may use the formula $r = \frac{i}{p}$.

Following are two methods of thinking how to find the rate of interest earned by this bond:

First Method	*Second Method*
Think: \$65 is what per cent of \$965. $\frac{65}{965}$ or $965\overline{)65.00}$ .067+ or 6.7+ % per year $965\overline{)65.000}$ 5790 7100 6755 345	Use the formula $r = \frac{i}{p}$ or $r = i \div p$ \$965 = p \$ 65 = i r = what? $r = \frac{i}{p}$ $r = \frac{65}{965}$, or $965\overline{)65.00}$ .067+ or 6.7+%

The bond will yield a little over 6.7%. Note that both methods use the same computation, but in the second method you use the formula or rule to guide you in doing the work.

In Problem I the formula $r = \frac{i}{p}$ gave us the rate of interest per year because in the problem $t = 1$. Let us see if it will work again.

Problem II

Mr. Franklin: I have a chance to make a good investment of $1500. I expect to get $120 a year income from it.

Mrs. Franklin: $120 a year interest would mean that our money was working for us at what rate?

Mr. Franklin (estimating rate): Oh, about 8% a year.

Mrs. Franklin: I don't see how you figure that.

John Franklin (the son): I do. I'll show you two methods of finding the rate. Here they are:

First Method	*Second Method*
We want to know what per cent is earned per year. $120 is what per cent of $1500? Think: $\frac{120}{1500}$ or $1500\overline{)120.00}$ $.08 = 8\%$ $1500\overline{)120.00}$ $\underline{120.00}$	To find the rate per cent annually, use the formula $r = \frac{i}{p}$ or $i \div p$ Interest is $120 Principal (the investment) $1500. $.08 = 8\%$ $r = \frac{120}{1500}$ or $1500\overline{)120.00}$

Mr. Franklin's estimate of the rate was exactly right. Note that both methods give the same result. You learned long ago how to do this kind of work: finding what per cent one number is of another.

We see that the formula $r = \frac{i}{p}$ gave correct answers to two problems asking us to find the rate **per year.** You need to remember that this formula is correct only when the interest

is given **for 1 year.** Do not forget that when the formula $r = \frac{i}{p}$ is used to find the rate, i must stand for the **interest for exactly 1 year.** It is not the same as i in the formula $i = prt$, for in this case i stands for the interest for t (any given number of) years.

Remember the formula for finding the rate of interest when i is the interest for exactly one year: $r = \frac{i}{p}$

➽In solving percentage problems which require that the rate per cent be found, you may use either the first method shown on page 313, or the formula $r = \frac{i}{p}$. But when you use the formula $r = \frac{i}{p}$, i must be the **interest for exactly 1 year.**

QUESTIONS

1. How is the formula for finding the rate made from the interest formula $i = pr$?

2. What kind of percentage example are you working when you find the rate of interest?

3. Does the rate of interest mean the per cent per year, per month, or per day?

4. Interest for 1 year, \$12.00; principal, \$200. How would you find the rate?

5. Find the rate in the examples below:

Principal	*Interest for 1 year*
\$ 50	\$2.00
\$200	\$6.00
\$150	\$9.00

173. Problems in Finding the Rate. (WRITTEN WORK.) In the problems below, use the formula $r = \frac{i}{p}$, since the interest (or income) in all cases is given for one year. You will notice that these problems ask you to find the rate per cent which is earned.

1. Mr. Seward invested \$5000 in a business. His yearly income from this investment yielded him an average of \$312.50 a year. What was the rate of interest earned on the investment?

2. Mrs. Taylor has an income of \$465 a year from a pecan farm in which she has invested \$7750. Consider the amount of her investment as the principal and the income as the interest. Find the rate.

3. A bond house advertised the following bonds for safe investment. Find the rate yielded by each bond at the stated price.

(A)

A High Grade Industrial Bond.

Price \$1050. - To yield \$70.

(C)

A Bond of a Well-Established Public Utility.

Price \$920. - Yield \$65.

(B)

A Government Guaranteed Bond.

Price \$980. - To yield \$40.

(D)

A Long Term Railroad Bond.

Price \$1000. - Yield \$60.

4. The shares of stock of several of the largest industries in the United States were selling on Nov. 2, 1927, at prices

given below. Their **dividends** (amount of interest per year paid for each share) are also given. Find the **rate** of yield from the stocks of at least five of these companies on the price stated for each share. Find the rate per cent to the **nearest whole per cent.**

	Price	*Dividend*
(*a*) American Express	$171.	$6.
(*b*) American Telephone & Telegraph Co.	175.	9.
(*c*) Baltimore and Ohio Railroad	117.	6.
(*d*) Chrysler Corporation (Automobiles)	53.	3.
(*e*) Eastman Kodak Co.	159.	5.
(*f*) Goodrich Tire and Rubber Co.	53.	4.
(*g*) Hudson Motors	67.	5.
(*h*) Fox Films (Motion Pictures)	70.	4.
(*i*) Studebaker Corporation (Automobiles)	53.	5.
(*j*) Western Union Telegraph	160.	8.
(*k*) Woolworth (5 and 10 cent stores)	176.	4.
(*l*) Wrigley (Chewing Gum)	64.	3.

174. Practice in Choosing Correct Formulas. (CLASS WORK.) Most pupils find it wise to memorize these three formulas having to do with interest. You will need to know them for the exercise on page 317.

(1) $i = prt$

(2) $r = \frac{i}{p}$ or $i \div p$, where i = interest for 1 yr.

NOTE: The formula $r = \frac{i}{pt}$ must be used to find r when the interest is given for any other period of time than one year. The formula says: first multiply the principal by the time (expressed in years), and then divide the interest for the whole period by the product.

(3) $A = i + p$.

Three ways are given for working each example below, but in each case only one is correct. Study the examples carefully for a few minutes, so that you will be ready to tell which way is correct. Use the formulas on page 316 in deciding which way to choose.

EXERCISE

1. What is the rate per cent earned by an investment of \$800 which pays \$48 a year dividends?

(*a*) $\$800\times.48\times1$ (*b*) $800\overline{)48.00}$ (*c*) $48\overline{)800.00}$

2. Find the interest that is earned on a \$2500 loan @ $5\frac{1}{2}\%$ for $2\frac{1}{2}$ years.

(*a*) $2500\times.055\times\frac{5}{2}$ (*b*) $.055\overline{)2500}$ (*c*) $2500\times55\times\frac{5}{2}$

3. Find the amount of a loan of \$750 @ 6% for 6 mo.

(*a*) $.06\overline{)750.00}$ (*b*) $750\times.06\times\frac{1}{2}$ (*c*) $750+(750\times.06\times\frac{1}{2})$

4. What is the rate of interest earned on an investment of \$1200 which pays \$75 a year?

(*a*) $75\overline{)1200}$ (*b*) $1200\overline{)75.00}$ (*c*) $1200\times.075\times1$

5. What is the rate of interest per year paid on a deposit of \$600, when the interest is \$12 every **six months?**

(*a*) $600\overline{)24.00}$ (*b*) $600\overline{)12.00}$ (*c*) $600\times.24\times\frac{1}{2}$

6. An investment of \$2500 paid me \$1500 interest in 10 years. What was the average yearly rate of interest on the investment?

(*a*) $\$2500\times.15\times10$ (*b*) $25000\overline{)1500}$ (*c*) $2500\overline{)1500}$

7. Find the total interest earned on a $5\frac{1}{4}\%$ investment of \$900 for 5 years.

(*a*) $900\times.05\times5$ (*b*) $900\times.525\times5$ (*c*) $900\times.0525\times5$

SELF-TESTING DRILL NO. 24

175. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*

1630523
750296

2. *Find the sum of the areas:*

6 sq. yd. 8 sq. ft.
2 sq. yd. 3 sq. ft.
3 sq. yd.
5 sq. yd. 4 sq. ft.

3. *Subtract:*

13829.17
7380.78

4.

498
689
934
787
888
366

5. Four fifths plus four fifteenths plus one sixth equals what?

6. *Multiply:*

6872
7095

7. $.57\overline{)425.334}$

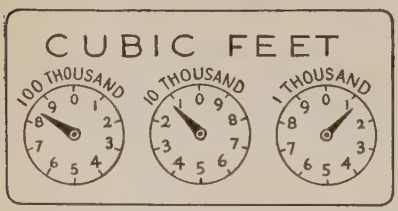
8. What is 200% of 350?

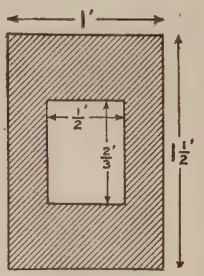
9. Multiply 6 sq. mi. 200 A. by 4.

10. Is $576 \times \frac{3}{8}$ about 1600, 220, or 70?

11. $3\frac{1}{3} \times 12 = ?$

12. Find the interest on \$116 @ $7\frac{1}{2}\%$ for 3 yr. 9 mo.





13. Give the reading on the gas meter.

14 What is the area of the shaded part of the figure?

15. 92.4 is what per cent of 847?

STANDARDS

Number correct	0–2	3	4	5	6	*	7	8	9	10–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

PROBLEM SCALE NO. XII

176. Problems of About Equal Difficulty. (WRITTEN WORK.) You can earn 55 points by working every problem correctly. **Estimating a reasonable answer will keep you from making many foolish mistakes.**

1. The girls in a sewing class made six aprons of different sizes and styles. The following amounts of cloth were used: $3\frac{1}{3}$ yd., 4 yd., $3\frac{3}{4}$ yd., $3\frac{5}{6}$ yd., $4\frac{1}{2}$ yd., and $3\frac{3}{4}$ yd. How much cloth was used in making the aprons? (5)

2. Before starting on an automobile ride in her father's car, Florence noticed that there were $13\frac{1}{2}$ gallons of gasoline in the tank. At the end of the ride there were $10\frac{3}{4}$ gallons. How much gasoline had been used during the ride? (5)

3. Five-eighths of a yard of canvas is needed for each uniform for the "Pirates" in a school play. How many of these uniforms can be made from $17\frac{1}{2}$ yd.? (5)

4. The back seat of a certain automobile is 3 ft. 7 in. long. This seat is for three people. How much width is allowed per person? (5)

5. Which example below gives the largest quotient? (5)
(*a*) $6.9\overline{)128.34}$ (*b*) $69\overline{)12.834}$ (*c*) $69\overline{)128.34}$ (*d*) $.69\overline{)128.34}$

6. A policeman's beat is from the Union Station to Ann Street. How long does it take him to cover his beat 7 times if he covers it in 1 hr. 10 min. on the average? (5)

7. Morris thought that the product of $\frac{5}{8}$ and $\frac{4}{15}$ was $\frac{1}{5}$. He was wrong. What is the correct answer? (6)

8. Find the interest on \$3500 @ 4% from January 1, 1914, to July 1, 1920. (6)

9. While Mary was in the seventh grade, she worked 323 problems correctly the first time that she tried them. This was 85% of all the problems she tried. How many problems did Mary try while in the seventh grade? (6)

10. How much money should Mr. Smith pay to Mr. Withers if he takes up his note on October 4, 1927? (7)

Cleveland, Ohio,
April 4, 1927.

$750.00

Six months after date, for value received, I promise to pay *Walter H. Withers* or order *Seven Hundred Fifty and no/100 Dollars*, with interest at *5½%* at the Republic National Bank.

John T. Smith.

STANDARDS

Superior work:	31–55 points	Slow work:	11–15 points
Good work:	25–30 points	Inferior work:	0–10 points
Average work:	16–24 points		

ESSENTIALS OF PERCENTAGE

177. Seven Questions About Percentage. (Class Work.) Spend 15 minutes in study. If there is any item you do not understand, be sure to have it explained.

Every seventh-grade pupil should be able to answer all of the questions before leaving this article.

1. What per cents are equal to the fractions below?

(*a*) $\frac{1}{2}$ of N = ... % of N
(*b*) $\frac{2}{3}$ of N = ... % of N
(*c*) $\frac{5}{4}$ of N = ... % of N
(*d*) $1\frac{3}{4} \times N$ = ... % of N
(*e*) $\frac{1}{20}$ of N = ... % of N
(*f*) $\frac{1}{50}$ of N = ... % of N
(*g*) $\frac{1}{6}$ of N = ... % of N
(*h*) $2\frac{1}{2} \times N$ = ... % of N
(*i*) $\frac{1}{8}$ of N = ... % of N

2. (*a*) $64\overline{)4.160}$ $.065 = \_\_\_\_\_\%$

(*b*) $93\overline{)116.25}$ $1.25 = \_\_\_\_\_\%$

(*c*) $21\overline{).105}$ $.005 = \_\_\_\_\_\%$

(*d*) $324\overline{)40.500}$ $.125 = \_\_\_\_\_\%$

(*e*) $103\overline{)247.2}$ $2.4 = \_\_\_\_\_\%$

(*f*) $84\overline{)1.68}$ $.02 = \_\_\_\_\_\%$

3. How can any fraction be changed to a decimal and then to per cent form?

SAMPLES

$\frac{7}{8} = \dfrac{.875}{8\overline{)7.000}} = 87.5\%$ or $87\frac{1}{2}\%$

$\frac{11}{13} = \dfrac{.846+}{13\overline{)11.000}} = 84.6+\%$ or $84\frac{3}{5}+\%$

Tell how you could change the fractions below to per cents, but do not do the work.

(*a*) $\frac{3}{8}$ (*b*) $\frac{9}{20}$ (*c*) $\frac{5}{6}$ (*d*) $\frac{7}{12}$ (*e*) $\frac{23}{60}$ (*f*) $\frac{17}{36}$

4. How would you solve each example below? Write the examples on the blackboard as shown in the samples.

SAMPLES

I. Find 4% of 376. *Write on board:* $\begin{array}{r} 376 \\ \times .04 \\ \hline \end{array}$

II. 86 is what % of 689? *Write on board:* $\frac{86}{689}$ or $689\overline{)86.00}$

III. If 42 is 8% of N, N=? *Write on board:* $.08\overline{)42.00}$

(*a*) Find 8% of 734

(*b*) What is N, if 411=13% of N?

(*c*) What is $6\frac{1}{2}\%$ of 2400?

(*d*) Find $3\frac{1}{4}\%$ of 96.72

(*e*) What % of 932 is 134?

(*f*) What is N, if 124% of N=92?

(*g*) 486 is what per cent of 350?

(*h*) If 6.2% of N=150, N=....?

5. What decimal, easy fraction, or whole number does each per cent below equal?

(*a*) 25% (*c*) 200% (*e*) $6\frac{1}{4}\%$ (*g*) $4\frac{1}{4}\%$ (*i*) $12\frac{1}{2}\%$
(*b*) 120% (*d*) $33\frac{1}{3}\%$ (*f*) $16\frac{2}{3}\%$ (*h*) 150% (*j*) $2\frac{1}{2}\%$

6. What are the meanings of the words and phrases below?

Before telling the meaning of each phrase, get clear and correct sentences well in mind.

discount for cash
discount for large amounts
discount for prompt payment
interest
rate of interest
commission
$r = \frac{i}{p}$ or $i \div p$
security
$i = prt$
savings bank
yield on an investment
per annum
interest table

7. How would you solve each problem below? Write the examples on the board as shown in the samples.

SAMPLES

I. Find interest on \$900 @ $6\frac{1}{2}\%$ for $2\frac{1}{2}$ yr. *Think:* $i = prt$.
Write on board: I. $\$900 \times .065 \times \frac{5}{2}$

II. At what rate was this money working? Invested \$1900. Yearly income \$110.

Think: $r = \frac{i}{p}$ or $p\overline{)i}$ *Write on board:* II. $\frac{110}{1900}$ or $1900\overline{)110.00}$

(*a*) What is the interest on \$1200 for 2 yr. 6 mo. at the rate of $3\frac{1}{2}\%$?

(*b*) What is the rate per cent when $p = \$1500$ and $i = \$70$ annually?

(*c*) What is the interest on \$2500 @ $7\frac{1}{2}\%$ for 3 yr. 9 mo.?

(*d*) What would it cost to borrow \$500 for 30 da. at 8%?

(*e*) At what rate must \$1500 be invested to earn \$80 per annum?

(*f*) What is r when $p = \$2800$ and $i = \$145$ (per year)?

(*g*) What does \$750 earn at 6% interest if invested for 4 yr. 3 mo.?

(*h*) At what per cent is \$800 invested if it yields \$30 a year interest?

(*i*) $p = \$3500$. $i = \$194$ per year. r equals what?

(*j*) What would be the interest on a loan of \$2500 @ $7\frac{1}{4}\%$ for 1 yr. 1 mo.?

PROFIT AND LOSS

178. The Meaning of Profit and Loss. (SILENT READING.) The words **profit** and **loss** are very common ones in business. In spite of the fact that they are often used, their meanings are misunderstood by many people.

It would seem a very simple matter to say that a store-keeper who bought a pocket knife for 77¢ and sold it for \$1.00 had made a profit of 23¢. To state that his profit was 23¢ would probably be quite misleading for a number of reasons. For example, we would not know whether the 77¢ included not only the amount he paid for the article but also the knife's share of the cost of carrying on the business.

The 77¢ that the store-keeper paid for the knife is not its entire cost to him as a man in business. To the 77¢ must be added the knife's share of business expense, such as freight, rent, the salaries of his clerks, and many other expenses of the business.

It is entirely possible that, when all costs of doing business have been considered, the knife was sold at a loss rather than a profit.

The following ideas are important in understanding profit and loss:

1. A business man has the right to expect to sell goods at a reasonable profit. He cannot carry on his business for a very long time at a loss. The store-keeper must pay for the goods he buys and for the work that others do for him in carrying on his business. He makes others lose money if he fails.

2. In determining the real cost of goods, a business man must consider two kinds of cost: (1) the amount actually paid for the goods, and (2) all costs of doing business (rent, light, heat, postage, freight, clerks' salaries, interest, etc.). These costs of doing business are called **overhead.** The actual cost of goods to be sold includes not only the original or first cost of the article but also that article's share of the total overhead expense.

3. If an actual profit is to be made, the selling price must be greater than the sum of the original cost and the expenses of selling the article (called the "overhead").

4. Considering the cost of the goods as the purchase price plus overhead, the selling price must be made high enough so that the profit is at least equal to what the money invested in the business would earn if loaned at interest.

Actually, a business man should earn somewhat more than the common rate of interest on his investment, since he must take many risks with his investment. Some of his risks are fire, theft, bad debts, failure to sell goods on

account of bad business conditions, and many others. If he does not earn more than the common rate of interest, he might just as well buy bonds, stocks, etc., with his money. Then he would not have to think, plan, and work as he must do if he carries on a business.

5. Overhead costs include such expenses as freight, postage, insurance, rent, taxes, wages of clerks and stenographers, money invested in equipment, heat, light, advertising, goods which cannot be sold, and interest on borrowed capital, if any.

In some kinds of businesses, like groceries, fruit, vegetables, and drugs, there are losses through the spoiling of the goods. In the case of shoes, clothing, automobiles, etc., the styles and models change and old goods must be sold at large discounts, sometimes for less than cost, because people do not like to buy things that are "out of date."

6. The difference between the selling price and the original cost was formerly called "gross profit." At the present time it is becoming more common to speak of this difference as **margin.**

We can sum up these facts by saying:

1. **Selling price = purchase price + margin**
2. **Selling price = purchase price + overhead + profit**
3. **Margin = selling price − purchase price**

7. Profit is sometimes computed on the cost price but more often on the selling price. Profits (or losses) are usually stated as per cents of the selling (sometimes the cost) price.

The profit may be a very small part of the margin. The margin is frequently as much as 25% or 30% but the actual profit is as little as 6% or 7%. The margin varies greatly in different kinds of business. A committee formed by the U. S. Congress made a study of retail businesses covering the years from 1913 to 1921. It reported the following facts:

Men's and boys' clothing expenses were	26.3% of sales
Hardware dealers' expenses were	19.8% of sales
Shoe dealers' expenses were	24.7% of sales
Groceries' expenses were	15.0% of sales
Drygoods stores' expenses were	26.6% of sales

Read these seven paragraphs at least once more very carefully. It will take some time and thought on your part really to understand business terms and business practices, but this time will be well spent if you master a few fundamental ideas. Business men like to employ boys and girls who understand business and who can see the business man's point of view.

QUESTIONS

1. Does a business man who fails cause others to lose money? How?

2. If a store-keeper buys a boy's coat for $12.00 and sells it for $16.00, is the difference of $4 all profit? What expenses must be paid from the $4 and from similar differences between cost and selling price in other things he sells?

3. A stock or a bond may pay you 6% interest on your money. Why may a store-keeper or a farmer expect to make more than 6% profit?

4. What is meant by margin?

5. Name all the things you can that would make up the overhead in a grocery business.

FINDING PROFIT

179. How Profit is Computed. (SILENT READING.) A few years ago a letter was sent to a large number of business men asking how they computed profit or loss. Those who sent the letter wanted to learn whether these business men figured their profit as a per cent of the cost of the things they sold or as a per cent of the selling price.

One hundred and ninety-two replies were received from this letter. The results are given below:

Number figuring profit on selling price	96 or 50.0%
Number figuring profit on first cost plus all costs of doing business	45 or 23.4%
Number figuring profit on first cost (or actual purchase price, disregarding overhead)	24 or 12.5%
Number figuring profit by some combination of the three preceding methods	27 or 14.1%

You can see from these replies that, although business methods vary considerably in figuring profit or loss, the most common method today is to compute profit on the **selling price.** Years ago when businesses were smaller and business costs were less, it was possible to use the actual first cost (ignoring costs of doing business) as a basis for figuring profit. Today it is more usual to compute profits on the selling price.

Since there are always overhead expenses in doing business, the manufacturer's or retailer's margin must always be large enough to yield a profit. Even in a simple case like a vegetable peddler there are business costs such as the cost of a horse and wagon, the feed for the horse, in-

terest on the money invested in the horse and wagon, etc. The per cent of overhead is probably smaller in such a case than would be true of a large department store with its expensive building and equipment.

On the other hand, a large store buys in great quantities and hence pays a smaller original cost for its goods. For this reason it often happens that a large business can actually sell goods for less money than a smaller business in spite of the greater overhead of the larger business.

It is not so important that you learn at this time to solve all of the different kinds of business problems concerned with profit and loss. In the eighth grade you will learn more about profit and loss, and how to work the more difficult kinds of problems in profit and loss. If you really understand Arts. 178 and 179, you can do the work called for in Art. 180.

180. Problems in Computing Profit and Loss. (Class and Written Work.) In these problems profit (or loss) is computed on the selling price. Study Problems I and II carefully and talk them over in class before doing the written work.

Problem I. During the year ending December 31, 1927, a grocer sold \$40,000 worth of goods. The purchase price of these goods was \$32,000. (*a*) How much was his margin on the year's business? (*b*) The margin was what per cent of the selling price?

Solution: Margin = selling price − purchase price.

Margin = \$40,000 − \$32,000 = \$8000 *Ans. to* (*a*).

$$\text{Per cent of margin} = \frac{\$8000}{\$40000} = 20\% \text{ } \textit{Ans. to } (b).$$

Notice that the margin was taken to be the selling price less the cost or purchase price. Notice also that the margin was computed on the selling price.

PROBLEM II. The business expenses of the grocer in Problem I for the year were as follows: wages, \$3300; freight, express, and postage, \$360; advertising, \$650; heat, light, and water, \$300; insurance and taxes, \$440; and miscellaneous expenses, \$550.

(*a*) What did his expenses total for the year?
(*b*) What was his profit for the year?
(*c*) What per cent of profit did he make?

Solution:

\$3300+\$360+\$650+\$300+\$440+\$500=\$5600 *Ans. to* (*a*).
Margin=\$8000 (from Problem I).
Profit =\$8000−\$5600=\$2400 *Ans. to* (*b*).

$$\text{Per cent of profit} = \frac{\$2400}{\$40000} = 6\% \text{ } \textit{Ans. to } (c).$$

Solve the problems below in the ways shown for Problems I and II.

1. A merchant buys neckties at \$6.60 a dozen and sells them at 75¢ apiece. He estimates his costs of doing business at 11¢ per necktie.

(*a*) What is his margin per tie?
(*b*) What is his profit per tie?
(*c*) What is his per cent of profit per tie?

2. An automobile dealer buys his cars at a cost of \$660 each at the factory. Freight and handling on each car amount to \$115. He figures that his other overhead costs amount to \$175 per car. He sells the cars for \$1095 each.

Figure (*a*) the margin, (*b*) the profit, and (*c*) the per cent of profit on each automobile.

3. The margin on a hat selling for \$6.00 is \$1.35. (*a*) What is the cost price? (*b*) If the profit is 65¢, what is the overhead (expense) on the sale of each hat?

4. A hardware dealer bought a shipment of oil stoves at \$7.40 each. He estimates his overhead at 15% of the selling price.

(*a*) What does he make or lose on each stove sold if he sells the stoves for \$8.00 each? (HINT: First find in dollars and cents the overhead on each stove.)

(*b*) What is the per cent of profit or loss?

5. On a box of apples selling for \$4.00, 65% of the selling price represents the purchase price, 25% the overhead, and 10% the profit.

(*a*) What per cent does the selling price equal?

(*b*) What is the margin in per cents?

(*c*) What is the margin in dollars and cents?

(*d*) State in dollars and cents the purchase price, the overhead, and the profit.

6. A real-estate salesman sold a house and lot for Alfred Brown for \$7500, charging a commission (on the selling price) of 5%. Mr. Brown paid \$6700 for the lot three months previously and had paid taxes, interest, and other charges to the amount of \$160.

(*a*) What was Mr. Brown's profit?

(*b*) What per cent of the selling price was this?

(*c*) What per cent was the profit if figured on the cost price (what Mr. Brown actually had invested in the house and lot at the time of sale)?

NOTE: In such problems as this one, profit is often computed on the cost price. The profit is looked at as being interest on the investment.

7. The overhead in the silk department of Nathan's Department Store is computed as 15% of the selling price. The cost of one kind of silk dress-goods is $4.25 per yard. It sells for $6.00 per yard.

Find (*a*) the margin, (*b*) the profit, and (*c*) the per cent of profit on this particular kind of silk goods.

8. A radio set costs the dealer $82.50. His overhead expenses are 30% of the selling price. He plans to earn 15% profit on each radio. For what price must the dealer sell each radio set?

HINTS: Remember these facts in working Problem 8: Margin = overhead + profit. Purchase price = selling price − margin. Purchase price = what % of the selling price?

9. The profit on a box of oranges which a commission merchant sold for $4.50 was 60¢. What was the per cent of profit?

10. A motor-car dealer sells a certain six-cylinder automobile for $1200. He computes his various business costs and his profit in per cents of the selling price as shown below:

Cost of each car at factory	70% of selling price
Freight and handling charges	9% of selling price
Rent, light, heat, etc.	$2\frac{3}{4}$% of selling price
Insurance, taxes, and interest	$3\frac{1}{4}$% of selling price
Salesmen's commissions, etc.	7% of selling price
Profit	8% of selling price

(*a*) Compute in dollars and cents the actual amount of each item of cost or overhead listed above.

(*b*) Find the margin in per cent.

(*c*) Find the margin in dollars and cents.

(*d*) What is the dealer's profit per car?

181. Rapid Drill. (ORAL WORK.) After a few minutes' study the examples below should be very easy for you. In division examples give only the first quotient figure.

	(A)	(B)	(C)	(D)	(E)	(F)
1.	701 ×2	384 −215	81)326	$1 is what % of $50?	.04+1.1 +2	*Best form for ans?* 5 hr. 105 min.
2.	139 −45	$\frac{5}{6}$ = what %?	92 ×8	*Reduce:* $\frac{25}{20}$ = ?	28)166	28.4 −16.6
3.	59)413	2 yd. 2 ft. ×3	471 −305	Meaning of factor?	29 ×.05	*Add:* 7, 6, 9, 9
4.	5.62 −5.62	$\frac{2}{3}\times\frac{3}{2}$	68)543	What is 10% of 32.5?	211 −17	$\frac{7}{8}+\frac{1}{4}$
5.	.6+.3 +.5	$4\frac{1}{2}\%$ = what decimal?	690 −125	1 lb. − 4 oz.	59 ×6	33)107
6.	150 −96	65 ×4	*Add:* 6, 5, 8, 7	38)299	119 −71	$\frac{2}{3}\div\frac{3}{2}$
7.	47)376	.8 − .4	Give the formula for interest.	9)2 gal. 1 qt.	7.2 ×4	5% of N = 50. N = ?
8.	$\frac{3}{4}-\frac{3}{8}$	86 ×6	112 −28	*Add:* 9, 8, 7, 7	73)230	54.1 −20.1

SELF-TESTING DRILL NO. 25

182. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*

1061893
234798

2.

297
4610
1676
1980
7689

3.

55.0
.9
78.0
46.4
56.0
53.7

4. $\frac{4}{9} \div 6 = ?$

5.

$95\overline{)682670}$

6. *Subtract:*

$62\frac{1}{4}$
$50\frac{5}{6}$

7. What is 125% of 364?

8. If 20% of $N = 3750$, 100% of $N = ?$

9. Find the product of 7.086 and 597.8.

10.* (*a*) $\frac{1}{6} =$ .....%. (*c*) 40 oz. = ...lb.
(*b*) 60 dimes = $..... (*d*) 1.45 = ...%.

11. What is the difference of 2 T. 500 lb. and 1 T. 1200 lb.?

12. If 168 = 100% of N, 147 = ...% of N.

13. $6\overline{)9 \text{ qt. } 1 \text{ pt.}}$

14. Dimensions of a rectangular solid are 1 yd., 2 yd. 1 ft., and 3 yd. What is the volume?

15. If $i = prt$, what is the formula for r?

*In Example 10 all parts must be correct to count the example correct.

STANDARDS

Number correct	0–1	2	3	4	5	*	6	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

183. Review of Mensuration. (ORAL WORK.) Study the questions below for 15 minutes in order to have correct answers in mind before the oral recitation begins.

➡Every seventh-grade pupil should be able to answer these questions before leaving this article.

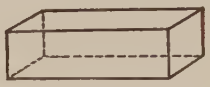
1. What is the name of each figure below?

(A)

(B)

(C)

(D)


(E)

(F)

2. What is the meaning of each item below?

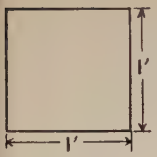
Before answering, get clear and correct sentences well in mind.

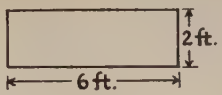
(*a*) scale drawing	(*h*) perpendicular	(*n*) parallel lines
(*b*) dimensions	(*i*) $A = ab$	(*o*) formula
(*c*) $V = lwh$	(*j*) square measure	(*p*) $A = \frac{1}{2}ab$
(*d*) altitude	(*k*) change dimensions to same unit	(*q*) a cubic inch
(*e*) obtuse angle		(*r*) acute angle
(*f*) area	(*l*) radius	(*s*) volume
(*g*) base	(*m*) diameter	(*t*) circumference

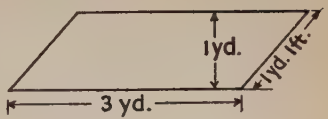
3. What should be on each blank?

(*a*) 4 ft. = ... in.	(*e*) 1 sq. ft. = ... sq. in.
(*b*) 4 ft. = ... yd.	(*f*) 1 cu. yd. = ... cu. ft.
(*c*) 15 in. = ... ft.	(*g*) 8 in. = ... ft.
(*d*) $9\frac{1}{3}$ yd. = ... ft.	(*h*) 10 ft. = ... yd.

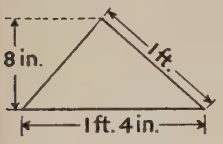
4. What should be on the blanks below?

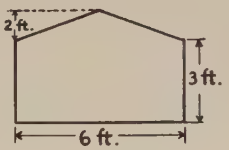


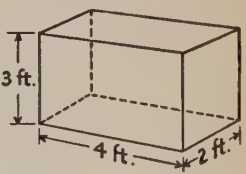




(*a*) $A =$ .....? (*b*) $A =$ .....? (*c*) $A =$ .....?







(*d*) $A =$ .....? (*e*) $A =$ .....? (*f*) $V =$ .....?

PROBLEM SCALE NO. XIII

184. Problems in Mensuration. (WRITTEN WORK.) You can earn 70 points by working every problem correctly.

1. Find the number of boxes of Excello Shoe Polishing Outfit (retail size) that can be shipped in a packing box whose inside dimensions are 12 in. by 16 in. by 28 in. A retail box of Excello is shown in Figure 1. (5)

FIG. 1

2. A figure which is 12 inches square is how many times as large as a figure whose area is 12 square inches? (6)

3. What is the volume of the solid shown in Figure 2? (7)

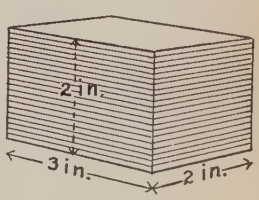


FIG. 2

4. Ruth was asked to estimate the number of bricks needed to lay a walk 4 ft. wide from the front door to the sidewalk, a distance of 27 ft. The side of a brick which will be walked on is 4 in. by 8 in. Ruth estimated that 36 bricks would be needed. Her estimate was very far from the correct answer. What is the correct answer? (Ignore space between bricks.) (7)

5. Find the number of acres in a triangular piece of land which has a base of 54 rd. and an altitude of 37 rd. (There are 160 sq. rd. in an acre.) (8)

6. The main floor in a certain railroad station is 75 ft. by 125 ft. It is covered with tiles, each of which is 3 in. by 5 in. How many tiles were used in making the floor? (Ignore space between tiles.) (8)

7. A map in a book has dimensions of 3 in. by 4 in. The map is drawn to a scale of 1 in.: 12 miles. How much actual area is pictured in the map? (9)

8. From a study of the steel beam, shown in Figure 3, compute the weight of a beam 72 in. long, if 1 cu. in. of steel weighs 3 lb. (10)

FIG. 3

9. A 4-in. cube of dry soil weighed 18 oz. After it was thoroughly soaked with water it weighed 32 oz. What would you expect a cubic foot of similar dry soil to weigh after it is thoroughly soaked with water? (10)

STANDARDS

Superior work:	16–70 points	Slow work:	7–10 points
Good work:	13–15 points	Inferior work:	0– 6 points
Average work:	11–12 points		

185. Practice in Finding and Correcting Errors. (CLASS WORK.) The examples below contain errors made by seventh-grade pupils. Study each example to find its error. Be careful not to make notes in your books, but you may make notes on paper to help you in the oral recitation after the study period.

Tell where each error is and how it should be corrected.

EXERCISE

1. What is the formula for finding the area of a triangle?
   Ans. $A = ab$

2. The area of a rectangle 8″ by 2 ft. = ?
   Ans. 16 sq. ft.

3. The formula for finding the rate when the principal and yearly interest are known is.....
   Ans. $r = ip$.

4. What is 4% of 936?

   The work:

   936
   ×.4
   374.4 *Ans.*

5. What is $3\frac{1}{4}\%$ of 1200?

   The work:

   1200 ×.325 or .325 ×1200
   65000
   325
   390.000 = 390 *Ans.*

6. $5236 \div 17 = ?$

   The work:

   38
   17)5236
   51
   136 38 *Ans.*
   136

7. 850 is what per cent of 170?

   The work:

   .2 = 20% *Ans.*
   $\frac{170}{850}$ or 850)170.0
   170 0

8. Find the interest on \$900 @ 7% for 30 da.

The work: \$900×.07×30

×.07

63.00

×30

\$1890.00 \$1890 *Ans.*

9. Find the price after discount is taken off a bill of \$750. Discount allowed, 8%.

The work: 750

×.08

\$60.00 \$60 *Ans.*

10. Find the total amount due on a loan of \$600 @ 6% for 6 mo.

The work: 600×.03×$\frac{1}{2}$ (.06)

600

×.03

\$18.00 \$18 interest. *Ans.*

SELF-TESTING DRILL NO. 26

186. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1.

893
382
362
933
744
948

2. *Subtract:*

1057.126
293.827

3. Betty said that 53,709 was the correct difference in this example: 712,294 minus 658,585. Was she right? If not, give the correct difference.

4. Find the difference of 1,526,330 and 891,736.

5. 2.53)24.1109

6. $6\frac{3}{4}\times 8\frac{1}{4}=?$

7. Find the earnings on \$350 @ 3% for 1 yr. 6 mo.

8. *Multiply:*

4 pk. 2 qt. 1 pt.
9

9.

$\frac{5}{6}+2+1\frac{7}{8}+2\frac{1}{2}=?$

10. *Find the sum:*

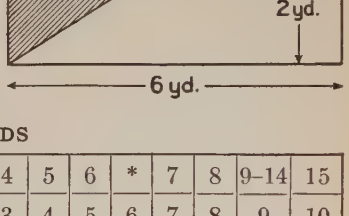
2 yd. 2 ft. 8 in.
3 yd. 9 in.
6 yd. 1 ft.
4 yd. 2 ft. 11 in.

11. What per cent of 600 is 492?

12. Find the product of 3489 and 1468.

13. Is $.809\times16$ about: 1200, 120, or 12?

14. Find the area of the shaded part of the figure.



15. If 0.5% of a number equals 0.770, what is 100% of the number?

STANDARDS

Number correct	0–1	2	3	4	5	6	*	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Do you need to use the *Self-Help Index*, page 471, for any examples in this drill?

SUMMARY

187. General Review. (Written Work.) This article gives you an excellent chance to find out how well you understand certain important things that you have been studying.

1. $4.36+6.07+14+.89+.04$

2. 8.76×13.7

3. $.43\overline{)17.458}$

4. $8073\div.69$

5. 8% of 916

6. 6.37 is what % of 49?

7. If \$283.88 is 47% of a sum of money, what is the sum?

8. Find the interest on \$500 @ $6\frac{1}{2}\%$ for 2 yr. 6 mo.

9. Find the amount saved on a bill of \$97 by paying cash when a 3% discount is allowed for prompt payment.

10. What commission should be paid on a sale of \$2400, if the rate of commission is 20%?

11. At what rate must \$900 be invested to yield \$29.25 per year?

12. If a scale of a map is 1″:10 mi., a distance on the map of 2.5 in. stands for what real distance?

13. Find the area of a rectangle whose dimensions are 3′6″ by 9′8″.

14. Find the area of a triangle whose base is 14 ft. and altitude is $11\frac{1}{2}$ ft.

15. Find the volume of a rectangular solid whose dimensions are $7' \times 11' \times 16'$.

16. Define: investment, yield, stocks, bonds, dividends, capital, corporation, shares, stockholder, and par value.

17. Why is the interest yield of a stock or bond figured upon the actual cost rather than the face or par value?

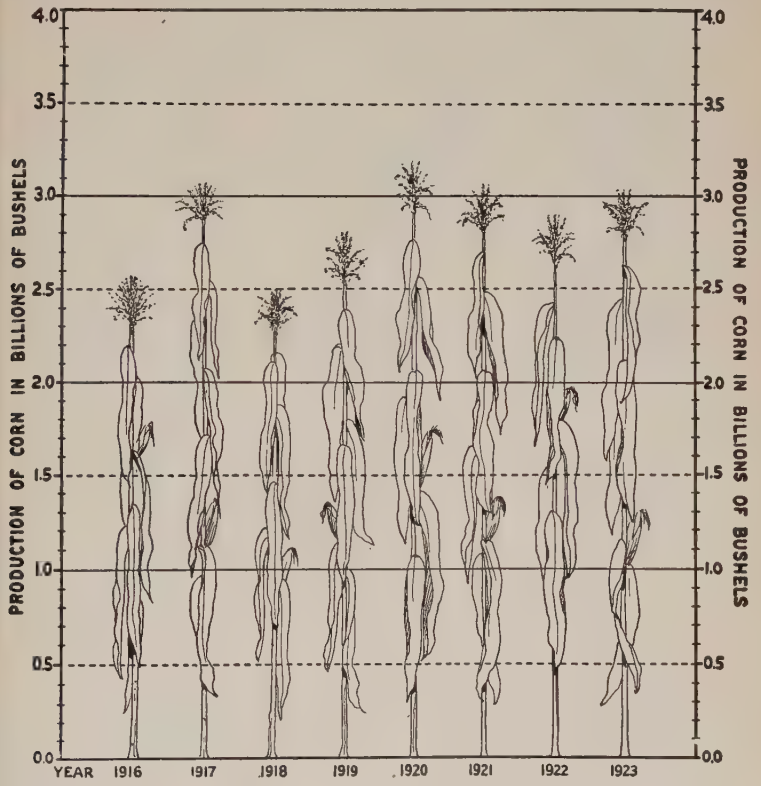
18. Make out a promissory note. Be sure that you include every necessary fact which the law requires. Compute the interest and the amount.

19. Study Graph 1 on the next page and then answer these questions:

(*a*) In what year was the production of corn the greatest shown? The smallest?

(*b*) About how many billions of bushels of corn were raised in the U. S. in 1916? In 1919? In 1923?

(*c*) About how much did the production of corn decrease from 1917 to 1918? Express this decrease in per cents. (Use 1917 as the basis for figuring the per cent of decrease, reading the graph as accurately as you can. You should be able to read to the nearest one-tenth of a billion bushels.)



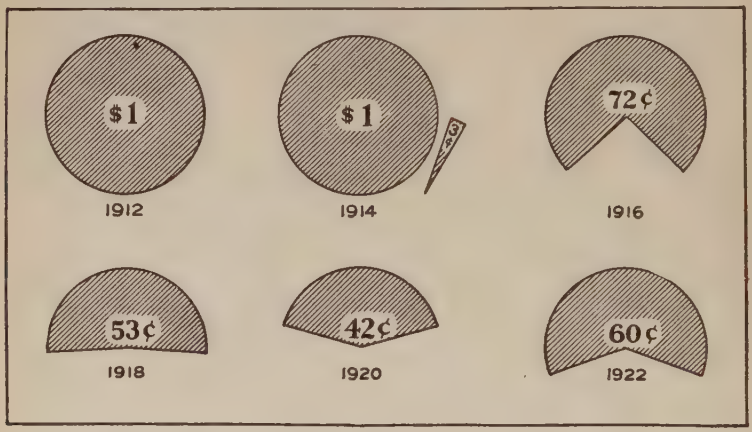
GRAPH 1. The production of corn in the United States from 1916 to 1923 in billions of bushels

(*d*) Why is Graph 1 better than a table of numbers which would show the same facts?

20. Study Graph 2 below carefully.

(*a*) In what year was the purchasing power of the farmer's dollar the greatest? The least?

(*b*) In what year was the purchasing power of the dollar about two-fifths of the value it had in 1912 and 1914?



GRAPH 2. Showing the purchasing power of the farmer's dollar for alternate years from 1912 to 1922. (Figures are from the United States Department of Agriculture). The average purchasing power of a dollar for the period from 1909 to 1914 is taken as the basis of comparison, i.e., as $1.00.

(*c*) In what year was the purchasing power of the dollar about one-half of that for 1914?

(*d*) Why were circles and parts of circles used to represent the purchasing power of the farmer's dollar for different years instead of lines or bars as is the case with most graphs?

CHAPTER IX

ANGLES. THE MEASUREMENT OF DIRECTION. THE PROTRACTOR AND THE COMPASS. GETTING READY FOR EIGHTH GRADE

THE USE OF THE ANGLE

188. The Use of the Angle in the Measurement of Direction. (Class Work.) In reading books and magazines you will often find such statements as the following:

1. The house is set at an angle to the street.

2. The new 60 degree motor is very powerful.

3. The axis of the earth is inclined at an angle of $23\frac{1}{2}°$ to the plane of the earth's orbit.

4. The rock layers have been tilted at an angle of 30 degrees to the horizontal.

5. The highway crosses the railroad at an angle of about 20 degrees.

In order to describe completely the position of any object, we need to know two things:

1. The **distance** the object is from some known point, and

2. The **direction** the object is from some known point.

Sometimes we wish to know only **distance**; at other times we may need to determine only **direction**; and very often we must know both distance and direction in order to describe the location of objects.

Study the four figures below and note the need of a knowledge of **angles** to understand the printed statements accompanying each figure.

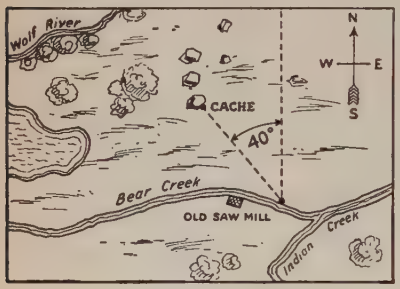


FIG. A. A rough map from a prospector's diary which reads: "My supplies are cached (hidden) under a large boulder near Wolf River. To reach the **cache,** move eastward along Bear Creek to a point midway between the old saw-mill and the junction of Indian Creek. From this point proceed $2\frac{1}{2}$ mi. at an angle of 40° west of exact north. The cache is under the southernmost of the three large boulders."

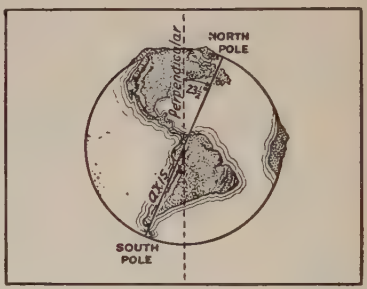


FIG. B. Showing how the earth moves around the sun with its axis inclined (tipped) at an angle of $23\frac{1}{2}$ degrees. The heavy line represents the earth's axis, and the dotted line shows the perpendicular.

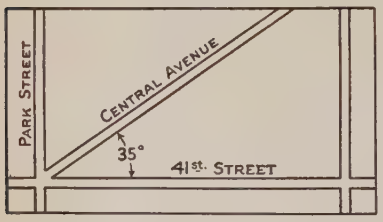


FIG. C. (From a city guide book) Central Avenue begins at the intersection of 41st Street and Park Street and runs northeast at an angle of 35 degrees to 41st Street.

FIG. D. Showing the rock layers tilted at angles of approximately 30° from the horizontal, because of folding of the earth's crust.

QUESTIONS

1. The directions for finding the cache in Fig. A give the distance $2\frac{1}{2}$ mi. and the angle 40°. What other needed fact is given?

2. If the prospector's diary had left out "west," what would one have had to do before he could find the cache?

3. Can you find the angle which Central Avenue makes with Park Street?

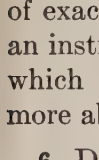
HINT: Park St. makes an angle of 90 degrees with 41st St.

4. Starting from a given point, what two facts must you know about the location of an object if you are to find it?

5. In order to locate the cache in Fig. A, how could you determine the line which is "at an angle of 40° west of exact north"? To do this you would need a **compass,** an instrument which resembles a watch, but the needle of which points to the north, or nearly so. We will learn more about the compass in a few days.

6. Do you see that you will need some sort of device for drawing and measuring angles if you are to try to draw a diagram like Fig. A or Fig. C? The instrument for measuring angles is called a **protractor.** You will learn about it in Art. 190. But first let us learn what **angles** are.

ANGULAR MEASUREMENT



189. Angles and Angular Motion. (CLASS WORK.) On the next page are shown four pictures, each one like the face of a clock with the minute hand removed, leaving only the hour (small) hand. Look at Figure 1. This shows where the hour hand would be at exactly twelve o'clock.

One hour later (one o'clock), the hour hand would be in the position shown in Fig. 2. The dotted line shows where the hand started an hour before. The hand has moved

FIG. 1.

FIG. 2. Angle $A = \frac{1}{12}$ of Circle

FIG. 3. Angle $A = \frac{4}{12}$ of Circle

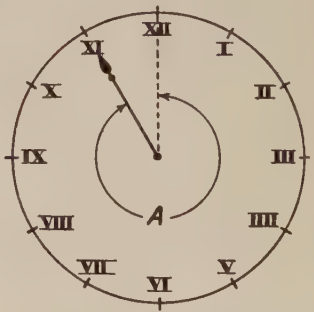


FIG. 4. Angle $A = \frac{11}{12}$ of Circle

$\frac{1}{12}$ of the distance around the circumference of the clock face. There are a great many ways in which we can describe the motion of the hand of the clock during the hour which has passed. For example:

1. The hour hand has moved from XII to I.

2. The hour hand has moved $\frac{1}{12}$ of a complete revolution, or circuit.

3. The tip of the hour hand has traced a part of a circumference. (A part of a circumference is called an **arc.**)

4. The hour hand of the clock can be thought of as a moving radius of a circle.

5. The hour hand has traveled 30 **degrees** (30°) of **angular motion.**

Statement 5 above contains two new ideas: (*a*) **degrees,** and (*b*) **angular motion.** These two ideas are really very simple, but you should take care to understand them, because the degree is the unit of a new kind of measurement —of angular movement or motion.

A circle is thought of as being made up of 360 equal angles, each of which is an angle of one degree (1°).

The hour hand in making one complete revolution from XII around to XII again, passes through an angle of 360 degrees. In each hour it passes through an angle of $\frac{1}{12}$ of 360°, or 30°. Do you see that the hour hand in changing from the position shown in Fig. 1 to that of Fig. 2 has passed through an angle of 30 degrees? ($\frac{1}{12}$ of $360° = 30°$)

Look at Fig. 3. Do you see that all of the following statements are true?

1. The hour hand has moved from XII to IIII.
2. The hand has moved $\frac{4}{12}$ of a complete revolution.
3. The hand has traveled 120°. ($\frac{4}{12}$ or $\frac{1}{3}$ of 360°)

Look at Fig. 4. Can you answer these three questions?

1. The hour hand has moved from .....?..... to .....?.....
2. The hour hand has moved $\frac{?}{12}$ of a complete revolution.
3. The hand has moved through an angle of ....?....degrees.

1. The *degree* is the unit of measurement of angles and of angular motion.

2. Every circle contains 360 degrees (360°). We may say that every circle is made up of 360 angles of 1° each.

Fig. 5 below will show you that a circle is made up of 360 degrees. Study Fig. 5 for a few seconds and then see if you can answer all the questions below Figure 5.

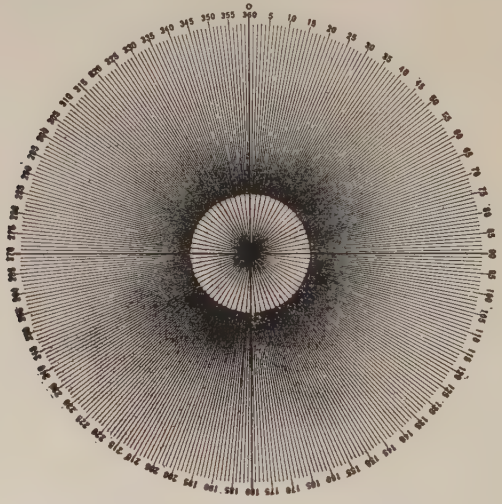


FIG. 5. Degrees in a Circle

QUESTIONS

1. How many degrees are there in a circle?

2. How many right angles are there in a circle?

3. How many degrees are there in a right angle? In two right angles? In three right angles? In four right angles?

4. How many angles of 45° are there in a right angle?

5. How many angles of 10° would make a right angle? How many such angles in a complete circle?

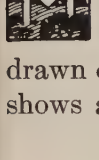
6. Through how many degrees does the hour hand of a clock pass between 6 A.M. and 11 A.M.? Between 6 A.M. and 3 P.M.?

7. Through how many degrees does the **minute** hand of a clock pass in one hour? In one-half hour?

8. How many degrees are there in the angle formed by two radii of a circle when these two radii form a diameter of the circle?

9. 60°+35°+20° equals 1 rt. ∠, and how many degrees more?

10. What is the name of the unit in which angular motion (**angular magnitude**) and angles are measured?



190. The Measurement of Angles—The Protractor. (CLASS WORK.) We need some instrument for the measurement and construction of angles. The usual method of measuring angles drawn on paper is by means of the **protractor.** Fig. 1 below shows a common type of protractor.

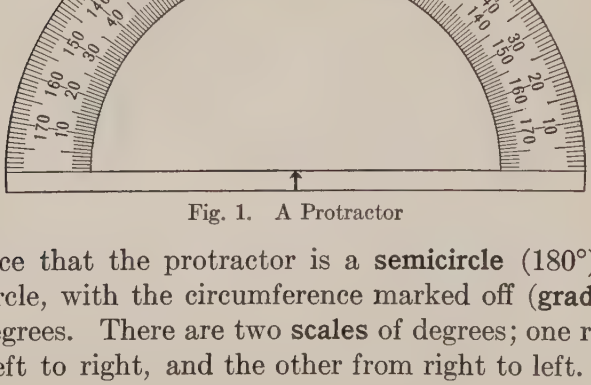


Fig. 1. A Protractor

Notice that the protractor is a **semicircle** (180°), or a half circle, with the circumference marked off (**graduated**) into degrees. There are two **scales** of degrees; one running from left to right, and the other from right to left.

To use the protractor properly, you must note these facts:

1. The circumference of the semicircle is marked off in degrees of an **arc** from 0° to 180°.

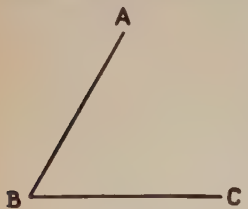


FIG. 2

2. There are two separate scales so that you can measure in either direction.

3. The center of the protractor is marked with an arrow.

Now study the following steps in the measurement of angle ABC (Fig. 2).

STEP I. Place the base of the protractor along one of the sides of the angle; for example, along the side BC with the arrow exactly at B. (B is called the **vertex** of the angle.)

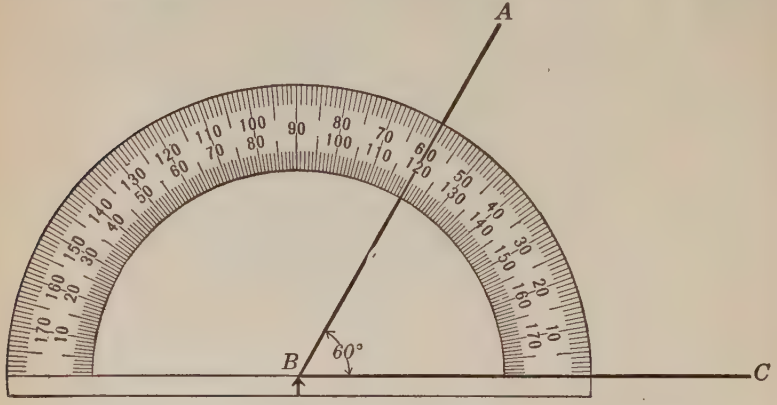


FIG. 3. Showing the protractor in proper position for reading the angle ABC

STEP II. Read the place where line AB cuts across the outer scale. If you have placed the protractor in the correct position, you will get 60° as your answer. We would say that angle ABC is an angle of 60° (see Fig. 3).

EXERCISE

If necessary, read again the two steps given on page 350 for the measurement of an angle.

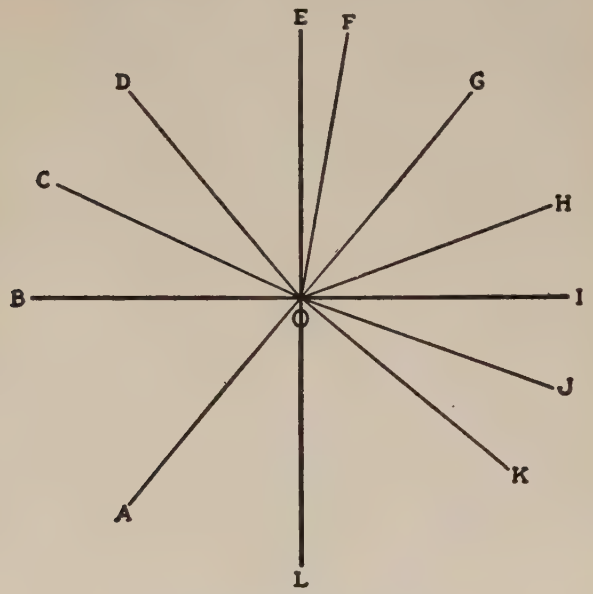


FIG. 4. Angles to be measured

1. Measure these angles (Fig. 4) to the nearest 5 degrees:
(*a*) *HOI* (*c*) *BOD* (*e*) *BOA* (*g*) *GOB* (*i*) *LOH*
(*b*) *GOI* (*d*) *BOE* (*f*) *FOI* (*h*) *BOK* (*j*) *AOE*

2. What is the sum of angles *IOJ* and *JOL*?

3. What is the sum of $\measuredangle$ *EOD*, *DOB*, and *BOL*?

4. What is the sum of $\measuredangle$ *BOC*, *COG*, *GOK*, and *KOB*?

5. What angle added to $\angle IOK$ would make a rt. $\angle$?

6. What angle added to angle *EOA* would make an angle of 180°?

191. The Construction of Angles. (Class and Written Work.) If you can measure the angles called for in the exercise on page 351, you will have no trouble in drawing angles of any size.

To draw any given angle, follow these three steps:

1. With a ruler or the edge of your protractor, draw a straight line which will form one side, or **leg,** of the angle.

2. Place the protractor along this line with the arrow at the point which is to be the vertex of the angle. Find the desired degree of the angle on the semicircular scale, and make a dot to show where the second side, or leg, of the angle should pass through.

3. Connect this dot with the vertex-point of the first line by means of a straight line. The result will be the desired angle.

Figs. 1 to 3 below show these three steps in the construction of an angle of 40°.

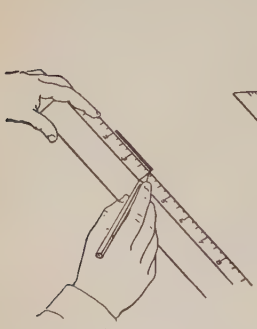


Fig. 1

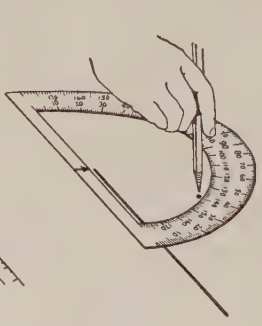


Fig. 2

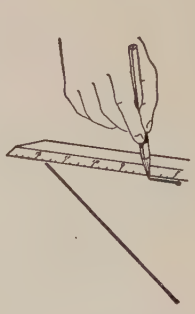


Fig. 3

Read the directions for drawing an angle again, study Figs. 1 to 3 once more carefully, and then draw the angles called for in the exercise on the next page.

1. Draw the following angles:

(*a*) An angle of 45°.
(*b*) An angle of 90°.
(*c*) An angle of 120°.
(*d*) An angle of 135°.
(*e*) An angle of 180°.
(*f*) An angle of 35°.
(*g*) An angle of 22°.
(*h*) An angle of 75°.
(*i*) An angle of 270°.
(*j*) An angle of 300°.

2. Draw a right angle and divide it into three equal angles. (Hint: How many degrees in each angle?)

3. Draw an angle of 180° and divide it into three equal angles.

4. Show by a drawing the number of right angles in a circle.

5. Draw a circle and divide it into ten equal parts. Think: How many degrees will there be in each of the ten angles?

192. The Vocabulary of Angles. (Study Period.) There are a number of words and expressions which you will need to know in connection with angles. Look over the list below and decide which ones you already know and which ones you need yet to learn.

1. Right angle
2. Radius
3. Straight angle
4. Acute angle
5. Obtuse angle
6. Vertex
7. Diameter
8. Circumference
9. Arc
10. Degree
11. Protractor
12. Angular motion
13. Perpendicular
14. Semicircle
15. Side (of an $\angle$)

In order to help you master each one of the above terms, the definitions on the next page are given. It is not necessary to learn these "by heart," but you should be able to define each in your own words, both with and without the use of diagrams.

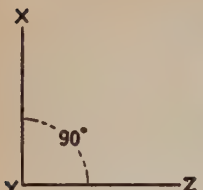


FIG. A

1. A **right angle** (a square corner) is an angle of 90°. Another way of defining a right angle is to say that the sides of a right angle are perpendicular to each other (Fig. A).

2. A **radius** is any straight line connecting a point on the circumference of a circle with the center of the circle (Fig. B). The surface swept over by a radius turning through an angle of 360° is a **circle.**

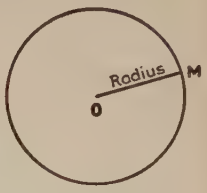


FIG. B

FIG. C

FIG. D

FIG. E

3. A **straight angle** is an angle of 180°. In other words, the sides of a straight angle form a **straight line** (Fig. C).

4. An **acute angle** is any angle of less than 90° (Fig. D).

5. An **obtuse angle** is any angle of more than 90° (Fig. E).

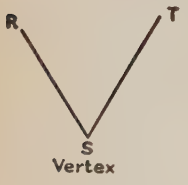


FIG. F

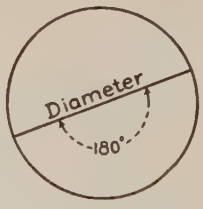


FIG. G

FIG. H

6. The **vertex** of an angle is the point where the two lines (**sides**) forming the angle meet (Fig. F).

7. A **diameter** is any straight line drawn through the center of a circle and which connects two points on the circumference. A diameter is also two radii placed so as to form an angle of 180°, a straight angle, (Fig. G).

8. The **circumference** is the closed curved line which forms the boundary of a circle. The circumference is also the line traced by the moving end of a radius in turning through an angle of 360° (Fig. H).

9. An **arc** is any portion of a circumference. An arc can also be said to be the curved line formed by the moving end of a radius in turning through an angle of any size (Fig. I).

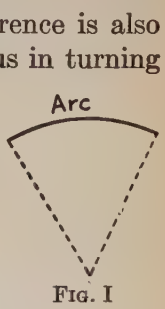
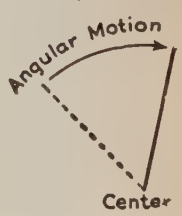


FIG. I

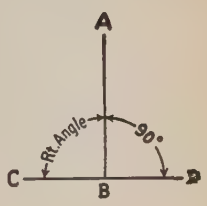
10. A **degree** is the angle formed by the turning of a radius through $\frac{1}{360}$ of a complete revolution about its center. There are therefore 360 degrees in any circle (See Fig. 5 on page 348.)

11. A **protractor** is a semicircular scale graduated (marked off) into degrees for the purpose of measuring and constructing angles.

12. **Angular motion** is circular motion, or the motion of a line moving around one end as a **center.** Thus, the hands of a clock and the spokes of a wheel have angular motion.



13. A line is said to be **perpendicular** to another line when the two lines form right angles (90°). Line AB is perpendicular to line CD.



14. A **semicircle** is half of a circle. It is also that part of a circle bounded by any diameter and an arc of 180°.

NOTE: **Arcs** are also measured in degrees. The arc contains as many degrees as does the angle formed by the radius in tracing the arc. The arc is a **line,** the angle is a **surface.**

15. The **leg** of an angle is a side, or one of the two lines forming the angle.

193. Practice with Angular Measure. (WRITTEN WORK.) Write the answers or make the drawings called for in each of the problems below.

1. Which of the following angles are acute angles? Which are obtuse angles? Which is a right angle? Which is a straight angle?

(*a*) 35°	(*d*) 90°	(*g*) 245°	(*j*) 45°	(*m*) 60°
(*b*) 180°	(*e*) 160°	(*h*) $90\frac{1}{2}°$	(*k*) 182°	(*n*) 325°
(*c*) 245°	(*f*) 16°	(*i*) 270°	(*l*) 210°	(*o*) 179°

2. Draw the following angles, using your protractor:

(*a*) 90°	(*c*) 280°	(*e*) 45°
(*b*) Straight angle	(*d*) $22\frac{1}{2}°$	(*f*) 135°

3. Draw a line 3″ long and label it AB. Draw a line BC which is perpendicular to AB at the point B.

4. Draw an arc of 90°.

5. Draw an angle of 75°, and label the vertex.

6. Draw a semicircle, and label the diameter, the arc, and the number of degrees in the angle at the center.

HINT: What kind of angle at the center is formed by the diameter of a circle?

194. Rapid Drill. (ORAL WORK.) Soldiers in the army take a set of physical exercises for a few minutes each morning, to use the different muscles in their bodies. The examples below are good "arithmetic exercises."

	(A)	(B)	(C)	(D)	(E)	(F)
1.	$82\overline{)691}$	$366 - 98$	*Add:* 9, 6, 8, 8	20 = what % of 200?	13.2×3	$1.41 - .58$
2.	$715 - 204$	47×1000	$\frac{5}{6} - \frac{1}{4}$	$1.6 - .05$	7 gal. = ...qt.	$49\overline{)442}$
3.	$19\overline{)114}$	$177 - 85$	$5\overline{)2}$ hr.	Formula for area of circle?	$.9 + .7 + .6$	Find 7% of 905
4.	$2 \div \frac{3}{5}$	$57 \times .5$	$3\frac{1}{3} = \frac{?}{6}$	$126 - 47$	15 = 150% of what number?	$37\overline{)261}$
5.	*Add:* 6, 2, 6, 4	$\frac{7}{8}$ = what %?	$96\overline{)895}$	Factors of 21 are 3 and	59×7	$942 - 810$
6.	63×4	$58.8 - 28.6$	What is $\frac{2}{3}$ of 60?	$13\overline{)80}$	$214 - 15$	2 bu. 3 pk. $\times 5$
7.	$262 - 84$	*Reduce:* 288 sq. in.	$26\overline{)182}$	$66\frac{2}{3}\%$ = what fraction?	*Add:* 5, 4, 9, 7	$85 \times .8$
8.	43×5	$151 - 63$	$2\frac{1}{2} + 1\frac{1}{4}$	$12.5 - 2.7$	$47\overline{)143}$	$.05 + .05 + .1$

SELF-TESTING DRILL NO. 27

195. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1. *Add:*

51
5637
8734
4167
3656

2. *Find the difference:*

18416.51
9390.96

3. *Multiply:*

9081
4957

4. *Find the sum:*

$2\frac{11}{12}$
$5\frac{3}{4}$
$3\frac{3}{8}$

5. *Divide:* 36.5668 by .452.

6. What number is sixteen times as large as $1\frac{5}{6}$?

7. Find the income from \$800 @ 5% for 1 yr. 6 mo.

8. *Multiply:*

5.67
67.98

9. *Multiply:*

3 gal. 3 qt.
5

10. If 24.9% of a number is 115, is 100% of the number about 4600, 2225, or 460?

11. *Find the sum:*

2 pk. 7 qt. 1 pt.
6 pk. 2 qt.
3 pk.
8 pk. 4 qt. 1 pt.

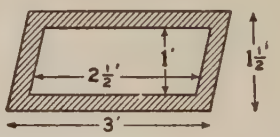


FIG. 1

12. What is the area of the shaded part of Figure 1?

13. What amount invested at $4\frac{1}{2}$% will pay my taxes which total \$421.65?

14. If 36 = 100% of a number, .18 = what per cent of the number?

Date	Checks in Detail			Date	Deposits	Date	Balance
	Balance brought forward ☛ $116.89						
Aug. 2	2.00	7.75		Aug. 2		Aug. 2	117.14
Aug. 6	13.50			Aug. 6		Aug. 6	103.64
Aug. 9	1.75	3.00		Aug. 9		Aug. 9	98.89
Aug. 15				Aug. 15	50.00	Aug. 15	148.89
Aug. 23	5.00	1.85	2.35	Aug. 23		Aug. 23	140.69
Aug. 27	4.16			Aug. 27		Aug. 27	136.53
Aug. 31	1.00			Aug. 31		Aug. 31	135.53

15. Check the final balance for August 31, 1927 ($135.53), as shown on the monthly statement above. If the final balance is correct, write "correct" as your answer. If a mistake has been made, write the correct final balance as your answer.

STANDARDS

Number correct	0–1	2	3	4	*	5	6	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

You are nearly through with seventh-grade arithmetic. These last drills are giving you a good "picture" of how able you will be to do good work in grade eight. It would be wise for you to study every mistake and use the *Self-Help Index*, page 471, to find needed review work.

THE MAGNETIC COMPASS

196. The Compass as an Instrument for Determining Direction. (Silent Reading.) Most of you have heard of the compass which sailors, woodsmen, hunters, aviators, and others use to tell direction. Probably you have seen a compass and have wondered how it works. Do you know that you can make a compass in a few minutes? Farther on in this article,

directions will be given for a simple experiment with a home-made compass.

A **compass** (better called a **magnetic compass**) is shown in Fig. 1 below. It looks like a watch with the directions (called **points** of the compass) marked on the dial instead of hours. There is a hand which turns on a pin, or pivot, in the center. Unlike the hand of a watch or clock, the hand on the compass is a magnet, like the horseshoe or bar magnet such as you can buy at small cost in the stores. You will understand how the compass works if you know something of the history of the compass.

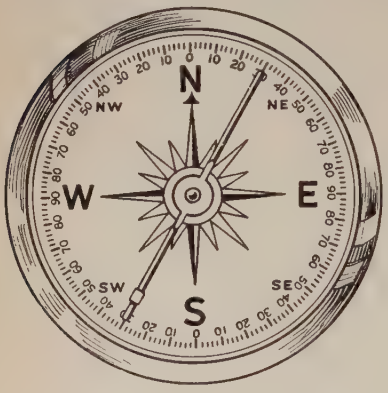


FIG. 1. A HUNTER'S COMPASS

Many centuries ago it was discovered that a certain kind of iron ore acted like a magnet; that is, it attracted bits of iron or steel. About the thirteenth century sailors began to use this curious ore (called "lodestone") in determining the directions by which to steer their ships. A needle made of lodestone will point almost north and south, if mounted on a pivot as shown above in the hunter's compass. You can make a compass at home from a sewing needle and a small piece of cork, provided you have an ordinary magnet with which to magnetize the needle.

197. An Experiment with a Home-made Compass. (HOME WORK.) Take a sewing needle and rub it hard for several minutes with the end of an ordinary horseshoe, or bar, magnet. It is best to stroke the needle with the magnet

always in the same direction. This will magnetize the needle and cause it to point nearly north and south when mounted on a piece of cork, now to be described.

Cut a thin piece of cork about an eighth of an inch thick. Run the needle through the cork as shown in Fig. 2. Place the cork and needle gently on the surface of some water, which nearly fills a shallow dish, such as a saucer or soup plate.

The needle, if magnetized, will turn slowly into a nearly north and south direction. Now try turning the needle in other directions and watch it return to the north and south position.

NOTE: A compass doesn't point directly toward the north pole, but rather toward the magnetic north pole, which is located near Hudson Bay. In the central States, the compass points within 5° or 10° of true north.

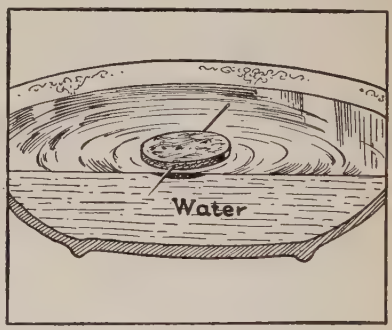


FIG. 2. A HOME-MADE COMPASS

In the Pacific States the compass will point 15° to 20° east of north. In the Eastern States it will point from 10° to 20° west of true north. Any one using a compass a great deal soon learns about how many degrees away from true north the needle of a compass points in his own neighborhood.

198. A Project with a Hunter's Compass. (CLASS WORK.) Borrow a hunter's compass (or other small pocket compass) from some one in your town or city. "Line up" the north and south poles of the compass as nearly in a straight line north and south as you can. Then read the hand and see how many degrees east or west of true north it stands.

From the facts below you can see about what the reading should be for your part of the United States.

If your results do not agree fairly closely with the figure given for the city nearest where you live, the error will be due chiefly to the fact that it is difficult to line up the N and S on the compass in a true north and south line.

Baltimore	7° W	Omaha	10° E
Boston	15° W	Philadelphia	9° W
Chicago	3° E	Portland (Maine)	16° W
Columbus	2° W	Portland (Oregon)	23° E
Denver	15° E	San Antonio	10° E
New Orleans	6° E	San Francisco	18° E
New York City	11° W	Savannah	0°

7° W (as given for Baltimore) is read: The compass points 7° west of true north.

3° E (as given for Chicago) is read: The compass points 3° east of true north.

QUESTIONS

1. How many degrees are there on the dial of a compass?

2. Why are the degrees shown as they are on the compass on page 360?

3. How many degrees apart are true east and true north?

4. Do you see that, in general, all cities west of a line running straight south of Hudson Bay (which is near the magnetic north pole) have compass readings which are east of true north, and that all cities east of this line show compass readings which are west of true north? Can you explain this fact?

199. Review of Angular Measurement. (WRITTEN WORK.) Notice that there are two parts to this article. Part I reviews the ideas about angles, and Part II asks you to work some problems and make some drawings.

PART I

Number your papers carefully and write the word or words that should be on each blank.

1. To describe the position of an object, we must know both the distance and the.....from a known point.

2. At 3 o'clock the hands of a clock form an angle of .....degrees.

3. In twelve hours the hour hand of a watch passes through an angle of.....degrees.

4. The figure formed by the tip of a hand of a clock in making a complete revolution is called a.....

5. At exactly six o'clock the hands of a clock form a .....angle.

6. The unit for measuring angles or angular motion is the.....

7. The circumference of any circle contains..... degrees.

8. There are.....degrees in the arc of a semi-circle.

9. There are.....right angles in a straight angle.

10. One line drawn at a 90° angle to another line is said to be.....to that line.

11. The hand of a clock or watch can be thought of as a.....of a circle.

12. There are.....angles of 45° in a circle.

13. Angles drawn on paper may be measured by means of an instrument called a.....

14. A protractor is graduated into.....degrees.

15. An angle of less than 90° is called an..... angle.

16. An angle of more than 90° is called an..... angle.

17. An angle of 90° is called a.....angle.

18. An angle of 180° is called a.....angle.

19. The point where the two sides of an angle meet is called the.....

20. Any portion of the circumference of a circle is called an.....

21. Two radii which form a diameter make an angle of .....degrees.

22. The sides of a straight angle form a ....

23. Any line which connects two points on the circumference and which passes through the center of a circle is a.....

24. The pointer or hand on a compass is really a.....

25. The compass in your neighborhood points approximately.....degrees.....of true north.

PART II

1. With a ruler draw two lines which intersect (cross) near their mid-points. (Do not make the angles right angles.) Measure all four angles formed. Are any two of the angles equal?

2. Draw a figure which shows that a straight angle equals two right angles.

3. Draw an angle of 45°. One of 75°. One of 135°.

4. Draw a right triangle of the dimensions: one side, 5″; one side, 3″; and the third side, 4″. Measure the two

angles other than the right angle. (*a*) What is the sum of these two angles? (*b*) Are they acute or obtuse angles?

5. Draw a triangle with one angle an obtuse angle.

6. See if you can draw a triangle with two angles obtuse. Can this be done?

7. See if you can draw a triangle with two angles that are right angles. Can this be done?

8. See if you can draw a triangle with all three angles acute. Can this be done?

9. Draw the face of a clock showing the hands at eight o'clock. Measure the angle formed by the hands.

10. What angle must be added to each of the following angles to make a right angle?

(*a*) 15° (*b*) 35° (*c*) $22\frac{1}{2}°$ (*d*) 90° (*e*) 60° (*f*) 75°

11. What angle must be added to each of the following angles to make a straight angle?

(*a*) 90° (*b*) 45° (*c*) 135° (*d*) $22\frac{1}{2}°$ (*e*) 165° (*f*) 180°

12. (*a*) Through how many degrees does the hour hand of a clock pass between 6.00 A.M. and 6.00 P.M.? (*b*) Through how many degrees does the minute hand pass in the same length of time?

13. What is the sum of the four angles of a rectangle?

14. Does increasing the length of the sides of an angle increase the number of degrees in the angle?

15. Write a short paragraph telling how a hunter's compass works and why it does so work.

200. Practice in Estimating. (WRITTEN WORK.) Do not work the examples below. Write as your answers the best estimate to be found in the column at the right. If the estimate is a denominate number, name it.

1. $.93 \times 8082$
2. What sum equals a $6\frac{1}{2}\%$ discount on \$186?
3. Area of a triangle with base of 80′ and altitude of 20′.
4. 54 is what per cent of 450?
5. $\frac{2}{5}$ of $6\frac{1}{8}$ = ...
6. If $228 = 19\%$ of N, N = ...
7. Find 250% of 302.
8. Interest on \$530 for 3 mo. @ 6%.
9. $.24\overline{).19032}$
10. Find the volume of a rectangular solid with dimensions $3\frac{2}{3}' \times 4' \times 7\frac{3}{4}'$.

(a) .8
(b) 120
(c) 800
(d) 12
(e) $2\frac{1}{2}$
(f) 12%
(g) 8000
(h) 750
(i) 8.00
(j) 1200

GETTING READY FOR THE EIGHTH GRADE

201. A Project in Self-Direction. (CLASS WORK.) There is no new work in Arts. 202-211. These articles are important, however, because they use processes and ask for facts which you must have well in mind in order to do good work in the eighth grade. If you can work Arts. 202-211 correctly without help from anyone, you are ready for the arithmetic work of Grade Eight.

It will be a good plan to watch your work in arithmetic very carefully the next few weeks. Some pupils have found it

helpful to make a record of their weak points and then to make themselves do a little extra work on these weak points.

A good way to do this is to keep "notes" on your work. These serve as a guide for extra practice. The chart below shows a part of the notes of a seventh-grade pupil. Florence put on her chart the number of each example she missed. Then she decided what articles to study by using the Self-Help Index on page 471.

Arithmetic Notes — If found please return to Florence White, Room 7, Wilson School.

Date	Art. No.	Examples missed	What To Do
May 10	Art. 202	3	Practice on Arts. 216, 217
	Drill 28	12	Practice on Art. 280
May 12	Art. 204	3	Study Arts. 99, 100
	Problem Scale XIV	8	Misread prob. Study Arts. 18, 105
May 14	Art. 207	5	Study Art. 279
	Drill 29	7	Study Art. 236
May 17	Art. 210	6	Study Art. 236
	Drill 30	9	Study Arts. 264, 265
May 19	Art. 211	2	Study Arts. 288, 289
		8	Study Arts. 38, 40
		15	Study Art. 109

You will enjoy directing your extra practice if you make your notes thoughtfully.

SELF-TESTING DRILL NO. 28

202. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. **Even if you rate high on this drill, you should be sure that you understand just what your mistakes were, if any.**

1. *Subtract:*

170400
97163

2. *Subtract:*

88.0488
30.4689

3. *Add:*

709.2
80.5
209.8
11.0
518.2
92.2
172.9

4. *Divide:* $4\frac{3}{8}$ by $3\frac{3}{4}$

5. *Subtract:*

19 ft. 3 in.
14 ft. 10 in.

6.

$452\overline{)217412}$

7. $39\frac{3}{8}$ is how much less than $62\frac{1}{6}$?

8. *Multiply:*

34.59
9.75

9. Find $87\frac{1}{2}\%$ of 784.

10. $66\frac{2}{3}\%$ of a certain number = 1654. 100% of that number = ?

11. If 67 = 100% of a number, .67 equals what per cent of the number?

12. 9 yd. 2 ft. divided by 9 = ?

13. Read the thermometer shown at the left.

14.* On the next page are given three formulas. Below them are six problems. Which problem in the list below could be solved by formula *a*? By formula *b*? By formula *c*?

Formulas:

(*a*) $i = prt$ (*b*) $A = ab$ (*c*) $V = lwh$

Problems:

(1) To find the volume of a room.

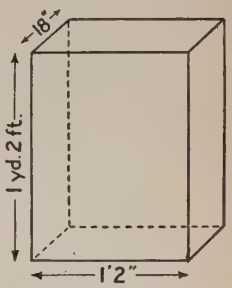
(2) To find the circumference of a circle.

(3) To find the area of a triangle.

(4) To find the area of a rectangle.

(5) To find the rate of interest.

(6) To find interest.



15. Find the volume of the figure at the right.

*To count Example 14 correct, each part must be correct.

STANDARDS

Number correct	0–1	2	3	4	5	*	6	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

What notes should you make on this drill?

PROBLEM SOLVING

203. Thinking About Problems. (Class Work.) Spend 10 minutes in studying the questions below. Be ready to tell just how you would answer each question.

1. If a problem asked you to find the sum of several numbers, what process would you use?

2. If 15 is a certain per cent of a number, how would you find the number?

3. If each of a certain number of pupils gave the same amount of money, how would you find the total amount given?

4. If you knew the dimensions of a room, how could you find its volume?

5. How can you check a division example?

6. How can you find each one's share, if you know the amount to be distributed, the number who are to share, and that all are to share equally?

7. How would you find the average of a list of 8 numbers?

8. How would you make very sure that the answer to a problem using subtraction is correct?

9. If you knew how much money was invested and how much money the investment earned per year, how would you find the per cent at which the money was invested?

10. If a problem asked you to find how much more one number is than another, would you add, subtract, multiply, or divide?

11. How would you find a given per cent of 75?

12. How would you find the interest, if you knew the sum of money on which the interest is reckoned, the per cent, and the length of time the money was at interest?

13. If you know the dimensions of a triangle, how can you find its area?

14. How would you find the money to be paid a salesman if you knew the amount of his sales and the per cent he was allowed as commission?

15. How would you find what per cent n is of h? How would you find what per cent h is of n? (NOTE: Think of n as being some number, and h as being another number.)

PROBLEM SCALE NO. XIV

204. Problems Requiring Careful Reading. (WRITTEN WORK.) You can earn 65 points by working every problem correctly. **Take time to get clearly in mind what each problem asks you to find.**

1.* Figure the prices for the Annual Marked-Down Sale. Use these facts and report your answers as shown. (5)

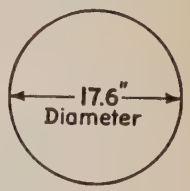
Regular Price	*Reduction*	*Marked-Down Price*
(*a*) Goods A, $ 5.00	$\frac{1}{2}$ off	.....
(*b*) Goods B, $ 3.00	$\frac{1}{3}$ off	.....
(*c*) Goods C, $24.00	20% off	.....
(*d*) Goods D, $42.00	$37\frac{1}{2}$% off	.....

2. A can of spinach weighs 1 lb. 1 oz., a can of clingstone peaches 1 lb. 13 oz., a can of sliced peaches 10 oz., a can of prunes 1 lb. 12 oz., and a can of fancy corn 1 lb. 4 oz. Three boys took the following amounts of canned goods for food on a canoe trip:

5 cans of spinach
4 cans of clingstone peaches
6 cans of sliced peaches
2 cans of prunes
7 cans of fancy corn

How much did the boys' canned goods weigh? (6)

3. Find the length of the circumference of the circle shown at the right. The figure gives the length of the diameter. (6)



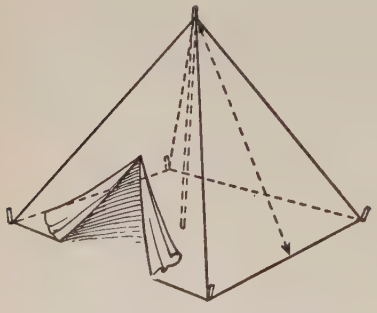
4. The question: "What state is next south of Kentucky?" was asked of a large number of pupils. 38%, or 1150, of these pupils got the correct answer. How many pupils were asked the question? (Ignore fraction in ans.) (7)

*All parts must be correct for Problem 1 to count as correct.

5. The Dane Coal Company borrowed $4500 for ninety days @ $6\frac{1}{2}\%$ in order to take advantage of certain discounts in buying from a mine. How much did the loan cost the company? (7)

6. Nineteen hundred twenty-four pupils in various seventh grades were asked in geography classes this question: "In what state is Mt. Shasta located?" Only 17.5% of these pupils gave the correct answer. How many pupils failed to answer this question correctly? (Ignore the fraction in the answer.) (8)

7. The base of a certain triangle is 4 miles. The altitude from this base is 7 miles. What is the area of the triangle? (8)



8. Calculate the amount of canvas it took to make this tent. The tent is 10 ft. by 10 ft. at the bottom. The dotted line running from the top of the tent down one side represents an altitude of 14 ft. No extra canvas is needed for the door, and there is no canvas on the floor of the tent. (9)

9. The three sides of a right triangle are 9 ft., 12 ft., and 15 ft. What is the area of the triangle? (It may help you to make a rough drawing.) (9)

STANDARDS

Superior work:	31–65 points	Slow work:	11–13 points
Good work:	21–30 points	Inferior work:	0–10 points
Average work:	14–20 points		

MIXED REVIEW

205. Mixed Review of Important Facts. (ORAL WORK.) There are three parts to this article. Study each part before the oral recitation begins.

After a few minutes of study every seventh-grade pupil should be able to give the correct answer to each item in this article.

PART I. WHOLE NUMBERS

1.	2.	3.	4.
(*a*) $7\times5+6$	(*a*) $53-48$	(*a*) $37\div6$	(*a*) $17\overline{)102}$
(*b*) $7\times7+6$	(*b*) $47-42$	(*b*) $52\div7$	(*b*) $50\overline{)400}$
(*c*) $8\times6+7$	(*c*) $41-36$	(*c*) $84\div9$	(*c*) $58\overline{)464}$
(*d*) $9\times4+8$	(*d*) $61-54$	(*d*) $75\div8$	(*d*) $19\overline{)171}$

5. $\begin{array}{r} 489 \\ \times 306 \\ \hline \end{array}$ Would be checked by.....

6. $36\overline{)14437}$ $401\frac{1}{36}$ Would be checked by.....

7. $\begin{array}{r} 83172 \\ -4185 \\ \hline 78987 \end{array}$ Would be checked by.....

8. The number 8,476,341 is read as follows:.....

9. In the number 8527: (*a*) the figure in the tens' place is....., (*b*) in the hundreds' place is....., (*c*) in the units' place is....., (*d*) in the thousands' place is.....

10. How can addition be checked?

PART II. FRACTIONS

1. Give the common denominator to use:

(*a*) $\frac{}{4}+\frac{}{6}$ (*d*) $\frac{}{4}+\frac{}{10}$ (*g*) $\frac{}{6}+\frac{}{10}$
(*b*) $\frac{}{3}+\frac{}{12}$ (*e*) $\frac{}{12}-\frac{}{8}$ (*h*) $\frac{}{16}-\frac{}{8}$
(*c*) $\frac{}{8}-\frac{}{6}$ (*f*) $\frac{}{4}-\frac{}{12}$ (*i*) $\frac{}{10}+\frac{}{15}$

2. Reduce the following to good forms for answers:

(*a*) $\frac{9}{2}$ (*c*) $1\frac{7}{2}$ (*e*) $6\frac{10}{4}$ (*g*) $5\frac{11}{3}$ (*i*) $\frac{8}{12}$
(*b*) $2\frac{6}{5}$ (*d*) $\frac{3}{9}$ (*f*) $2\frac{27}{12}$ (*h*) $7\frac{21}{9}$ (*j*) $7\frac{12}{4}$

3. Change the following to improper fractions:

(*a*) $4\frac{1}{2}$ (*c*) $11\frac{2}{5}$ (*e*) $9\frac{5}{12}$ (*g*) $3\frac{3}{4}$ (*i*) $6\frac{2}{3}$
(*b*) $1\frac{1}{4}$ (*d*) $3\frac{1}{3}$ (*f*) $6\frac{5}{6}$ (*h*) $7\frac{1}{2}$ (*j*) $3\frac{5}{8}$

4. Tell what canceling would be possible in each:

(*a*) $\frac{4}{15}\times\frac{5}{24}$ (*c*) $8\times\frac{19}{4}$ (*e*) $\frac{7}{8}\times\frac{12}{49}$
(*b*) $\frac{7}{2}\times\frac{8}{7}$ (*d*) $\frac{6}{11}$ of $\frac{5}{8}$ (*f*) $12\times\frac{33}{8}$

5. Read each of the following as multiplication examples:

(*a*) $\frac{2}{3}\div\frac{4}{5}$ (*c*) $\frac{3}{8}\div 6$ (*e*) $7\frac{1}{3}\div 2\frac{3}{4}$
(*b*) $8\div\frac{3}{7}$ (*d*) $4\frac{1}{2}\div 2\frac{1}{3}$ (*f*) $5\frac{1}{6}\div 1\frac{7}{8}$

6. Tell what should be on each blank:

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)	(*i*)	(*j*)	(*k*)	(*l*)	(*m*)	(*n*)
Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{5}{8}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{2}{3}$	$\frac{1}{8}$	$\frac{2}{4}$	$\frac{3}{5}$	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{1}{1}$
Decimal	....	....	....	....	....	....	....	....	....	....	....	....	....	....
Per cent	....	....	....	....	....	....	....	....	....	....	....	....	....	....

7. Which of the fractions is the larger in each pair?

(*a*) $\frac{1}{2}$ or $\frac{1}{3}$ (*c*) $\frac{4}{5}$ or $\frac{5}{4}$ (*e*) $\frac{3}{5}$ or $\frac{1}{2}$ (*g*) $\frac{1}{9}$ or $\frac{1}{8}$
(*b*) $\frac{3}{4}$ or $\frac{2}{3}$ (*d*) $\frac{7}{8}$ or $\frac{8}{9}$ (*f*) $\frac{6}{7}$ or $\frac{6}{8}$ (*h*) $\frac{4}{6}$ or $\frac{3}{4}$

PART III. DECIMALS

1. Read the correct answers. The figures are correct.

(*a*)
$$\begin{array}{r} 43.6 \\ 72.8 \\ .09 \\ 4.72 \\ \hline 12121 \end{array}$$

(*b*)
$$\begin{array}{r} 427.8 \\ -3.26 \\ \hline 3952 \end{array}$$

(*c*) $4.26 \times 70.3 = 299478$

(*d*) $3.41\overline{)247.566}$ — 726

2. In the number 7812.345:

3 is in the.....place. 1 is in the.....place.

8 is in the.....place. 4 is in the.....place.

5 is in the.....place. 7 is in the.....place.

2 is in the.....place. The number is read.....

3. Tell what should be on each blank. Give fractions in lowest terms.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)
Decimals	.5	.6	1.00	$.16\frac{2}{3}$	.01	.875	.95
Fractions	.....	.....	.....	.....	.....	.....	.....
Per cents	.....	.....	.....	.....	.....	.....	.....

	(*h*)	(*i*)	(*j*)	(*k*)	(*l*)	(*m*)	(*n*)
Decimals	$.33\frac{1}{3}$	.84	.05	2.50	.75	.005	1.25
Fractions	.....	.....	.....	.....	.....	.....	.....
Per cents	.....	.....	.....	.....	.....	.....	.....

4. Tell which example in each of the pairs below will give the larger answer.

(*a*) *Multiply:*

$$\begin{array}{r} 47.6 \\ 7.8 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 47.6 \\ .78 \\ \hline \end{array}$$

(*b*) $3.48\overline{).92176}$ or $348\overline{)921.76}$

(*c*) $78.1\overline{)4670.17}$ or $.0781\overline{)46.7017}$

(*d*) *Multiply:*

$$\begin{array}{r} 9084 \\ .04 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 9084 \\ .004 \\ \hline \end{array}$$

206. Rapid Drill. (ORAL WORK.) Here is the last rapid drill in the seventh grade. If you worked all the others carefully, this one will not be hard to do. In the long division examples, give only the first quotient figure.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	Find 6% of \$250	$\begin{array}{r} 432 \\ \times 2 \\ \hline \end{array}$	$63\overline{)256}$	64 oz. = ...lb.	*Add:* 10, 15, 6	$\begin{array}{r} 913 \\ -230 \\ \hline \end{array}$
2.	$\begin{array}{r} 12.0 \\ -8.4 \\ \hline \end{array}$	How many degrees in a circle?	$\begin{array}{r} .8 \\ +1.8 \\ +1.8 \\ \hline \end{array}$	$\begin{array}{r} 142 \\ -75 \\ \hline \end{array}$	$29\overline{)143}$	45 = what % of 15?
3.	1 mi. = ...ft.	$27\overline{)135}$	$\begin{array}{r} 324 \\ -69 \\ \hline \end{array}$	$2\frac{1}{2} \div 4$	$\begin{array}{r} 894 \\ -104 \\ \hline \end{array}$	$\begin{array}{r} 9.6 \\ \times 5 \\ \hline \end{array}$
4.	$\frac{3}{8}$ = what %?	$\begin{array}{r} 120 \\ \times .01 \\ \hline \end{array}$	What is an acute angle?	$48\overline{)427}$	$\begin{array}{r} 113 \\ -84 \\ \hline \end{array}$	$\begin{array}{r} .07 \\ +2. \\ +.06 \\ \hline \end{array}$
5.	$74\overline{)299}$	$\begin{array}{r} 112 \\ -96 \\ \hline \end{array}$	*Best form for ans.?* $\frac{3}{2}\frac{5}{5}$	3 T. − 3000 lb.	$\begin{array}{r} 24 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 4.21 \\ -.65 \\ \hline \end{array}$

	(A)	(B)	(C)	(D)	(E)	(F)
6.	*Add:* 7, 9, 7, 9	43% = what decimal?	94 ×3	.136 −.068	$\frac{1}{3}+\frac{3}{10}$	$18\overline{)149}$
7.	15.3 −6.9	$3\times\frac{4}{5}$	$36\overline{)252}$	*Add:* 9, 5, 5, 6	95.7 ×.01	145 −57
8.	$26\overline{)79}$	319 −187	4 × 9 mo. = ... yr.	$4\frac{3}{4}-1\frac{3}{4}$	35 = 70% of N Find N	95 ×9

SELF-TESTING DRILL NO. 29

207. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself.

1. *Subtract:*
1379484
945897

2. *Subtract:*
113749.8
36289.7

3. *Add:*
89110
770869
881001
494817

4. *Multiply:*
6859
241

5. *Multiply:*
2 bu. 3 pk.
8

6. *Add:*
$2\frac{2}{3}$
$\frac{7}{9}$
$3\frac{5}{6}$

7. $.987\overline{)343.476}$

8. What is the total area of the three farms whose areas are listed at the right?

1 sq. mi. 250 A.
1 sq. mi. 526 A.
1 sq. mi. 319 A.

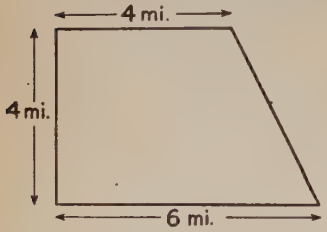
9. $3\frac{5}{6}\times 9\frac{3}{4}=?$

10. What is 300% of 310?

11. What is 125% of 292?

12. 126 is what per cent of 450?

13. Is $16.5029 \div 8.24$ about 200, 2, or .2?



14. Find the area of the figure at the left.

15. Fill in the blanks below for the facts given in the problem. Do not solve the problem.

PROBLEM: Find the income made on an investment of $750 @ $7\frac{1}{2}\%$ for 2 yr. 6 mo. $i =$ ..... $\times$ ..... $\times$ .....

STANDARDS

Number correct	0–1	2	3	4	5	6	*	7	8	9–14	15
Rating	0	1	2	3	4	5	6	7	8	9	10

Your mistakes on this drill will show what review study you should do.

208. Mixed Review of Important Facts. (ORAL WORK.) The two parts of this article will give you a chance to review denominate numbers and percentage. Study for a few minutes and be ready with the answers.

PART I. DENOMINATE NUMBERS

1. Tell what should go on each blank:

I. 4 ft. = ... inches.
II. 1 cu. yd. = ... cu. ft
III. 17 in. = ... ft.
IV. 16 qt. = ... pk.
V. 10 bu. = ... pk.
VI. 13 quarters = $ .....
VII. 9 qt. = ... gal.
VIII. $9\frac{1}{3}$ yd. = ... ft.
IX. 1 mi. = ... ft.
X. 109 yd. = ... ft.
XI. 8 oz. = ... lb.
XII. 90 min. = ... hr.
XIII. $\frac{1}{2}$ ft. = ... in.
XIV. 1 sq. ft. = ... sq. in.
XV. 7 ft. = ... yd.
XVI. 9 qt. = ... pt.
XVII. $7\frac{1}{2}$ dimes = ... cents
XVIII. $1\frac{1}{4}$ lb. = ... oz.

2. Tell what the black-faced numbers will become as the division is being performed:

(*a*) $4\overline{)1\text{ ft. }\mathbf{8}\text{ in.}}$ (*c*) $5\overline{)6\text{ lb. }\mathbf{3}\text{ oz.}}$ (*e*) $6\overline{)8\text{ gal. }\mathbf{3}\text{ qt.}}$

(*b*) $3\overline{)4\text{ bu. }\mathbf{2}\text{ pk.}}$ (*d*) $2\overline{)7\text{ qt. }\mathbf{1}\text{ pt.}}$ (*f*) $4\overline{)9\text{ yd. }\mathbf{2}\text{ ft.}}$

3. Reduce these numbers to good forms for answers:

(*a*) 8 ft. 17 in. (*c*) 4 qt. 7 pt. (*e*) 5 lb. 21 oz.

(*b*) 3 bu. 7 pk. (*d*) 3 yd. 11 ft. (*f*) 3 sq. ft. 150 sq. in.

PART II. PER CENTS

1. Tell what ought to be on each blank. You need not give an answer where a cross (x) stands.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)
Per cents	50%	100%	$66\frac{2}{3}\%$	7%	25%	$12\frac{1}{2}\%$	$6\frac{1}{4}\%$
Decimals	.....	.....	.....	.....	.....	.....	.....
Fractions	.....	.....	.....	.....	.....	.....	x

	(*h*)	(*i*)	(*j*)	(*k*)	(*l*)	(*m*)	(*n*)
Per cents	120%	$16\frac{2}{3}\%$	$37\frac{1}{2}\%$	$3\frac{1}{2}\%$	.005%	210%	1%
Decimals	.....	.....	.....	.....	.....	.....	.....
Fractions	.....	.....	.....	x	x	.....	.....

2. At the top of the next page are some questions in percentage. Restate each question in example form. The sample above the questions will show you how you are to give your answers.

SAMPLE: What is 4% of 96? Answer in words: "Point zero 4 times ninety-six."

(*a*) What is 9% of 316?
(*b*) What % of 84 is 31?
(*c*) 484 is 6% of N, N = ...?
(*d*) 580 is what % of 970?
(*e*) What is 125% of 347?
(*f*) 84 is what % of 64?
(*g*) What is N when 87 = 140% of N?
(*h*) If 37 = $1\frac{1}{2}$% of N, N = ...?

3. Make questions in percentage which would be answered by these examples:

(*a*) $\frac{37}{148}$ or $148\overline{)37.00}$
(*b*) $.06\overline{)\$452}$
(*c*) $.72 \times \$724$
(*d*) $1.42\overline{)4372.}$
(*e*) $.005\overline{)3075}$
(*f*) $\frac{73}{21}$ or $21\overline{)73.}$
(*g*) $1.125 \times \$936$
(*h*) $\frac{1}{4}$ or $4\overline{)1.00}$
(*i*) $\frac{421}{1236}$ or $1236\overline{)421.00}$

4. Restate each question in example form. You will find it useful to know the formulas $i = prt$ and $r = \frac{i}{p}$ or $i \div p$.

(*a*) What is the interest on \$745 for 3 yr. 6 mo. @ 7%?

(*b*) What is the interest on \$1200 for 3 mo. @ 4%?

(*c*) What is the interest on \$500 at $6\frac{1}{2}$% for 2 yr.?

(*d*) If \$500 earns \$40 a year interest, at what rate is it invested?

(*e*) \$80 a year dividend is paid on an investment of \$1200. At what per cent is the money invested?

(*f*) What is the rate of interest earned on an investment of \$5000 which pays \$350 dividend per annum?

(*g*) What is the interest on \$750 @ $4\frac{1}{4}$% for 30 da.?

(*h*) What is the interest on \$1000 @ $5\frac{1}{4}$% for 8 mo.?

(*i*) What is the rate per cent on an investment of \$800 which pays \$54 annually?

209. Twenty-Five Review Examples. (ORAL WORK.) All seventh-grade pupils who expect to do good work in the eighth grade should be able to give the correct answer to each example below without having to use paper and pencil. Answers must be reduced when possible.

1. *Add:*

49
36
42

2. *Subtract:*

437
149

3. *Multiply:*

87
6

4. $9\overline{)982}$

5. $\frac{1}{4}+\frac{2}{3}$ **6.** $4\frac{1}{2}-2$ **7.** $\frac{2}{3}\times\frac{5}{8}$ **8.** $\frac{4}{5}\div 6$

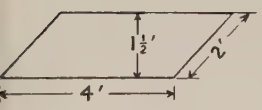
9. $4.3+.14$ **10.** $6.28-.163$ **11.** $.7\times 3.4$ **12.** $.8\overline{)656}$

13. 4 ft. 11 in.
4 in.
2 ft. 3 in.

14. *Subtract:*
2 yd. 1 ft.
1 yd. 2 ft.

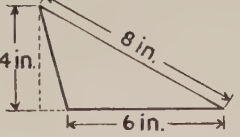
15. *Multiply:*
2 qt. 1 pt.
7

16.



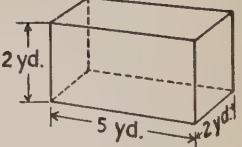
$A =$ .....?

17.



$A =$ .....?

18.



$V =$ .....?

19. $3\overline{)5 \text{ gal. } 1 \text{ qt.}}$ **20.** 6% of 400 **21.** 4 is what % of 12?

22. If $9 = 25\%$ of N, $N =$ .....?

23. $i =$ .....when $p = \$200$, $r = 6\%$, $t = 2$ yr. 6 mo.

24. $r =$ .....when $p = \$100$ and $i = \$7$ per year.

25. 6×4.49 equals about: 2700, 270, 27, or 2.7.

SELF-TESTING DRILL NO. 30

210. Directions. Time allowed: 15 minutes. Follow the directions given for Drills 1 to 5. Rate yourself. After you have marked your rating on your Chart, take time to study the errors you have made. Be sure you understand just what your trouble was.

1. *Add:*

\$ 5.10
43.44
6.54
22.56
25.78

2. If 50% of $N = 900$, 100% of $N = ?$

3. $8.71\overline{)61135.49}$

4. *Subtract:*

$62\frac{1}{6}$
$39\frac{3}{8}$

5. $869\overline{)543125}$

6. Martha said that 6897 multiplied by 578 equals 3,984,466. If she was correct, write "Correct" for your answer. If she was wrong give the correct product.

7. *Find the product:*

1.934
690.8

8. Find the cost of borrowing \$750 for 3 mo. @ $3\frac{1}{2}\%$.

9. $6 \div 2\frac{7}{12} = ?$

10. *Subtract:*

9 sq. yd. 3 sq. ft.
6 sq. yd. $6\frac{1}{2}$ sq. ft.

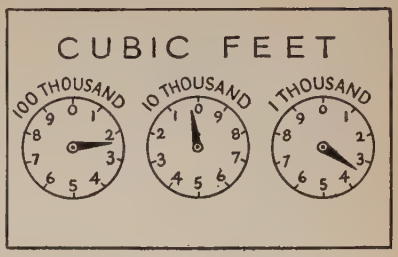
11.* (*a*) $6\frac{1}{2}$ ft. = ... in.
(*b*) 5 pk. = ... qt.
(*c*) $\frac{4}{5}$ equals what decimal?
(*d*) 15,840 ft. = ... mi.

12. What per cent of 1300 is 585?

13. $7\overline{)6 \text{ ft. } 11 \text{ in.}}$

14. Find the volume of a wall safe, the inside dimensions of which are $2'8'' \times 1$ yd. 1 ft. $\times 3'$.

15. The gas meter shown at the right reads..... cubic feet.



*To count Example 11 correct, each part must be correct.

STANDARDS

Number correct.....	0	1	2	*	3	4	5	6	7	8–14	15
Rating.....	0	1	2	3	4	5	6	7	8	9	10

REVIEW OF IDEAS

211. Inventory of Arithmetic Ideas. (Class Work.) Below are some questions and little problems to use in reviewing important ideas in arithmetic. If you have done good work in seventh-grade arithmetic, you will find it very interesting. Make sure to find out the answers to any questions which bother you.

1. Can you name three places in arithmetic where we have to change the form but not the value of a number? (Anyone will be able to think of reducing fractions and answers, but there are many other places.)

2. Which of the five examples below is the right one to use in finding $5\frac{1}{4}\%$ of 435?

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
535	435	$435 \div 5\frac{1}{4}$	435	$.0525\overline{)435}$
$\times .525$	$\times .0525$		$\times 5\frac{1}{4}$	

3. Which of the five examples below is the right one to use in answering this question: What per cent of 386 is 655?

(*a*) $\frac{6}{3}\frac{5}{8}\frac{5}{6}$ or $386\overline{)655.00}$ (*b*) $\begin{array}{r} 386 \\ \times .655 \end{array}$ (*c*) $\begin{array}{r} 386 \\ \times 6.55 \end{array}$

(*d*) $\frac{3}{6}\frac{8}{5}\frac{6}{5}$ or $655\overline{)386.00}$ (*e*) $655-386$

4. Which example of the four below would you use in finding 100% of a number if you knew that $12\frac{1}{2}\%$ of the number was \$75?

(*a*) $1.25\overline{)75.00}$ (*b*) $\begin{array}{r} 75 \\ \times 1.25 \end{array}$ or $75\times\frac{1}{8}$ (*c*) $.125\overline{)75.000}$ (*d*) $\begin{array}{r} 75 \\ \times 12\frac{1}{2} \end{array}$

5. What important ideas in arithmetic do each of the following words suggest? *indorsed discounted cashed receipted deposited balanced security amount principal promissory commission.*

6. Be ready to go to the board and explain clearly how a remainder in the quotient in a long division example is used in checking.

7. What are good questions to ask oneself when solving arithmetic problems?

8. Can you tell the things that a citizen should know about electric meters and gas meters? (Can you give your answer without making any mistakes in grammar?)

9. What questions in percentage would the following examples, if worked correctly, answer?

(*a*) $456\times.07$ (*b*) $\frac{9}{3}\frac{8}{8}\frac{8}{9}$ or $389\overline{)98.00}$ (*c*) $.64\overline{)470.00}$

10. What important ideas do each of the following words suggest? *deposit slip deposit book monthly statement check stub.*

11. Think out carefully just how you would reply to this request: "Please give me the main ideas about budgets and budget making."

12. Mr. Bailey was "all mixed up" about the meanings of the word capacity. Can you explain the different ways in which this word is used?

13. You have learned five important facts about the formula. Be ready to explain these if you are called upon to do so.

14. What important ideas do each of the following words suggest to you? *square measure cubic measure radius base and altitude perpendicular circumference diameter* π.

15. Suppose one of your friends hadn't the least idea about the meaning of the word commission. How would you explain the meaning of this word to your friend?

16. What special meaning has the word solid as used in arithmetic?

17. How would you find the volume of a rectangular prism whose three dimensions were: 2 feet $\times$ 2 yards $\times$ 5 yards?

18. The superintendent of schools was visiting Mary White's class when she happened to be giving a two-minute recitation on the topic "some important facts about interest." When Mary was through, the superintendent turned to the teacher and said, "That is one of the finest recitations I have ever heard a seventh-grade pupil make."

What would you say about interest if you were called upon to discuss that topic?

19. What question in interest would the following example answer if it were worked correctly? $i = \$375. \times .065 \times \frac{7}{2}$.

20. What is the special meaning of the word amount?

21. The formula $i = prt$ would not mean anything to a fourth-grade child. What does it mean to you?

22. For what is an interest table used?

23. What are eight facts shown on a promissory note?

24. When a bank lends money, it often "discounts the loan." What is the meaning of bank discount?

25. Be ready to go to the board to illustrate what is meant by: *a bar graph a circle graph a broken line graph a picture graph or pictogram.*

26. Show on the board how you would use the formula $r = \frac{i}{p}$ or $p\overline{)i}$ in solving this problem: An investment costs \$920 that yields \$65 a year, and we want to find the rate of interest the investment earns.

27. Many people think that if a store-keeper pays \$60 for a dining room table and sells it for \$100, he makes a profit of \$40. Why is the profit sure to be less than \$40?

28. Margin is the difference between the cost and selling price of an article. Give some illustrations that show that you understand this meaning of the word margin.

29. What important ideas do each of the following suggest? *right angle protractor semicircle degree obtuse angle acute angle angles as measures of direction.*

30. Two seventh-grade pupils were arguing about the value of estimating answers. One thought it was a good thing to do, and the other thought that it was a silly waste of time. Tell what you think about it, and why.

SELF-HELP STUDY AND PRACTICE LESSONS

212. How to Use the Self-Help Study and Practice Lessons. (CLASS WORK.) While you are doing seventh-grade work, you may need to review some of the things you learned in the previous grades. It will not be very often though, that all of the pupils in your class will need to review these things at the same time. Sometimes you will need to study and practice on some of these lessons while other pupils are working on different lessons. That is why these lessons are called **Self-Help Lessons.** You must learn how to help **yourself.**

Look at Article 214. What does it tell you about? Notice that it is for self-help **study.** You can study everything in this article without the help of your teacher.

Now look at Articles 225 and 231. They are also articles to be **studied.**

What is the name of Article 228? Do you see that it is for self-help **practice?** It gives you some good practice on what you **studied** in Article 225.

Important: Remember that all of these Self-Help Study and Practice Lessons are to be used by you to **help yourself.** You can't **help yourself** if you look at the answers in the exercises and in the Answer Key before you have written the answers to the examples in any article. Be fair in playing this "game" with **yourself.** If you are fair, you will find that you can teach yourself many things that you may have forgotten.

ADDITION OF WHOLE NUMBERS

213. How to Use the Lessons on Addition. (SILENT READING.) Articles 213 to 217 are for pupils who want to help themselves improve in addition. The little chart below can be used in finding the kind of work you need.

Art. 214 shows you six places in addition which often cause trouble. Study Art. 214 if you do not understand addition very well.

Art. 215 gives you useful practice on "little" addition examples which are needed for accurate work. Use this article if you are troubled with mistakes when you add columns of figures.

Arts. 216 and 217 give practice on harder examples. These articles give good practice for those who understand addition and can do very easy work but need more practice on harder kinds of work.

214. Six "Trouble Spots" in Addition. (FOR SELF-HELP STUDY.) You will probably read this article if you are having trouble with addition. After studying it, if there is anything you do not understand, you should talk over your addition troubles with your teacher.

I.

Keep your place in the column.

$$\begin{array}{r} 3 \\ 4 \\ 4 \\ 4 \\ 7 \\ 7 \\ \hline \end{array}$$

In a long column it is easy either to skip a figure or to add a figure twice.

Some pupils find it a help to put their pencil point by the side of each figure as it is added.

II.

Carry the correct number.

$$\begin{array}{r} 38 \\ 25 \\ 85 \\ 93 \\ 64 \\ \hline 5 \end{array}$$

What number is carried to the tens' column? Many mistakes are made by carrying a 1 when a 2 or even a 3 should be carried.

Always be careful to carry the correct number.

III.

Write the correct figure.

In Example II on page 388, the sum of the units' column is 25. This means you should write the 5 under the 4 and carry the 2.

What would you say to a pupil who kept doing the wrong thing, such as writing a 2 instead of the 5, or even forgetting to write any number at all?

IV.

Stay in the proper column.

38
25
85
93
64
5

After you have carried 2 to the 6, think 8 and then add 9; think 17 and add 8 to the 17; etc.

Some pupils let their eyes wander into the wrong column, and add numbers in the wrong column.

V.

Add all columns.

473
241
26
70
25
32

Some pupils will add the units' and tens' columns correctly, and then forget to add the hundreds', because there are no numbers near the bottom.

Always look all the way up the column farthest to the left in any example before leaving it. By doing this you can always make sure that you have not forgotten to add some numbers at the top of the last column.

VI.

When numbers are left out, don't be puzzled.

(*a*)	(*b*)	(*c*)
272	272	272
6	116	006
72	172	072
493	493	493

Look at Examples (*a*) and (*b*) above. Most pupils would find Example (*a*) the harder. When there is a gap, it means 0.

We do not use 0 at the left of whole numbers. But Example (*a*) above can be thought of as it is shown in Example (*c*).

215. "Little" Addition Facts. (For Self-Help Practice.) The addition facts in this article are harder ones that you find "hidden" in addition examples. You may have been having trouble with addition because you do not know these facts well enough. Practice on them makes

them easy. Put a slip of paper over the answers when you want to test yourself.

	(*A*)	(*B*)	(*C*)	(*D*)
1.	23+8=**31**	34+9=**43**	28+7=**35**	49+7=**56**
2.	11+9=**20**	18+4=**22**	38+2=**40**	29+2=**31**
3.	39+4=**43**	26+8=**34**	58+8=**66**	48+5=**53**
4.	16+0=**16**	49+9=**58**	27+3=**30**	14+6=**20**
5.	36+7=**43**	16+6=**22**	48+3=**51**	28+8=**36**
6.	72+9=**81**	29+1=**30**	16+9=**25**	37+9=**46**
7.	64+7=**71**	49+8=**57**	26+4=**30**	46+5=**51**
8.	65+9=**74**	28+6=**34**	15+7=**22**	25+8=**33**
9.	37+8=**45**	39+6=**45**	24+8=**32**	25+6=**31**
10.	17+4=**21**	43+9=**52**	33+7=**40**	22+8=**30**
11.	47+6=**53**	29+3=**32**	27+7=**34**	35+0=**35**
12.	27+5=**32**	39+5=**44**	38+9=**47**	87+9=**96**

216. Addition Examples with Answers. (FOR SELF-HELP PRACTICE.) Every seventh-grade boy and girl should be able to find the sums of the addition examples in this article without help and without mistakes. **Cover the answers while working.** Let the answers really help you by using them as a check **after** the work is finished.

1.	2.	3.	4.	5.	6.
2					16
9	66	53	36	78	45
8	55	83	57	31	44
0	78	54	47	34	86
3	97	29	80	65	77
22	**296**	**219**	**220**	**208**	**268**

7.	8.	9.	10.	11.	12.
					873
516	87	20	290	62	305
136	29	899	752	210	879
135	92	541	630	709	334
179	97	164	919	110	827
825	18	768	759	537	269
1791	**323**	**2392**	**3350**	**1628**	**3487**

217. Helping Yourself in Addition. (For Self-Help Practice.) Can you work and check these examples? Art. 236, page 410, explains how to check addition.

1.	2.	3.	4.	5.
98	67	87	18	8865
72	524	6	30	1547
39	499	57	85	6978
48	747	88	69	2991
45		48	79	3368

6.	7.	8.
$89.79	7965	$8.56
4.95	6349	.37
45.55	1896	9.37
32.46	8167	2.84
34.26	2958	7.95
		5.66

☛ When you are through, compare your answers with those in the Answer Key on page 463. Correct all your mistakes.

Questions to ask yourself about addition:

(*a*) Did I check or "test" my work?
(*b*) Did I "carry" the correct numbers?
(*c*) Did I add all the figures or did I overlook some?
(*d*) Did I name answers when necessary (Exs. 6 and 8)?

SUBTRACTION OF WHOLE NUMBERS

218. How to Use the Lessons on Subtraction. (SILENT READING.) Articles 218-222 are for pupils who want to help themselves improve in subtraction. The little chart below can be used in finding the kind of work you need.

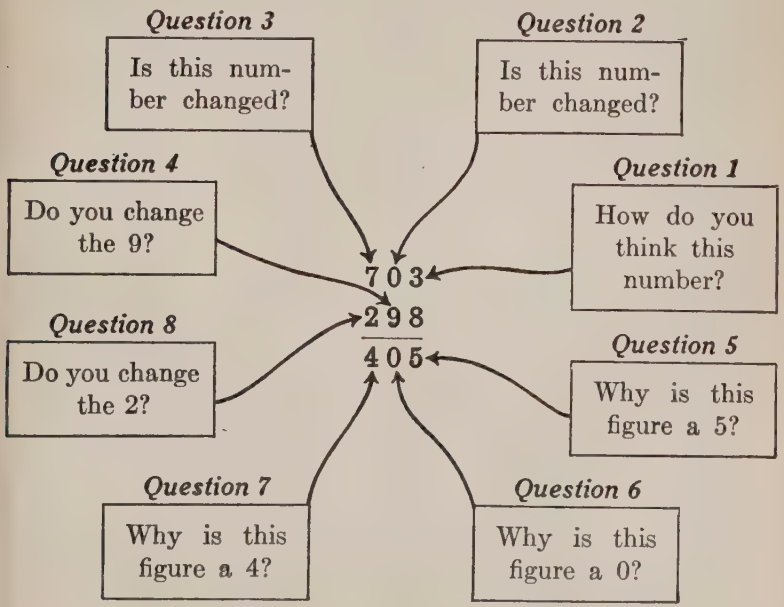
> Art. 219 gives practice on "little" subtraction examples needed in accurate work in harder examples.
> Art. 220 asks questions about things to do in hard examples.
> Arts. 221 and 222 give practice on harder examples.

219. "Little" Subtraction Examples. (FOR SELF-HELP PRACTICE.) If you make little mistakes in subtraction, study the examples below until each one is very easy for you. Cover answers with a slip of paper while practicing.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)	(*G*)	(*H*)
1.	19	26	31	20	27	30	30	27
	10	19	22	9	17	14	11	18
	9	**7**	**9**	**11**	**10**	**16**	**19**	**9**
2.	32	30	41	22	25	21	83	31
	14	16	25	9	19	14	48	23
	18	**14**	**16**	**13**	**6**	**7**	**35**	**8**
3.	35	40	31	32	26	78	38	32
	18	22	12	15	18	10	29	28
	17	**18**	**19**	**17**	**8**	**68**	**9**	**4**
4.	36	23	63	45	94	31	34	43
	36	15	59	30	27	17	19	27
	0	**8**	**4**	**15**	**67**	**14**	**15**	**16**

5.	35	54	30	32	62	34	51	45
	17	39	13	13	47	18	40	36
	18	**15**	**17**	**19**	**15**	**16**	**11**	**9**
6.	30	61	34	54	70	33	61	56
	20	48	25	26	58	24	39	37
	10	**13**	**9**	**28**	**12**	**9**	**22**	**19**
7.	63	70	59	62	31	30	47	47
	46	47	49	46	16	5	29	0
	17	**23**	**10**	**16**	**15**	**25**	**18**	**47**

220. "Hard Spots" in Subtraction. (For Self-Help Study.) Study these questions carefully. If you are not sure of the answers, ask your teacher for help.



Be sure you know how to get each figure in each difference below. The differences are all correct.

(*a*)

607
389
218

(*b*)

426
198
228

(*c*)

3002
1935
1067

221. Subtraction Examples with Answers. (For Self-Help Practice.) Notice that the answers are given in the examples below. Here is a way to use the answers so that they will really help you:

1. Study until you see **why** each answer is correct.
2. Then cover the answers and work the examples.
3. Compare your own answers with the printed ones. Be sure to make yourself correct any mistakes.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	124	$6.73	113	$3.05	128	856
	75	1.04	39	2.18	69	279
	49	**$5.69**	**74**	**$.87**	**59**	**577**
2.	100	723	$1.11	941	134	671
	61	318	.63	304	58	384
	39	**405**	**$.48**	**637**	**76**	**287**
3.	$8.63	883	$8.93	925	264	631
	6.79	785	8.08	479	186	555
	$1.84	**98**	**$.85**	**446**	**78**	**76**
4.	806	693	934	958	625	703
	429	197	679	203	466	403
	377	**496**	**255**	**755**	**159**	**300**

222. Final Practice in Subtraction. (FOR SELF-HELP PRACTICE.) Every seventh-grade pupil should be able to work these examples accurately and easily. The way to check subtraction is explained in Art. 236, page 410. Be careful to name answers when necessary.

1. 10580 5946	2. 8129 5590	3. 59650 24560	4. 5058 4701
5. 15332 13480	6. 5090 2338	7. 7217 1398	8. 57745 1328
9. 10632 2923	10. 15551 7962	11. 11313 1627	12. $121.85 76.01
13. 5236 1530	14. 17352 7108	15. $137.32 46.01	16. 11003 4699
17. 8774 4675	18. 6598 5729	19. 9624 8208	20. 12868 7648
21. 17879 9721	22. $59.41 35.54	23. 11946 9368	24. 10120 7785

☛ After you have finished, check your work by comparing your answers with those given on page 463.

An important question to ask yourself: Can I subtract well enough to do good seventh-grade work?

If you can do the work in Art. 222 correctly and easily, subtraction will not trouble you much in seventh grade.

MULTIPLICATION OF WHOLE NUMBERS

223. How to Use the Lessons on Multiplication. (SILENT READING.) Arts. 223 to 228 are given here for pupils who want to help themselves improve in multiplication. The little chart below will be useful in finding the kind of work that is needed.

> Art. 224 gives practice on the "little" examples which are needed for accurate work in harder multiplication.
>
> Art. 225 shows the "hard spots" in multiplication. It is important that pupils understand clearly everything in this article.
>
> Arts. 226-228 give good practice for those who understand how to multiply, but need more practice.

224. "Little" Examples Used in Multiplication. (FOR SELF-HELP PRACTICE.) The examples below and on the next page are "little" facts which are very helpful in "big" multiplication.

Do you see where the "little" facts in the box at the right are helpful in the "big" example?

$4\times6=24$	296
$4\times9+2$	$\times84$
$4\times2+3$	1184
$8\times6=48$	2368
$8\times9+4$	24864
$8\times2+7$	

It will save time this year if you will practice on the "little" facts below and on the next page until each one is as easy for you as $2+2=4$. Cover the answers. The answers will help you most if you use them as a check of your own work.

(*A*)	(*B*)	(*C*)	(*D*)
$7\times7+6=\mathbf{55}$	$6\times3+5=\mathbf{23}$	$9\times2+7=\mathbf{25}$	$9\times8+7=\mathbf{79}$
$9\times4+6=\mathbf{42}$	$9\times3+7=\mathbf{34}$	$9\times8+6=\mathbf{78}$	$9\times6+6=\mathbf{60}$

(A)	(B)	(C)	(D)
$8\times7+3=59$	$9\times7+8=71$	$7\times6+5=47$	$8\times0+7=7$
$8\times9+5=77$	$9\times0+6=6$	$8\times0+4=4$	$9\times7+6=69$
$9\times7+3=66$	$9\times4+7=43$	$9\times8+8=80$	$9\times5+7=52$
$6\times9+4=58$	$9\times7+4=67$	$9\times1+8=17$	$8\times6+3=51$
$7\times6+4=46$	$8\times9+6=78$	$8\times9+7=79$	$9\times6+8=62$
$8\times7+5=61$	$9\times6+7=61$	$7\times5+6=41$	$9\times9+7=88$
$7\times8+6=62$	$8\times6+6=54$	$8\times7+4=60$	$9\times5+8=53$
$9\times9+5=86$	$9\times4+8=44$	$9\times9+6=87$	$8\times6+7=55$
$9\times0+4=4$	$5\times3+4=19$	$8\times7+6=62$	$9\times9+8=89$
$6\times2+5=17$	$9\times7+7=70$	$9\times3+8=35$	$6\times7+4=46$
$9\times5+6=51$	$8\times8+7=71$	$7\times0+5=5$	$6\times9+5=59$
$7\times6+3=45$	$7\times9+5=68$	$8\times1+6=14$	$9\times6+3=57$
$8\times8+5=69$	$9\times2+8=26$	$9\times3+4=31$	$8\times2+5=21$
$7\times7+4=53$	$7\times8+4=60$	$6\times3+4=22$	$6\times6+4=40$
$6\times8+5=53$	$9\times4+5=41$	$7\times4+5=33$	$5\times8+4=44$
$9\times9+4=85$	$8\times8+6=70$	$8\times2+6=22$	$9\times9+3=84$
$7\times4+6=34$	$8\times3+7=31$	$9\times5+5=50$	$8\times8+4=68$
$9\times8+4=76$	$9\times3+6=33$	$9\times8+3=75$	$8\times4+7=39$
$9\times6+5=59$	$9\times2+6=24$	$6\times8+4=52$	$8\times2+7=23$
$7\times7+5=54$	$7\times9+4=67$	$7\times9+6=69$	$4\times9+3=39$

225. Studying Some "Hard Spots" in Multiplication. (For Self-Help Study.) On the next page are two multiplication examples with certain "hard spots" in them which may be causing you trouble in your work.

A careful study of these two examples should clear up your difficulties if you know the "little" facts of multiplication and addition.

Let us go through each step in multiplying 609 by 376, as shown in Sample A. Begin by multiplying 609 by 6.

(*A*)

```
  609
  376
 3654
4263
1827
228984
```

$6\times9=54$. Write the 4 under the 6 of 376 and carry the 5.

$6\times0+5=5$. Write the 5. Nothing to carry.

$6\times6=36$. Write the 36, and you have 3654 as the first step in your work.

The hard spot was in knowing what to do in the little example $6\times0+5$.

Now let us see how we get the 4263.

$7\times9=63$. Write the 3 directly under the 7 you are multiplying by; this is **important.** Carry the 6.

$7\times0+6=6$. Write the 6. Nothing to carry.

$7\times6=42$. Write the 42, and you have the 4263.

Here are the "little" examples needed to get the 1827:

$3\times9=27$. Write the 7 and carry the 2. Notice that the 7 goes directly under the 3 you are multiplying by.

$3\times0+2=2$. Write the 2. Nothing to carry.

$3\times6=18$. Write the 18, and you have the 1827.

In Example B you have these "little" multiplications:

(*B*)

```
  779
  208
 6232
15580
162032
```

$8\times9=72$. Write the 2 and carry the 7. Notice that the 2 goes under the 8 that you are multiplying by. This is **important.**

$8\times7+7=63$. Write the 3 and carry the 6.

$8\times7+6=62$. Write the 62, and you have the 6232.

Now notice what happens when you multiply by 0. Just write a zero directly under the 0 in the multiplier and then go ahead and multiply by 2.

$2\times9=18$. Write the 8 in front of the 0 and it comes directly under the 2 you are multiplying by. Carry the 1.

$2\times7+1=15$. Write the 5 and carry the 1.
$2\times7+1=15$. Write the 15.

If you know your "little" addition and multiplication facts, multiplying numbers of any size is easy. Remember that the first figure you get in each part goes directly under the number you are multiplying by.

226. Easy Multiplication with Answers. (For Self-Help Practice.) You should practice on the work below until it is very easy for you. Cover the answers with a slip of paper when you are working the examples.

Of course, this practice will not help you much if you use the answers while you are working.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	314	501	415	916	527	72¢
	8	7	9	6	8	6
	2512	**3507**	**3735**	**5496**	**4216**	**$4.32**
2.	308	$ 8.37	709	326	207	470
	8	6	5	7	9	7
	2464	**$50.22**	**3545**	**2282**	**1863**	**3290**
3.	609	405	936	$ 2.89	709	708
	8	6	9	7	7	9
	4872	**2430**	**8424**	**$20.23**	**4963**	**6372**

227. Multiplication Examples with Answers. (For Self-Help Practice.) When you practice, place your paper over the answers or copy some examples and close your book.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)
1.	346	$5.79	389	217	380
	8	7	14	32	67
	2768	**$40.53**	**5446**	**6944**	**25460**

	(A)	(B)	(C)	(D)	(E)
2.	120	608	436	9728	857
	71	95	321	906	158
	8520	**57760**	**139956**	**8813568**	**135406**
3.	8905	291	1364	564	$71.09
	203	659	507	496	804
	1807715	**191769**	**691548**	**279744**	**$57156.36**

228. Final Practice in Multiplication. (For Self-Help Practice.) Every seventh-grade pupil should be able to work the examples below accurately and easily. The way to check multiplication is explained in Art. 236, page 410.

1. 683	2. 902	3. 147	4. 289	5. 547
247	151	328	140	496
6. 863	7. 289	8. 702	9. 145	10. 360
158	579	134	605	229
11. 291	12. $8.25	13. 459	14. 607	15. 461
369	582	748	478	407
16. 608	17. 91	18. 76¢	19. 387	20. $3.54
136	823	350	279	307

☛After you have worked and checked every example, compare your answers with those given in the Answer Key on page 463.

An important question to ask yourself: Can I multiply well enough to do good work in the seventh grade?

If you can do the work in Art. 228 correctly and easily, multiplication will not trouble you in seventh grade.

DIVISION OF WHOLE NUMBERS

229. How to Use the Lessons on Division. (SILENT READING.) Arts. 230-235 contain review work on division. The chart below will help you to find the lessons on division.

> Art. 230 reviews short division.
>
> Art. 231 explains each of the four steps used in working long division.
>
> Arts. 232 and 233 explain how to find quotient figures, and give practice.
>
> Art. 234 explains zeros in the quotient, and gives practice.
>
> Art. 235 gives general practice on division for those who understand how to divide but need more practice.

230. Easy Division Practice. (FOR SELF-HELP PRACTICE.) Lay the edge of your paper **above** the examples to cover the quotients, and write the answers.

	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
1.	**30** 6)180	**323** 3)969	**51** 5)255	**71 3/4** 4)287	**$1.11** 7)$7.77	**31** 9)279
2.	**52 1/2** 2)105	**$.30** 5)$1.50	**41** 6)246	**81** 7)567	**$.80** 8)$6.40	**52** 4)208
3.	**$ .41** 8)$3.28	**70** 7)490	**71 2/3** 9)645	**91** 6)546	**51** 7)357	**71** 6)426
4.	**69** 6)414	**$1.01** 5)$5.05	**79** 4)316	**$.49** 7)$3.43	**89 3/4** 8)718	**58** 7)406
5.	**35** 9)315	**$2.00** 6)$12.00	**38 2/5** 5)192	**49** 8)392	**78** 7)546	**203** 3)609

231. How to Work Long Division. (For Self-Help Study.) You will want to use this article if you are having trouble in understanding how long division work is done.

Study the work below. It will show you each step necessary in dividing 6468 by 14, and will make clear **how** long division examples are worked.

```
  462
14)6468
  56
  86
  84
  28
  28
   0
```

THE STEPS IN LONG DIVISION

Step 1. Divide.

$64 \div 14$. 4 is the largest number of times 14 will go into 64. Write 4 above the 4 of 64.

Step 2. Multiply.

$4 \times 14 = 56$. Write the 56 under the 64. The product (56) must be smaller than the number above it (64).

Step 3. Subtract.

$64 - 56 = 8$. The number you get by subtracting must be **smaller** than the divisor.

Step 4. Bring Down.

Bring down the 6 from 6468. Now start with Step 1 again: $86 \div 14$, etc.

Think through Samples A, B, and C below. Notice that in Sample B the remainder is written in fraction form. The remainder becomes the numerator and the divisor becomes the denominator. In Sample C a decimal point and zero are added in the dividend and the remainder is carried out in decimal form.

```
(A)  32          (B)  32 3/23        (C)  3.5+
    23)736           23)739              27)95.0
       69               69                 81
       46               49                 140
       46               46                 135
        0                3                   5
```

FINDING QUOTIENT FIGURES

232. How to Find Quotient Figures. (For Self-Help Study.) Some quotient figures are not so easy to find as those you have studied. You probably will find this is true when you try to get the quotient figures in Sample A. See if you can tell how to finish Sample A.

Sample A

$$\begin{array}{r} 1? \\ 48\overline{)768} \\ \underline{48} \\ 288 \end{array}$$

The first figure in the quotient is easy, but you must then find how many 48's there are in 288. It looks as if there might be 7, for there are seven 4's in 28.

But $7 \times 48 = 336$, which cannot be subtracted from 288. If, then, 7×48 is too large, try 6×48; this gives you 288, which can be subtracted from 288. Use 6, then, as the correct quotient figure, and multiply.

$$\begin{array}{r} \cancel{7} \\ 16 \\ 48\overline{)768} \\ \underline{48} \\ 288 \\ \cancel{336} \\ \underline{288} \\ 0 \end{array}$$

Too large - erase or cross out

Notice the sample at the left. It shows a good way to arrange your work.

When you try 7 as the quotient figure, write it small and very lightly above the correct place; also write the 336 which you get by multiplying, very lightly, as shown.

When you see that the 7 and the 336 are too large, cross them out or erase them. Then write 6 in the quotient and the 288 under the 288.

Later you will be able to find difficult quotient figures by thinking correctly, without writing them lightly. But until you know well how to find quotient figures, it is helpful to write the **trial** figure in the quotient. When you find the

correct quotient figure to use, it is called the **true** or **correct quotient figure.**

Study Samples B, C, and D below, and see if you understand every step shown in the work. Check the answer to each sample.

SAMPLE B

$$\begin{array}{r} 3 \\ 72\tfrac{43}{54} \\ \hline 54\overline{)3931} \\ 378 \\ \hline 151 \\ 162 \\ 108 \\ \hline 43 \end{array}$$

Too large, Erase

SAMPLE C

$$\begin{array}{r} 4 \\ 39\tfrac{12}{65} \\ \hline 65\overline{)2547} \\ 260 \\ 195 \\ \hline 597 \\ 585 \\ \hline 12 \end{array}$$

Too large, Erase

SAMPLE D

$$\begin{array}{r} 69 \\ 58\tfrac{29}{78} \\ \hline 78\overline{)4553} \\ 468 \\ 390 \\ \hline 653 \\ 702 \\ 624 \\ \hline 29 \end{array}$$

Too large, Erase

Too large, Erase

In Sample B 3 was a trial quotient figure because 5 into 15 goes 3 times. But 3 is too large, because $3 \times 54 = 162$. 162 is larger than 151.

In Sample C you know that the trial quotient figure 4 is too large, because $4 \times 65 = 260$. 260 is larger than 254.

In Sample D both trial figures were too large.

☞Often you will write a figure in the quotient above the place where it belongs, and then, when you multiply, you will find that the figure you have written is correct.

You should then write it in its proper place and cross out or erase the first figure that you wrote.

This sentence is worth remembering:

In finding quotients, if the trial quotient figure which you use is too large, try the next smaller figure.

233. Finding Harder Quotient Figures. (FOR SELF-HELP STUDY AND PRACTICE.) The samples in this article give help in finding the right quotient figures to use.

Remember:

1. First try the quotient figure which seems right.
2. If it is too large, try the next smaller one.
3. Keep coming down 1 at a time until you get the correct figure.
4. Do as much of the work as you can without writing. You will save erasing.

Study these samples:

SAMPLE A

```
  7   Think, but
  6   do not
  5   write:
29)145  7×29=203
 145    6×29=174
 ---
```

SAMPLE B

```
  9   Think, but
  8   do not
  7   write:
38)277  9×38=342
 266    8×38=304
 ---
```

SAMPLE C

```
  7   Think, but
  6   do not
  5   write:
    4  7×17=119
17)71  6×17=102
  68   5×17= 85
  --
```

Be sure you understand every part of these samples:

SAMPLE D

```
    9
   58
   47 11/38
38)1797
  152
  ---
  277
  266
  ---
   11
```

SAMPLE E

```
    7
   66
   53
   42
17)714
  68
  --
  34
  34
  --
   0
```

SAMPLE F

```
   8 7
  726
  615
29)17835
  174
  ---
   43
   29
   --
  145
  145
  ---
    0
```

The examples below give practice in finding quotients.

1. $67\overline{)610}$	4. $36\overline{)321}$	7. $57\overline{)511}$	10. $23\overline{)205}$	13. $46\overline{)413}$
2. $16\overline{)142}$	5. $29\overline{)238}$	8. $28\overline{)236}$	11. $19\overline{)158}$	14. $39\overline{)317}$
3. $62\overline{)430}$	6. $84\overline{)754}$	9. $14\overline{)57}$	12. $54\overline{)482}$	15. $86\overline{)573}$

☛ You will find the correct quotients on page 463.

234. Zero in the Quotient. (For Self-Help Study and Practice.) Zero (0) in the quotient sometimes troubles pupils. If you study this article carefully, zeros in the quotient should not trouble you.

Study Examples A to E in this article, and see if you can decide what to do. Watch zeros closely here.

Example A

```
          6
      46)27738
New     276
Dividend→13
```

```
          ↓
         60
      46)27738
        276
         13
```

```
         603
      46)27738
        276
        138
        138
          0
```

In Example A, after 3 is brought down from the dividend, we have 13 as a **new dividend.**

But 46 will not go into 13 even once. We must show this fact in the quotient. Notice the use of 0 in the quotient.

We do not need to multiply by the 0, but we bring down the 8 next. 46 will go into **138**; so you can finish the example as shown.

Copy Example B on a piece of paper and see if you can finish it by yourself. The correct answer is given on page 464.

Example B

```
          8
      73)58692
New     584
Dividend→ 29
```

Study these three rules and see if they are followed in Examples C, D, and E below.

1. If the divisor will not go into the new dividend, be sure to write 0 in the quotient.
2. Then bring down the next figure of the dividend.
3. The last figure of the quotient (not counting the remainder) must always be above the last figure of the dividend.

Be sure you understand each zero in these examples:

(C)
$$\begin{array}{r} 906 \\ 72\overline{)65232} \\ 648 \\ \hline 432 \\ 432 \\ \hline 0 \end{array}$$

(D)
$$\begin{array}{r} 301 \\ 84\overline{)25284} \\ 252 \\ \hline 84 \\ 84 \\ \hline 0 \end{array}$$

(E)
$$\begin{array}{r} 405\frac{7}{58} \\ 58\overline{)23497} \\ 232 \\ \hline 297 \\ 290 \\ \hline 7 \end{array}$$

Practice with Zero in the Quotient

If you think carefully, you will have no trouble with these examples and problems. Some have remainders.

1. $67\overline{)40334}$ 2. $73\overline{)15045}$ 3. $92\overline{)46828}$ 4. $89\overline{)53700}$

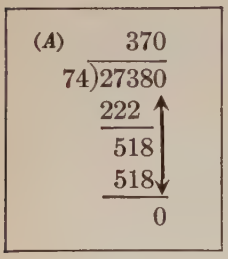
5. $43\overline{)34486}$ 6. $65\overline{)32955}$ 7. $97\overline{)87995}$ 8. $56\overline{)11540}$

9. The Martin Motor Sales Company sold 82 cars last month. These cars cost them \$66,010. If all the cars were alike, what did each one cost?

☛ The answers are given on page 464.

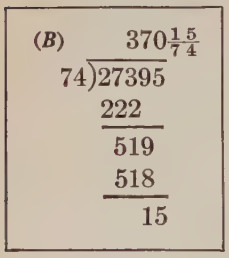
FINAL ZERO IN QUOTIENT

Examples A, B, and C below need careful study. They show you a new thing to watch for in long division. Make your later work easy by understanding these examples now.



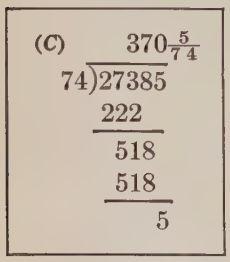
In Sample A, after we subtract 518 from 518, notice that there is still a number to be brought down. In this example it is 0.

74 will not go into 0. Notice the 0 in the quotient to show this. The last figure of the quotient is always over the last figure of the dividend.



In Sample B we put 0 in the quotient to show that 74 will not go into 15. 15 is the remainder. Remember what to do with the remainder.

Be sure to write 0 in the quotient before you write the fraction part of the quotient.



Sample C also has 0 in the quotient. It shows that 74 will not go into 5. 5 is the remainder.

Remember that you must write 0 above the last figure in the dividend **before** you write the fraction part of the quotient.

Study Samples A, B, and C again to see if the rules at the top of the next page would be good ones.

When the divisor will not go into the last new dividend:

1. Write 0 in the quotient.

2. Write the fraction part, if any, of the quotient.

3. See that the last figure of the quotient (not counting the fraction part) is above the last figure of the dividend.

4. Take time to check your work.

Practice with Final Zeros in the Quotient

1. $54\overline{)44820}$	2. $93\overline{)26992}$	3. $89\overline{)55180}$	4. $66\overline{)47525}$
5. $62\overline{)35960}$	6. $83\overline{)79687}$	7. $57\overline{)23961}$	8. $21\overline{)2730}$

9. A book store sold 36 geographies for $50.40. The books were the same price. What did each one sell for?

☛ The answers are given on page 464.

235. Final Practice in Division. (For Self-Help Practice.) If you can do these examples correctly and easily, division will not trouble you much in the seventh grade.

1. $17\overline{)819}$	7. $84\overline{)5208}$	13. $38\overline{)3382}$	19. $12\overline{)276}$
2. $52\overline{)3848}$	8. $59\overline{)3599}$	14. $70\overline{)5040}$	20. $60\overline{)9600}$
3. $60\overline{)2400}$	9. $40\overline{)3800}$	15. $31\overline{)1557}$	21. $14\overline{)994}$
4. $69\overline{)6003}$	10. $51\overline{)4912}$	16. $63\overline{)1638}$	22. $40\overline{)3320}$
5. $59\overline{)1369}$	11. $72\overline{)43272}$	17. $95\overline{)38760}$	23. $52\overline{)4420}$
6. $38\overline{)2774}$	12. $83\overline{)3403}$	18. $27\overline{)24313}$	24. $96\overline{)5671}$

☛ The answers to the examples are given on page 464.

CHECKING FOR ACCURACY

236. Checking. (For Self-Help Study.) This article shows you how to make fairly sure that your arithmetic work is done correctly. Pupils who are careful workers take time to check their work.

Check your addition like this:

*1872**	(*Check answer*)
237	
89	
634	
912	
1872*	(*First answer*)

First: Add **upward** and write answer below the bottom line.

Second: Add **downward** and write the answer above the columns. When the two numbers marked with stars (*) are the same, you may be pretty sure you have made no mistakes.

Check your subtraction like this:

Subtract: 3274*

327 } Check by
2947 } addition.

*3274**

Add subtrahend and difference. The sum should equal the minuend.

When the two starred (*) numbers are the same, you may be pretty sure that you have the correct answer.

Check your multiplication like this:

The Work	*The Check*
47	24
24	47
188	168
94	96
1128*	*1128**

Exchange multiplicand and multiplier, and multiply again.

When the two starred (*) numbers are the same, you may be pretty sure the work is correct.

Check your division like this:

The Work	*The Check*
$41\frac{1}{3}$	41
$27\overline{)1116}$*	$\times 27$
108	287
36	82
27	1107
9	+9
	*1116**

FIRST: Multiply the whole number part of the quotient by the divisor.

SECOND: To this product, add the numerator of the unreduced fractional part of the quotient.

CHECKING DECIMALS

Check decimals just as you check whole numbers. The decimal points must be correctly handled.

When you check division of decimals, there is one thing of which you must be careful: **The remainder must be pointed off.** Be sure to put a decimal point in the remainder directly below the decimal point in the dividend, inserting zeros in any vacant places. Then add the remainder to the product of the quotient and divisor. Study the samples.

The Work	*The Check*	*The Work*	*The Check*
4.6 6		5.0 7	
7 9)3 6 8.3 1*	4.6 6	5.7)2 8.9 3 0*	5.0 7
3 1 6	7 9	2 8 5	5.7
5 2 3	4 1 9 4	4 3 0	3 5 4 9
4 7 4	3 2 6 2	3 9 9	2 5 3 5
4 9 1	3 6 8.1 4	.0 3 1	2 8.8 9 9
4 7 4	+.1 7		+.0 3 1
.1 7	3 6 8.3 1*		2 8.9 3 0*

Work and check the following (Answers are on page 464):

1. $2.664 \div 3.7$ 2. $32.7 \div 46$ 3. $2.482 \div 6.4$

ESTIMATING

237. Self-Help Lesson on Estimating. (For Self-Help Study and Practice.) What one really does when he estimates an answer, is to think through the example quickly in order to see **about** what the answer must be. The best way to do this is to change the numbers to numbers that are of about the same size but are easier to handle. The samples below will show you how this may be done.

Sample 1. 493
×247

We could change these numbers to 500 and 250. We can multiply 500 and 250 much more easily "in our heads" than 493 and 247. We can see quickly that $500 \times 250 = 12{,}500$. Therefore, the answer to Sample 1 must be somewhere near 12,500. If we know this, we will be kept from making a big error in the example 247×493.

Sample 2. $462\overline{)13078}$

We could change this example to $450\overline{)13000}$. We could see quickly that our answer would be a two-figure number. That would help us some; but we could also see that it would be pretty close to 30. So 30 would be a good estimate for $13078 \div 462$.

Sample 3. 77.83
×8.97

We could change to 80
×9

"In our heads" we would quickly get our estimate of 720 as being about what the answer should be to the example 8.97×77.83.

Sample 4. $6.89\overline{)23.076}$

Change to $7\overline{)23}$, and we can tell quickly that our answer must be a little more than 3.

In estimating answers to examples and problems you will find these suggestions helpful:

1. Take a very careful look at the example to get its numbers clearly in mind.

2. Change these numbers to numbers that are of about the same size, but easier to work with "in your head"; then work the "easy" example.

3. Work the example for which you are getting the estimate and compare the answer with the estimate.

➡Estimating will help keep you from making "big" or foolish mistakes. Checking helps you to keep from making "little" mistakes.

PRACTICE

Change the examples below to easier ones that you could use to get an estimate.

1. 58×31

2. $22\overline{)789}$

3. 892×41

4. $46+31+79$

5. 519×23

6. $\$82.93+\$47.59$

7. $\$22.50 \times 12$

8. $12\overline{)\$469.83}$

9. $5764-3543$

10. $41\overline{)2364}$

Below are estimates of the examples above. They are not in order. Choose a reasonable estimate for each example.

1. 40	3. $47	5. $240	7. 36,000	9. 1800
2. 10,400	4. 2200	6. 155	8. $130	10. 60

☛On page 464 you will find how these examples could be changed in order to estimate, and what reasonable estimates would be.

WORDS USED IN ARITHMETIC

238. Review of Vocabulary. (For Self-Help Study.) Below are explained some important words that are used in arithmetic.

248
72
316
636←***Sum*** (*Answer*)

476←***Multiplicand***
×24←***Multiplier***
1904
952
11424←***Product*** (*Answer*)

$636 \div 3 = 212$, the **average** of the three numbers in the example above.

$206\frac{19}{24}$←***Quotient*** (*Ans.*)
Divisor→24)4963←***Dividend***
48
163
144
19←***Remainder***

4367←***Minuend***
−2136←***Subtrahend***
2231←***Difference*** (*Ans.*)

1 is a **whole number** or **integer.**

$\frac{1}{2}$ is a **common fraction** (less than 1).

$\frac{3}{2}$ is an **improper fraction** (more than 1).

$1\frac{1}{2}$ is a **mixed number** (a whole number and a fraction).

In the fraction $\frac{3}{4}$, **3** is the **numerator,** and **4** is the **denominator.** When the denominators of two or more fractions are the same, they are called **common denominators.**

A **reciprocal** is a fraction inverted ("turned upside down"). The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$. $\frac{1}{9}$ is the reciprocal of 9 or $\frac{9}{1}$.

.39 and 24.56 are **decimal fractions,** or **decimals.**

Any number that is named, as 9 miles, 6 bushels, 7 feet, is a **denominate number.**

ADDITION OF FRACTIONS AND MIXED NUMBERS

239. How to Use These Lessons. (SILENT READING.) The chart below will guide you in using the Self-Help Lessons on addition of fractions and mixed numbers.

Art. 240 explains common denominators.
Art. 241 gives practice in finding common denominators.
Arts. 242 and 243 explain and give practice in reducing of fractions.
Arts. 244 and 245 explain and give practice in changing improper fractions to good forms for answers.
Art. 246 explains how to add mixed numbers.
Art. 247 gives practice in adding mixed numbers.

240. How to Find Common Denominators. (FOR SELF-HELP STUDY.) Sometimes it is hard to see just what the common denominator ought to be in an example. In such cases use the rule below:

***Rule:* To find a common denominator multiply the denominators. The product will always be a common denominator.**

What does the rule tell you to do to find a common denominator for the example at the right? Can $\frac{1}{8}$ and $\frac{2}{3}$ be changed to $\frac{}{24}$? See if the following is done correctly:

$\frac{1}{8}+\frac{2}{3}=?$

$$\frac{1}{8}=\frac{1\times 3}{8\times 3}=\frac{3}{24} \qquad \frac{2}{3}=\frac{2\times 8}{3\times 8}=\frac{16}{24} \qquad \frac{3}{24}+\frac{16}{24}=\frac{19}{24}$$

$$\text{So, } \frac{1}{8}+\frac{2}{3}=\frac{19}{24}$$

When we change $\frac{1}{8}$ to $\frac{3}{24}$ we change the **form** but not the **value** of $\frac{1}{8}$, because $\frac{1}{8}=\frac{3}{24}$.

241. Finding Common Denominators. (For Self-Help Practice.) What common denominators would you use in examples with denominators as shown below?

1. $\frac{}{2}+\frac{}{4}$
2. $\frac{}{9}+\frac{}{3}$
3. $\frac{}{4}+\frac{}{6}$
4. $\frac{}{8}+\frac{}{3}+\frac{}{6}$
5. $\frac{}{8}+\frac{}{3}$
6. $\frac{}{2}+\frac{}{4}+\frac{}{6}$
7. $\frac{}{2}+\frac{}{6}+\frac{}{3}$
8. $\frac{}{4}+\frac{}{6}+\frac{}{3}$
9. $\frac{}{6}+\frac{}{8}$
10. $\frac{}{6}+\frac{}{12}$
11. $\frac{}{2}+\frac{}{5}$
12. $\frac{}{6}+\frac{}{6}+\frac{}{4}$
13. $\frac{}{3}+\frac{}{4}$
14. $\frac{}{5}+\frac{}{4}$
15. $\frac{}{6}+\frac{}{2}+\frac{}{4}$
16. $\frac{}{8}+\frac{}{4}+\frac{}{6}$

☛The answers are given on page 464.

242. Reducing Fractions in Answers. (For Self-Help Study.) In adding fractions watch the **forms** of the answers. Answers must be reduced to simplest, or lowest, terms.

A fraction is reduced by dividing both the numerator and the denominator by the same number. When this is done correctly, the value of the fraction is not changed. Thus:

$\frac{4}{6}=\frac{2}{3}$ because $\frac{2)\overline{4}}{2)\overline{6}}=\frac{2}{3}$ $\qquad$ $\frac{8}{12}=\frac{2}{3}$ because $\frac{4)\overline{8}}{4)\overline{12}}=\frac{2}{3}$

$\frac{9}{15}=\frac{3}{5}$ because $\frac{3)\overline{9}}{3)\overline{15}}=\frac{3}{5}$ $\qquad$ $\frac{6}{8}=\frac{3}{4}$ because $\frac{2)\overline{6}}{2)\overline{8}}=\frac{3}{4}$

1. $\frac{1}{2}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$, *Answer.* Let us agree to change $\frac{4}{6}$ to $\frac{2}{3}$, for $\frac{2}{3}$ is $\frac{4}{6}$ in lowest terms.

2. $\frac{2}{3}+\frac{5}{6}=\frac{9}{6}=1\frac{3}{6}=1\frac{1}{2}$, the correct form of the answer.

3. $4\frac{1}{12}+1\frac{1}{4}+5\frac{4}{12}=10\frac{8}{12}$. Is there a better form for the answer than $10\frac{8}{12}$? What is it?

4. $5\frac{1}{2}+2\frac{1}{4}+1\frac{1}{8}+3\frac{1}{8}=11\frac{8}{8}$. Best form for this answer?

A fraction is in lowest terms when the numerator and denominator cannot be divided without remainder by the same whole number (except 1).

CAUTION: Sometimes you may have to divide both terms more than once.. Thus, $\frac{12}{30}=\frac{6}{15}=\frac{2}{5}$.

243. Reducing Fractions. (FOR SELF-HELP PRACTICE.) Change the examples below to correct forms for final answers. The correct forms are given on page 464.

1. $\frac{4}{6}$	**3.** $8\frac{3}{6}$	**5.** $\frac{8}{8}$	**7.** $1\frac{6}{24}$	**9.** $\frac{6}{9}$	**11.** $4\frac{2}{6}$
2. $2\frac{8}{12}$	**4.** $6\frac{6}{12}$	**6.** $7\frac{2}{4}$	**8.** $3\frac{6}{8}$	**10.** $5\frac{10}{16}$	**12.** $3\frac{9}{24}$

244. Changing Improper Fractions to Good Forms for Answers. (FOR SELF-HELP STUDY.) You will need to know how to change an improper fraction to a mixed number in these three kinds of examples:

FIRST USE: When the sum of several fractions is an improper fraction, thus, in adding fractions: ⟶

$$\begin{array}{r} 6\frac{3}{8} \\ 2\frac{7}{8} \\ 4\frac{5}{8}\downarrow \\ \hline 12\frac{15}{8}=13\frac{7}{8},\ Ans. \end{array}$$

An improper fraction in the sum must always be changed to a mixed number.

SECOND USE: When an unreduced product is an improper fraction. Thus: ⟶

$$2\tfrac{1}{2}\times 5\tfrac{1}{3}=\frac{5}{\cancel{2}_{1}}\times\frac{\overset{8}{\cancel{16}}}{3}=\frac{\downarrow 40}{3}=13\tfrac{1}{3},\ Ans.$$

Never leave the answer of a multiplication example in the form of an improper fraction.

THIRD USE: When an unreduced quotient is an improper fraction. Thus: ⟶

$$\frac{3}{5}\div\frac{2}{7}=\frac{3}{5}\times\frac{7}{2}=\frac{\downarrow 21}{10}=2\tfrac{1}{10},\ Ans.$$

Always change an improper fraction in a quotient to a mixed number.

Rule: **Improper fractions can be reduced to mixed numbers by dividing the numerator by the denominator.**

$\frac{7}{2} = 3\frac{1}{2}$ because $2\overline{)7}^{3\frac{1}{2}}$ $\qquad$ $\frac{14}{3} = 4\frac{2}{3}$ because $3\overline{)14}^{4\frac{2}{3}}$

If the fraction in the mixed number that you get by changing the improper fraction can be reduced, **reduce it without fail.** Thus:

$\frac{14}{4} = 3\frac{2}{4} = 3\frac{1}{2}$ $\qquad$ $\frac{20}{6} = 3\frac{2}{6} = 3\frac{1}{3}$ $\qquad$ $\frac{15}{10} = 1\frac{5}{10} = 1\frac{1}{2}$

245. Changing Improper Fractions to Correct Form. (For Self-Help Practice.) Change the following improper fractions to good forms for answers. Use the answers on page 465 as a check.

1. $\frac{22}{3}$ 2. $\frac{15}{8}$ 3. $\frac{27}{5}$ 4. $\frac{16}{2}$ 5. $\frac{18}{4}$ 6. $\frac{21}{6}$ 7. $\frac{30}{12}$

246. How to Add Mixed Numbers. (For Self-Help Study.) If you are having trouble with the addition of mixed numbers this article will help you.

1. The work in the box below shows the correct way to do this example: $3\frac{3}{4} + 4\frac{7}{8} + 4\frac{5}{8} = ?$

$3\frac{3}{4}$	$\frac{6}{8}$
$4\frac{7}{8}$	$\frac{7}{8}$
$4\frac{5}{8}$	$\frac{5}{8}$
$2\frac{1}{4}$	$\frac{18}{8} = 2\frac{2}{8} = 2\frac{1}{4}$
$13\frac{1}{4}$	

First: Add the fractions. Denominators must all be 8; so we get

$$\frac{5}{8} + \frac{7}{8} + \frac{6}{8} = \frac{18}{8} = 2\frac{2}{8} = 2\frac{1}{4}$$

Second: Add the $2\frac{1}{4}$ to the whole numbers.

2. See if this work is right:

$2\frac{2}{3}$	$\frac{8}{12}$
$\frac{1}{4}$	$\frac{3}{12}$
$2\frac{8}{12}$	$\frac{8}{12}$
$5\frac{7}{12}$	$\frac{19}{12} = 1\frac{7}{12}$

Is the $\frac{19}{12}$, the sum of the fractions, changed to its correct form?

The answer is $5\frac{7}{12}$. How did we get $5\frac{7}{12}$ instead of $4\frac{7}{12}$?

Remember to reduce improper fractions in answers.

Remember about adding the sum of the fractions to the whole numbers.

3. Are these five examples done correctly?

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
$2\frac{2}{3}$	$6\frac{1}{3}$	$4\frac{1}{2}$	$2\frac{1}{4}$	$4\frac{2}{3}$
$\frac{1}{2}$	4	$3\frac{4}{5}$	$5\frac{1}{3}$	$2\frac{1}{2}$
$3\frac{1}{6}$	$2\frac{1}{2}$	$2\frac{4}{10}$	$\frac{5}{12}$	$4\frac{3}{12}$
$6\frac{1}{3}$	$12\frac{5}{6}$	$10\frac{7}{10}$	8	$11\frac{5}{12}$

The answers are on page 465.

247. Adding Fractions and Mixed Numbers. (For Self-Help Practice.) The examples below and on the next page contain good practice for you. You can check your own work by using the answers on page 465.

Answers must be left in simplest form.

1. $\frac{5}{8} + \frac{3}{8}$
2. $5\frac{7}{12} + \frac{1}{2}$
3. Three tenths plus one half.
4. $\frac{2}{5} + \frac{1}{2}$
5. $4\frac{1}{2} + 1\frac{3}{4}$
6. $\frac{2}{3} + \frac{4}{9}$
7. Seven and three twelfths plus three and five twelfths.
8. $\frac{4}{9} + 11\frac{5}{6}$
9. $4\frac{2}{3} + 7\frac{1}{2}$
10. $\frac{3}{4} + \frac{6}{11}$
11. $\frac{9}{20} + \frac{3}{5}$

12. $\frac{2}{5}$
$\frac{1}{2}$
$\frac{3}{4}$

13. $\frac{1}{8}+\frac{1}{3}+\frac{5}{12}$

14. $4+\frac{1}{2}+\frac{1}{4}$

15. Two sevenths plus one half plus one fourteenth

16. $\frac{1}{3}+\frac{4}{5}+\frac{8}{15}$

17. $3\frac{1}{17}$
$\frac{7}{17}$
$6\frac{4}{17}$

18. $5\frac{5}{16}+\frac{1}{8}+\frac{1}{4}$

19. One sixth plus seven and three fourths plus one third.

20. $\frac{8}{21}+\frac{2}{21}+\frac{4}{21}$

21. $\frac{3}{50}$
$12\frac{9}{10}$
$\frac{16}{25}$

22. $\frac{2}{5}+\frac{3}{35}+\frac{6}{7}$

23. $\frac{1}{4}+8\frac{5}{6}+\frac{1}{3}$

24. *Add:* $\frac{1}{7}$
$\frac{1}{11}$

25. *Add:* $3\frac{2}{15}$
$\frac{3}{5}$

26. *Add:* $9\frac{7}{19}$
$4\frac{12}{19}$

27. $\frac{1}{6}+\frac{7}{18}$

28. $3\frac{1}{2}+4\frac{3}{10}$

29. $2\frac{3}{8}+3\frac{2}{5}$

30. $2\frac{1}{6}+1\frac{4}{5}$

31. *Write the reduced form of each:* $\frac{17}{3}$, $\frac{9}{9}$, $\frac{20}{8}$, $\frac{24}{6}$.

32. What denominator would you use with $\frac{}{4}+\frac{}{3}+\frac{}{6}$?

SUBTRACTION OF FRACTIONS AND MIXED NUMBERS

248. How to Use these Lessons. (Silent Reading.) The chart below will help you find the articles you need to study on subtraction of fractions and mixed numbers.

Art. 249 explains the finding of common denominators.

Art. 250 gives practice in using common denominators in easy subtraction.

Art. 251 explains how to subtract mixed numbers.

Art. 252 gives practice in subtracting fractions and mixed numbers.

249. The Common Denominator in Subtraction. (For Self-Help Study.) Do you remember that in adding fractions the denominators always have to be the same? The denominators of fractions when you are subtracting must be the same also.

Remember: In adding and subtracting fractions, the denominators must always be the same. This means that you sometimes have to change fractions in form. Changing the form of a fraction does not change its value (see Art. 240).

In the example $\frac{3}{4}-\frac{1}{4}$, we do not need to change the form of either fraction, because the fractions have the same (a common) denominator.

But in the example $\frac{3}{4}-\frac{1}{2}$, we must change the form (but not the value) of one of the fractions so that both will have the same denominator. In this example we would change $\frac{1}{2}$ to $\frac{2}{4}$.

250. Practice with Denominators in Easy Subtraction. (For Self-Help Practice.) You should be able to work these examples very easily and quickly. The answers are given on page 465.

Subtract:

1.	2.	3.	4.	5.	6.	7.
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{9}{10}$	$\frac{5}{6}$	$\frac{5}{6}$
$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{3}{4}$

8.	9.	10.	11.	12.	13.	14.
$\frac{7}{12}$	$\frac{2}{3}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{11}{12}$	$\frac{5}{6}$	$\frac{7}{8}$
$\frac{3}{10}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{3}{10}$	$\frac{7}{10}$

251. How to Subtract Mixed Numbers. (For Self-Help Study.) Below are three examples in the subtraction of mixed numbers. Examples B and C show the way to handle the two main kinds of "hard spots" in subtracting mixed numbers.

(*A*) $17\frac{2}{3}$ / $14\frac{1}{3}$ / $3\frac{1}{3}$ *Ans.*

(*B*) $24\frac{7}{9} \rightarrow \frac{7}{9}$ / $18\frac{1}{3} \rightarrow \frac{3}{9}$ / $6\frac{4}{9}$ $\frac{4}{9}$

(*C*) $16\frac{1}{4}$ *Think:* $15\frac{5}{4}$ / $12\frac{3}{4}$ / $3\frac{1}{2}$ *Ans.* $12\frac{3}{4}$ / $3\frac{2}{4} = 3\frac{1}{2}$

Example A above is very easy. The fractions already have common denominators. In Example B notice that it was necessary to find a common denominator. Example C contains the one "hard spot" in subtraction. The fraction in the minuend is smaller than the fraction in the subtrahend.

To work Example C we think of $16\frac{1}{4}$ as $15\frac{5}{4}$. A whole 1 is taken from the 16, changed to $\frac{4}{4}$ and added to the $\frac{1}{4}$. Thus the 16 becomes 15 and the $\frac{1}{4}$ becomes $\frac{5}{4}$. Think of this as changing the form but not the value of $16\frac{1}{4}$ to help us over a "hard spot." When you take a 1 from the whole number and change it to a fraction, don't forget to add the new fraction to the fraction already there. Thus, $\frac{4}{4} + \frac{1}{4} = \frac{5}{4}$.

Study the next examples. Notice Example E particularly. It contains a "hard spot" for many pupils, but it really is easy.

(*D*) $3\frac{1}{8}$ becomes $2\frac{9}{8}$ / $-1\frac{7}{8}$ / $1\frac{1}{4}$ *Ans.* $-1\frac{7}{8}$ / $1\frac{2}{8} = 1\frac{1}{4}$ *Ans.*

Remember that: $1 = \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \frac{7}{7}, \frac{8}{8}, \frac{9}{9}$, etc.

(*E*) 14 *Think:* $13\frac{5}{5}$ / $-8\frac{3}{5}$ / $5\frac{2}{5}$ *Ans.* $-8\frac{3}{5}$

(*F*) $21\frac{1}{3} \rightarrow 21\frac{4}{12} \rightarrow 20\frac{16}{12}$ / $-8\frac{3}{4}$ $-8\frac{9}{12}$ $-8\frac{9}{12}$ / $2\frac{7}{12}$ *Ans.*

252. Subtracting Fractions and Mixed Numbers. (For Self-Help Practice.) If you can work 24 of these examples correctly in 15 minutes, you will not have much trouble with subtraction of fractions and mixed numbers this year.

After you have worked the examples, check your answers with those on page 465.

Only answers in simplest form count as correct.

1. $\frac{5}{8}-\frac{1}{2}$ 2. $\frac{7}{9}-\frac{3}{4}$ 3. $1\frac{3}{5}-\frac{2}{3}$ 4. $9\frac{3}{11}-\frac{3}{11}$ 5. $6\frac{9}{20}-2\frac{7}{20}$

6. Ten and two ninths minus one sixth.

7. $\frac{7}{18}-\frac{1}{6}$ 8. $4\frac{5}{11}-2$ 9. $\frac{12}{17}-\frac{10}{17}$ 10. $1\frac{3}{4}-1\frac{2}{3}$ 11. $3\frac{1}{3}-1\frac{3}{7}$

12. $7\frac{3}{8}-6\frac{5}{24}$ 13. $1-\frac{1}{20}$ 14. $5\frac{3}{20}-2\frac{2}{5}$

15. Three and one sixth minus three fourths.

16. $7\frac{1}{6}-\frac{1}{2}$ 17. $\frac{2}{3}-\frac{1}{2}$ 18. $9\frac{3}{8}-\frac{3}{4}$ 19. $7\frac{3}{5}-\frac{1}{4}$

20. Nine and one sixth minus four and two thirds.

21. $10\frac{5}{10}-\frac{1}{2}$ 22. $5\frac{4}{7}-3$ 23. $6\frac{1}{2}-6\frac{2}{5}$ 24. $8\frac{9}{50}-\frac{29}{50}$

25. $9-3\frac{4}{11}$ 26. $\frac{7}{15}-\frac{4}{15}$ 27. Eight and two thirds minus four and one half.

28. $7\frac{1}{14}-\frac{4}{7}$ 29. $9\frac{5}{6}-4\frac{1}{4}$ 30. 7 minus $\frac{3}{22}$

MULTIPLICATION OF FRACTIONS AND SMALL MIXED NUMBERS

253. How to Use These Lessons. (SILENT READING.) The chart below will help you find the articles you may need to use in reviewing multiplication of fractions and small mixed numbers.

Art. 254 explains how to multiply fractions.
Art. 255 gives practice on multiplying easy fractions.
Art. 256 shows how to change mixed numbers to improper fractions.
Art. 257 gives practice on changing mixed numbers to improper fractions.
Arts. 258 and 259 explain how to cancel.
Art. 260 gives practice in canceling in easy multiplication.
Art. 261 explains how to multiply small mixed numbers.
Art. 262 gives practice in multiplication of small mixed numbers.

254. How to Multiply by Fractions. (FOR SELF-HELP STUDY.) If you are having trouble with the multiplication of fractions and small mixed numbers, you should study first of all the five things to remember in doing this kind of work.

The five things to remember in working an example in the multiplication of fractions are as follows:

1. Cancel whenever you can.

2. Multiply the numerators to obtain the numerator of the product.

3. Multiply the denominators to obtain the denominator of the product.

4. Reduce the product to simplest form.

5. When there is a whole number in the example, it is used as a numerator. Thus, think of 2 as $\frac{2}{1}$; 6 as $\frac{6}{1}$, etc.

These five points in multiplying fractions are illustrated below:

POINT 1. Cancel first: $\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{7}{\cancel{14}}}$

POINT 2. Multiply numerators: $\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{7}{\cancel{14}}} = \frac{2}{\quad}$

POINT 3. Multiply denominators: $\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{7}{\cancel{14}}} = \frac{2}{21}$

POINT 4. In an example like this, reduce the product:

$$\tfrac{9}{5} \times \tfrac{4}{5} = \tfrac{36}{25} = 1\tfrac{11}{25}.$$

POINT 5. A whole number is used as a numerator:

$$4 \times \tfrac{1}{5} = \tfrac{4}{1} \times \tfrac{1}{5} = \tfrac{4}{5}.$$

If you have forgotten how to cancel, study Arts. 258-259.

SAMPLES FOR STUDY

SAMPLE 1.
$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$, *Ans.*

1. In Sample 1, 2×4 gives us 8, the numerator of our answer.

2. How was the denominator 15 gotten?

3. Can the answer, $\frac{8}{15}$, be reduced?

SAMPLE 2.
$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$, *Ans.*

1. In Sample 2, does $1 \times 1 = 1$? (Some pupils write **2.** 2 is wrong.)

2. Which of the Five Points listed above are used in Sample 2?

SAMPLE 3.

$\frac{2}{3}$ of $8 = \frac{2}{3} \times \frac{8}{1} = \frac{16}{3} = 5\frac{1}{3}$, *Ans.*

1. In Sample 3, should seventh-grade pupils be expected to remember that "of" means "×" in multiplication?

2. Notice that the 8, a whole number, is used as a numerator.

3. If you really do not understand how $\frac{16}{3}$ is changed to $5\frac{1}{3}$, study Art. 244 now.

SAMPLE 4.

$$\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{8}}} = \frac{2}{12}, \textit{ Ans.}$$

1. Notice the canceling in Sample 4. 2 will go into both 4 and 8 an even number of times. 3 will go into both 3 and 9 an even number of times.

2. Could a better cancellation have been used? Do you see that it would have been simpler to think: 4 into 4 once and 4 into 8, 2 times?

3. Is the answer in the best form?

255. Easy Practice on Multiplication of Fractions. (FOR SELF-HELP PRACTICE.) After you have finished working the examples, use the answers on page 465 as a check.

1. $\frac{2}{3} \times \frac{4}{5}$	6. $\frac{5}{6} \times \frac{1}{2}$	11. $\frac{4}{7} \times \frac{4}{7}$	16. $\frac{3}{8}$ of 12
2. $\frac{3}{4}$ of $\frac{5}{7}$	7. $\frac{2}{3}$ of 5	12. $\frac{1}{2}$ of $\frac{1}{2}$	17. $\frac{5}{6}$ of 8
3. $\frac{2}{5} \times \frac{1}{5}$	8. $9 \times \frac{1}{2}$	13. $\frac{4}{5}$ of 6	18. $\frac{5}{8} \times \frac{3}{10}$
4. $\frac{4}{5} \times \frac{2}{3}$	9. $\frac{1}{4} \times \frac{1}{6}$	14. $\frac{1}{4} \times 12$	19. $\frac{7}{12}$ of 84
5. $\frac{1}{2}$ of $\frac{1}{4}$	10. $\frac{3}{5} \times 7$	15. $4 \times \frac{3}{5}$	20. $\frac{2}{3} \times 0$

256. How to Change Mixed Numbers to Improper Fractions. (FOR SELF-HELP STUDY.) There are two places in arithmetic where the ability to change mixed numbers to improper fractions is very helpful. These places are:

I. In multiplying small mixed numbers. Thus: $4\frac{1}{2} \times 3\frac{1}{3}$ becomes $\frac{9}{2} \times \frac{10}{3}$. Or, $7\frac{2}{5} \times 6\frac{3}{7}$ becomes $\frac{37}{5} \times \frac{45}{7}$.

II. In dividing small mixed numbers. Thus:

$10\frac{3}{4} \div 2\frac{5}{6}$ becomes $\frac{43}{4} \div \frac{17}{6}$, becomes $\frac{43}{4} \times \frac{6}{17}$.

Rule: **To change a mixed number to an improper fraction:**

1. Multiply the whole number by the denominator of the fraction.

2. Add this product to the numerator of the given fraction. This sum is the required numerator. The denominator does not change.

Thus: $3\frac{1}{2} = \frac{7}{2}$ because: $2 \times 3 = 6; 6 + 1 = 7$

$7\frac{3}{5} = \frac{38}{5}$ because: $5 \times 7 = 35; 35 + 3 = 38$

See if this rule works in the next four examples.

$4\frac{9}{16} = \frac{73}{16}$ $3\frac{1}{7} = \frac{22}{7}$ $8\frac{2}{9} = \frac{74}{9}$ $12\frac{1}{3} = \frac{37}{3}$

257. Practice in Changing Mixed Numbers to Improper Fractions. (For Self-Help Practice.) Change the numbers below to improper fractions. When you have finished, use the answers on page 466 to check your work.

1.	2.	3.	4.	5.	6.
$4\frac{1}{2}$	$6\frac{5}{12}$	$6\frac{1}{4}$	$3\frac{4}{7}$	$1\frac{1}{9}$	$3\frac{2}{3}$
7.	**8.**	**9.**	**10.**	**11.**	**12.**
$4\frac{7}{8}$	$2\frac{4}{5}$	$3\frac{3}{10}$	$9\frac{1}{6}$	$7\frac{6}{8}$	$10\frac{11}{16}$
13.	**14.**	**15.**	**16.**	**17.**	**18.**
$19\frac{1}{3}$	$8\frac{6}{9}$	$1\frac{4}{3}$	$11\frac{11}{12}$	$5\frac{3}{5}$	$12\frac{7}{12}$

258. How to Cancel. (For Self-Help Study.) The process of dividing out the same number from a numerator and denominator is called **cancellation.** To be able to

cancel quickly and accurately is very useful in multiplying fractions. Study the next four examples.

1.

$\frac{2}{5}\times\frac{1}{2}$ is worked:

$$\frac{\overset{1}{\cancel{2}}}{5}\times\frac{1}{\underset{1}{\cancel{2}}}=\frac{1\times1}{5\times1}=\frac{1}{5},\ \textit{Ans.}$$

2.

$\frac{3}{4}\times\frac{2}{3}$ is worked:

$$\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}}\times\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}=\frac{1\times1}{2\times1}=\frac{1}{2},\ \textit{Ans.}$$

In multiplication of fractions a **divisor** may be canceled from any numerator and denominator. If it is canceled from a numerator, it **must** be canceled from a denominator.

3. $\frac{\overset{2}{\cancel{4}}}{5}\times\frac{7}{\underset{5}{\cancel{10}}}=\frac{14}{25}$ $\quad 4\div2=2$, $10\div2=5$

4. $\frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{9}}}\times\frac{\overset{1}{\cancel{9}}}{\underset{7}{\cancel{14}}}=\frac{4}{7}$ $\quad 8\div2=4$, $14\div2=7$, $9\div9=1$, $9\div9=1$

➠ Always cancel when you can **before you multiply** the numerators and denominators. This allows you to work with smaller numbers when you multiply. It will also give you as your answer an easier fraction with which to work, and may save you the work of reducing the answer.

259. Repeated Canceling. (For Self-Help Study.) In multiplying by fractions, you can sometimes cancel more than once. Cancel as many times as you can; it makes the work simpler. Look at the examples below.

1.

$$\frac{3}{5}\times\frac{15}{36}=\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}}\times\frac{\overset{1}{\cancel{15}}}{\underset{4}{\cancel{36}}}=\frac{1\times1}{1\times4}=\frac{1}{4}$$

$5\div5=1$, $15\div5=3$, $3\div3=1$, $36\div3=12$, $3\div3=1$, $12\div3=4$

2.

$$\frac{\overset{1}{\cancel{3}}}{\underset{9}{\cancel{27}}}\times\frac{\overset{4}{\cancel{12}}}{\underset{7}{\cancel{14}}}=\frac{1\times4}{3\times7}=\frac{4}{21}$$

$12\div3=4$, $27\div3=9$, $6\div2=3$, $14\div2=7$, $3\div3=1$, $9\div3=3$

In canceling we can divide out the same number from the numerator of one fraction and the denominator of another (Example 3). Also, we can use the terms of only one fraction, as we do in reducing a fraction (Example 4). When you multiply more than two fractions, cancel just as you have learned to do with two fractions (Example 5).

3. $\frac{\overset{2}{\cancel{6}}}{7}\times\frac{4}{\underset{3}{\cancel{9}}}=\frac{8}{21}$ **4.** $\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}}\times 7=\frac{14}{3}=4\frac{2}{3}$ **5.** $\frac{\overset{1}{\overset{2}{\cancel{8}}}}{\underset{1}{\underset{2}{\cancel{12}}}}\times\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{4}}}\times\frac{\overset{1}{\cancel{6}}}{\underset{3}{\cancel{9}}}=\frac{1}{3}$

260. Practice in Cancellation. (For Self-Help Practice.) Check your answers with those shown on page 466.

1. $\frac{2}{3}\times\frac{3}{4}$	**7.** $\frac{5}{6}\times\frac{2}{3}$	**13.** $\frac{2}{5}\times\frac{1}{8}$	**19.** $\frac{3}{5}\times\frac{20}{21}$
2. $\frac{1}{2}\times\frac{1}{2}$	**8.** $\frac{7}{8}\times\frac{4}{5}$	**14.** $\frac{5}{9}\times\frac{3}{4}$	**20.** $\frac{2}{3}$ of $\frac{9}{10}$
3. $\frac{4}{5}\times\frac{1}{4}$	**9.** $\frac{1}{2}$ of $\frac{4}{5}$	**15.** $\frac{25}{36}$ of 18	**21.** $\frac{3}{4}$ of 24
4. $\frac{10}{12}$ of $\frac{1}{5}$	**10.** $\frac{9}{10}\times\frac{2}{3}$	**16.** $\frac{7}{8}\times 12$	**22.** $\frac{5}{16}\times\frac{3}{10}$
5. $\frac{1}{3}$ of 21	**11.** $5\times\frac{1}{5}$	**17.** $\frac{4}{25}\times 50$	**23.** $\frac{4}{5}\times\frac{5}{12}$
6. $\frac{2}{3}\times\frac{3}{4}$	**12.** $\frac{3}{8}\times\frac{1}{6}$	**18.** $\frac{1}{2}$ of 18	**24.** $\frac{1}{6}$ of $\frac{12}{25}$

261. How to Multiply Small Mixed Numbers. (For Self-Help Study.) You will have little if any trouble with the multiplication of mixed numbers if you will understand and **use carefully** the information which follows.

The multiplication of small mixed numbers becomes exactly like the multiplication of fractions if the mixed numbers are first changed to improper fractions.

$4\frac{1}{2}\times 8$. Change to $\frac{9}{2}\times\frac{8}{1}$. $6\times 7\frac{3}{5}$. Change to $\frac{6}{1}\times\frac{38}{5}$.

$3\frac{1}{3}\times 5\frac{1}{2}$. Change to $\frac{10}{3}\times\frac{11}{2}$. $6\frac{3}{4}\times\frac{2}{7}$. Change to $\frac{27}{4}\times\frac{2}{7}$.

Study Problems 1 and 2 below.

PROBLEM 1. I am going away for $2\frac{1}{2}$ months. A boy next door will take care of my hens while I am away. My hens eat $3\frac{3}{5}$ bu. of corn each month. How much corn should I leave?

The work: $2\frac{1}{2} \times 3\frac{3}{5}$. Change the mixed numbers to improper fractions: $2\frac{1}{2} = \frac{5}{2}$, $3\frac{3}{5} = \frac{18}{5}$.

Cancel and multiply: $\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{9}{\cancel{18}}}{\underset{1}{\cancel{5}}} = \frac{9}{1}$, or exactly 9 bu., *Ans.*

PROBLEM 2. My puppy, Blacknose, gains $1\frac{1}{2}$ lb. in weight a month. At this rate how much will he gain in 4 months?

The work: $4 \times 1\frac{1}{2}$ or $4 \times \frac{3}{2}$ or $\frac{\overset{2}{\cancel{4}}}{1} \times \frac{3}{\underset{1}{\cancel{2}}} = 6$ lb., *Ans.*

$4 \times \frac{3}{2}$ can be thought of as $\frac{4}{1} \times \frac{3}{2}$.

> ***Rule:*** **To multiply small mixed numbers, change to improper fractions and multiply. Remember to leave the product in simplest terms.**

262. Multiplying Fractions and Mixed Numbers. (FOR SELF-HELP PRACTICE.) These examples are a good test of whether you are "safe" with multiplication of fractions and small mixed numbers. Answers are given on page 467.

1. Four and two fifths times two and three elevenths.
2. $7 \times \frac{1}{2}$
3. $\frac{2}{3} \times \frac{1}{3}$
4. $\frac{3}{4} \times \frac{2}{7}$
5. $\frac{4}{5} \times \frac{5}{8}$
6. $2\frac{1}{4} \times \frac{16}{27}$
7. $4\frac{1}{2} \times \$3.50$
8. $\frac{7}{9}$ of $\frac{3}{14}$
9. $\frac{1}{2} \times 2\frac{1}{5}$
10. $\frac{5}{12}$ of 4
11. $5\frac{1}{7} \times 1\frac{9}{12}$
12. $5 \times \frac{4}{21}$
13. $8\frac{1}{3} \times \frac{2}{7}$

14. $\frac{5}{11}\times\frac{2}{3}$
15. $\frac{9}{16}\times 8$
16. $\frac{3}{16}\times\frac{4}{15}$
17. $\frac{3}{8}\times\frac{4}{9}\times 1\frac{1}{5}$
18. $1\frac{1}{4}\times\$1.60$
19. $\frac{1}{2}\times\frac{1}{5}\times 7$
20. $\frac{5}{12}\times\frac{3}{10}\times\frac{4}{5}$
21. $3\frac{1}{3}\times\frac{1}{4}\times 1\frac{1}{2}$
22. $\frac{1}{2}\times\frac{3}{4}\times\frac{2}{3}$
23. $1\frac{4}{5}\times\frac{1}{4}\times 15$
24. $1\frac{2}{9}\times\frac{3}{5}\times 1\frac{2}{3}$
25. $1\frac{1}{7}\times\frac{1}{6}\times 5\frac{1}{4}$
26. $\frac{1}{2}\times\frac{8}{15}\times\frac{5}{16}$
27. $7\times 2\frac{2}{3}\times\frac{3}{10}$
28. $\frac{7}{12}\times\frac{2}{7}$

DIVISION OF FRACTIONS AND MIXED NUMBERS

263. How to Use These Lessons. (Silent Reading.) Use the chart to help you find the work you may need.

Art. 264 explains how to divide by fractions and mixed numbers.

Art. 265 explains and gives practice in inverting divisors.

Art. 266 gives practice in dividing by fractions and mixed numbers.

Art. 267 gives you the main points in division of fractions and mixed numbers. It is a good summary for study and practice.

Art. 268 gives some good mixed practice with fractions.

264. How to Divide by Fractions and Mixed Numbers. (For Self-Help Study.) Study the next four samples.

Sample 1: $\frac{1}{4}\div\frac{2}{3}=?$

Step I. **Invert the fraction you divide by and change $\div$ to $\times$.**

Thus: $\frac{1}{4}\div\frac{2}{3}$ becomes $\frac{1}{4}\times\frac{3}{2}$. ($\frac{3}{2}$ is $\frac{2}{3}$ **inverted.**)

Notice: $\frac{2}{3}$ becomes $\frac{3}{2}$ and $\div$ is changed to $\times$.

$\frac{2}{3}$ is the **reciprocal** of $\frac{3}{2}$.

Step II. **Multiply.** $\frac{1}{4}\times\frac{3}{2}=\frac{3}{8}$, *Ans.*

SAMPLE 2: $\frac{3}{4} \div \frac{2}{5} = ?$

Step I. **Invert the fraction you divide by and change $\div$ to $\times$.**

Thus: $\frac{3}{4} \div \frac{2}{5}$ becomes $\frac{3}{4} \times \frac{5}{2}$. ($\frac{5}{2}$ is $\frac{2}{5}$ inverted.)

Step II. **Multiply.** $\frac{3}{4} \times \frac{5}{2} = \frac{15}{8} = 1\frac{7}{8}$, *Ans.*

SAMPLE 3: $5 \div \frac{3}{4} = ?$

Step I. **Invert the fraction you divide by and change $\div$ to $\times$.**

Thus: $5 \div \frac{3}{4}$ becomes $5 \times \frac{4}{3}$.

Step II. **Multiply.** $5 \times \frac{4}{3} = \frac{20}{3} = 6\frac{2}{3}$, *Ans.*

SAMPLE 4: $2\frac{1}{4} \div 3 = ?$

Step I. **Change mixed numbers to improper fractions.**

Thus: $2\frac{1}{4} \div 3$ becomes $\frac{9}{4} \div 3$.

Step II. **Invert the fraction you divide by and change $\div$ to $\times$.**

Thus: $\frac{9}{4} \div 3$ becomes $\frac{9}{4} \times \frac{1}{3}$ ($\div 3 = \times \frac{1}{3}$).

Step III. **Multiply. Cancel if you can.** $\frac{\overset{3}{\cancel{9}}}{4} \times \frac{1}{\underset{1}{\cancel{3}}} = \frac{3}{4}$, *Ans.*

Rule: To divide by a fraction, invert the fraction you divide by, change $\div$ to $\times$, and multiply. Mixed numbers are changed to improper fractions.

265. Inverting the Divisor. (For Self-Help Practice.) Examples in the division of fractions are changed to multiplication examples by **inverting** the **divisor** and changing $\div$ to $\times$. Study the two samples below.

(*a*) $\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3} =$ ..... (*b*) $3\frac{1}{2} \div 2\frac{1}{3} = \frac{7}{2} \div \frac{7}{3} = \frac{7}{2} \times \frac{3}{7} =$ .....

Change the divisors below to multipliers. Example 1 is already worked. Use the answers on page 467 as a check.

1. $\div \frac{1}{3} = \times \frac{3}{1}$
2. $\div 2\frac{1}{2} = \times$....
3. $\div 1\frac{4}{5} = \times$....
4. $\div \frac{1}{6} = \times$....
5. $\div 1\frac{1}{2} = \times$....
6. $\div \frac{1}{3} = \times$....
7. $\div 1\frac{1}{8} = \times$....
8. $\div \frac{3}{8} = \times$....
9. $\div 9\frac{1}{8} = \times$....
10. $\div 4\frac{1}{2} = \times$....
11. $\div 1\frac{2}{3} = \times$....
12. $\div 8\frac{1}{2} = \times$....
13. $\div 5\frac{1}{3} = \times$....
14. $\div 2\frac{1}{3} = \times$....
15. $\div 7 = \times$....
16. $\div 4\frac{1}{2} = \times$....
17. $\div 3\frac{3}{4} = \times$....
18. $\div 6\frac{1}{4} = \times$....
19. $\div 3 = \times$....
20. $\div 2\frac{2}{5} = \times$....
21. $\div 6\frac{1}{2} = \times$....
22. $\div \frac{2}{3} = \times$....
23. $\div 5\frac{1}{8} = \times$....
24. $\div 2 = \times$....

266. Dividing Fractions and Mixed Numbers. (For Self-Help Practice.) Correct answers are on page 467. If the examples below are hard for you, study Art. 267.

1. $\frac{1}{4} \div \frac{1}{3}$
2. $4\frac{1}{2} \div 2\frac{1}{2}$
3. $3\frac{1}{3} \div 1\frac{4}{5}$
4. $\frac{3}{4} \div \frac{1}{6}$
5. $1\frac{1}{5} \div 3$
6. $7 \div 9\frac{1}{8}$
7. $\frac{3}{4} \div 4\frac{1}{2}$
8. $1\frac{1}{5} \div 1\frac{2}{3}$
9. $\frac{5}{6} \div 8\frac{1}{2}$
10. $2\frac{2}{3} \div 5\frac{1}{3}$
11. $2\frac{1}{6} \div 3\frac{3}{4}$
12. $25 \div 6\frac{1}{4}$
13. $11\frac{1}{3} \div 4$
14. $7\frac{1}{2} \div 2\frac{2}{5}$
15. $3\frac{1}{4} \div 6\frac{1}{2}$
16. $10 \div \frac{1}{3}$
17. $12 \div 1\frac{1}{8}$
18. $11\frac{1}{4} \div \frac{3}{8}$
19. $\frac{14}{15} \div 2\frac{1}{3}$
20. $4\frac{2}{3} \div 7\frac{1}{3}$
21. $2 \div 4\frac{1}{2}$
22. $5 \div \frac{2}{3}$
23. $2\frac{1}{3} \div 5\frac{1}{8}$
24. $26 \div 2\frac{1}{6}$

267. Main Points in Division of Fractions and Mixed Numbers. (For Self-Help Study and Practice.) If division of fractions and small mixed numbers still bothers you after you have studied Art. 263, you may use the following work for a further study of division.

Division of fractions and division of mixed numbers are not hard, if you will take care to have 5 things clearly in mind. These five things are:

1. If there is a mixed number in the example, change it to an improper fraction. Thus:

$4\frac{2}{3} \div \frac{3}{7}$ becomes $\frac{14}{3} \div \frac{3}{7}$ $\quad \frac{2}{3} \div 4\frac{3}{7}$ becomes $\frac{2}{3} \div \frac{31}{7}$

2. Invert the terms of the **divisor** and change $\div$ to $\times$. Thus:

$\frac{14}{3} \div \frac{3}{7}$ means $\frac{14}{3} \times \frac{7}{3}$ $\quad \frac{2}{3} \div \frac{31}{7}$ means $\frac{2}{3} \times \frac{7}{31}$

3. Make sure which number is the divisor. This is easy in an example like $\frac{14}{3} \div \frac{3}{7}$ because $\div$ is read, "divided by." The divisor is the $\frac{3}{7}$. But you will have to think carefully which number is the divisor when the example is: "Divide $4\frac{2}{3}$ by $\frac{3}{7}$," or "What is the quotient when $4\frac{2}{3}$ is divided by $\frac{3}{7}$?"

What is the divisor in the problem below?

Problem: How many catcher's mitts can be made from $16\frac{1}{2}$ sq. ft. of horse hide if it takes $2\frac{3}{4}$ sq. ft. for each mitt?

4. Remember that in the division of fractions a whole number is written and used like a numerator. Thus:

$\frac{2}{3} \div 4$ means $\frac{2}{3} \div \frac{4}{1}$, or $8 \div \frac{5}{3}$ means $\frac{8}{1} \div \frac{5}{3}$.

5. After you have changed the division example to multiplication form, **multiply numerators to get the numerator**

of the answer, and multiply denominators to get the denominator of the answer. Before you multiply, cancel wherever you can.

Can you work the examples below correctly? Use the answers on page 467 as a check on your own work.

1. $4 \div \frac{2}{3} =$
2. $\frac{3}{5} \div 3 =$
3. $4\frac{2}{3} \div 7 =$
4. $4\frac{1}{3} \div 3\frac{1}{4} =$
5. $\frac{3}{7} \div \frac{1}{7} =$
6. $2\frac{1}{2} \div 6 =$
7. $8 \div 1\frac{2}{3} =$
8. $3\frac{1}{5} \div 3\frac{2}{3} =$
9. $6\frac{7}{12} \div 3 =$
10. $16 \div \frac{1}{2} =$
11. $\frac{1}{2} \div 2 =$
12. $6\frac{1}{5} \div 2\frac{1}{15} =$
13. $18 \div 4\frac{1}{2} =$
14. $31 \div \frac{1}{4} =$
15. $0 \div 2\frac{1}{5} =$

MIXED PRACTICE

268. Mixed Fundamentals of Fractions. (For Self-Help Practice.) No example in this article should trouble a seventh-grade pupil. Can you do them all correctly without help? After you have finished the examples, use the answers on page 468.

1. *Add:*
$2\frac{1}{2}$
$3\frac{3}{4}$
$6\frac{5}{8}$

2. *Subtract:*
$6\frac{1}{2}$
$2\frac{3}{4}$

3.
$7\frac{1}{4}$
$2\frac{5}{6}$
$6\frac{2}{3}$

4. *Find:*
$\frac{2}{3}$ of $\frac{9}{20}$

5.
$\frac{4}{5} \div 6 = ?$

6. *Subtract:*
$11\frac{1}{5}$
$4\frac{7}{10}$

7.
$2\frac{1}{3} \times 4\frac{1}{5} = ?$

8.
$4\frac{5}{12}$
$7\frac{1}{4}$
$1\frac{3}{8}$
$9\frac{5}{6}$

9.
$3\frac{5}{6} - 1\frac{3}{8} = ?$

10.
$\frac{7}{8} \div 3\frac{1}{3} = ?$

11.
$7\frac{1}{2} \div 1\frac{2}{3} = ?$

12.
$9 \times 4\frac{1}{4} = ?$

DECIMAL FRACTIONS

269. How to Use These Lessons. (SILENT READING.) Here are the articles that you may use when you need to review important facts about decimals:

> Art. 270 explains the "hard spot" in adding and subtracting decimal numbers.
> Art. 271 explains how to multiply decimal numbers.
> Art. 272 explains division of decimals.

270. The "Hard Spot" in the Addition and Subtraction of Decimal Fractions. (FOR SELF-HELP STUDY AND PRACTICE.) The one thing to remember about the addition and subtraction of decimal fractions is that the **decimal points must be kept in a straight line.**

The next samples show you what this means.

Addition

Correct	Wrong
47.6 .45 2.68 50.73	47.6 .45 2.68 7.89

Subtraction

Correct	Wrong
8.720 .236 8.484	8.720 .236 .6360

PRACTICE

Copy the next examples and work them. The answers are on page 468.

1. *Add:* 6.34, 17.2, 18.05, .75.
2. $17.367 - 8.072 = ?$
3. $243.6 - 121.43 = ?$
4. $75 - 24.25$
5. $7 + 2.7 + 3.5$
6. $81.004 - 75$

271. How to Multiply Decimals. (For Self-Help Study and Practice.) Multiplying by decimals is just like multiplying by whole numbers except that **we must know where to put the decimal point in the product.**

> ***Rule:*** **There are as many decimal places in the product as there are in the multiplier and multiplicand together.**

Study the two examples below. The rule is used for pointing off the products. Putting the decimal point in the right place is called **"pointing off."**

(*A*)	4.27	(2 decimal places)	**(*B*)**	3.26	(2 decimal places)
	6.34	(2 decimal places)		7.6	(1 decimal place)
	1708	(4, sum of decimal		1956	(3, sum of decimal
	1281	places)		2282	places)
	2562			24.776	(3 decimal places in
	27.0718	(4 decimal places)			the product)

PRACTICE

The products in the first four examples below are correct. Copy the answers and point off with the correct number of decimal places.

1.	34.6	**2.**	7232	**3.**	24.78	**4.**	424
	3.2		.142		.379		730
	11072		1026944		939162		309520

5. Multiply 726 by 4.7

6. Multiply 3.2 by .0025

7. Multiply $500 by .0425

8. .065 × $1250

☛ You will find the correct answers on page 468.

272. Directions for Dividing Decimals. (For Self-Help Study and Practice.) The directions and the samples below them will show you how to divide with decimals.

1. Move the decimal point in the divisor to the right as many places as are needed to make the divisor a whole number (Sample *b*).

2. Move decimal point in dividend to the right just as many places as you moved the decimal point in the divisor. Annex zeros to dividend where needed (Sample *c*).

3. Place decimal point in quotient exactly above the new decimal point in the dividend.

4. Then divide as you do with whole numbers. Zeros must sometimes be inserted in the quotient (Sample *d*).

5. If the divisor is a whole number to begin with, all you have to do is to place the decimal point in the quotient, and divide (Sample *a*).

See if the above rules are used in the next examples.

(*a*) $24\overline{)362.4}$ Divisor a whole number. Decimal point in dividend not moved. Point in quotient put right above the point in the dividend.

(*b*) $2.4\overline{)3.624}$ becomes $2.4\overline{)3.624.}$ Notice where the point is placed in the quotient.

(*c*) $.24\overline{)362.4}$ becomes $.24\overline{)362.40.}$ Notice zero needed.

(*d*) $48\overline{).144}$ Vacant places between decimal point and first quotient figure should be filled with zeros.

PRACTICE

The quotient figures in Examples 1 to 3 are correct. Copy the quotients and point them off correctly. Then work Examples 4, 5, and 6.

1. $.049\overline{).97118}$ 1982
2. $74\overline{)3.182}$ 43
3. $.0005\overline{)3.214670}$ 642934
4. $\$162.50 \div \$2500$
5. $362.4 \div .242$
6. $168 \div .32$

☛ You will find the answers on page 468.

DENOMINATE NUMBERS

273. How to Use the Lessons on Denominate Numbers. (Silent Reading.) If you are having trouble with denominate numbers, you will find the next articles very helpful. The chart below will guide you in selecting the articles you may need to study.

Art. 274 deals with changing the form of denominate numbers.

Art. 275 explains and gives practice in changing denominate numbers to good forms for answers.

Art. 276 explains how to add denominate numbers.

Art. 277 gives some hints on changing the form of answers for addition examples and practice in adding.

Art. 278 explains subtracting denominate numbers and gives practice.

Art. 279 explains multiplying denominate numbers and gives practice.

Art. 280 explains division of denominate numbers and gives practice.

274. Changing the Form of Denominate Numbers. (For Self-Help Study.) In seventh-grade arithmetic we sometimes have to change the form of a denominate number. This work is not hard if you can do everything in this article correctly and easily.

Important Fact Number 1. In order to change the form of denominate numbers, you must **know** the facts in the Tables of Measure on page 470. Do you really know these facts?

Important Fact Number 2. To change the form of denominate numbers you must be able to **use** the facts in the Tables of Measure. Is there anything in the next samples which you do not understand?

16 in. = **1 ft. 4 in.**	There are 12 inches in **1 foot.**
$\frac{1}{4}$ lb. = **4 oz.**	There are 16 oz. in 1 lb.
11 ft. = **3 yd. 2 ft.**	1 yd. = 3 ft.
$3\frac{1}{2}$ ft. = **42 in.**	1 ft. = 12 in. $\frac{1}{2}$ ft. = 6 in.
$2\frac{1}{2}$ pk. = **20 qt.**	There are 8 qt. in 1 pk.

When you know the Tables of Measure, turn back to page 8 and work Article 6 again.

275. The Form of Answers in Denominate Numbers. (For Self-Help Study and Practice.) In the example at the right, the answer should be reduced, for we do not leave 12 inches or more in answer form. We change amounts of 12 inches or more to feet or to feet and inches.

2 ft.	6 in.
4 ft.	9 in.
4 ft.	2 in.
10 ft.	17 in.

Since 17 in. equals 1 ft. 5 in., the correct form of the answer to this example is 11 ft. 5 in.

Whenever you have 12 inches or more in any part of an answer, change the inches to feet or to feet and inches. Whenever you have 3 ft. or more in an answer, change the feet to yards or to yards and feet, etc.

If you have two or three denominate numbers in an answer, change the numbers to larger units if possible. Thus: 2 yd. 7 ft. would be 4 yd. 1 ft. 1 gal. 7 qt. would be 2 gal. 3 qt.

You will see that this has been done in the six answers below. The numbers at the left are answers to addition examples as first found. The correct form for answers as final results are shown at the right.

First Found Answers	Final Results
(*a*) 4 yd. 7 ft. 3 in.	6 yd. 1 ft. 3 in.
(*b*) 1 yd. 7 ft. 20 in.	3 yd. 2 ft. 8 in.
(*c*) 6 qt. 11 pt.	2 gal. 3 qt. 1 pt.
(*d*) 4 hr. 100 min.	5 hr. 40 min.
(*e*) 3 pk. 14 qt. 8 pt.	5 pk. 2 qt.
(*f*) 6 lb. 40 oz.	8 lb. 8 oz. ($8\frac{1}{2}$ lb.)

PRACTICE

Change the following numbers to correct forms for final answers. Check your answers with the correct forms given on page 468.

1. 6 ft. 18 in.	**4.** 9 lb. 21 oz.	**7.** 6 sq. ft. 150 sq. in.
2. 4 yd. 7 ft.	**5.** 4 pk. 7 qt. 8 pt.	**8.** 4 sq. yd. 14 sq. ft.
3. 3 qt. 5 pt.	**6.** 6 hr. 110 min.	**9.** 2 cu. yd. 32 cu. ft.

276. How to Add Denominate Numbers. (For Self-Help Study.) On the next page is a problem that calls for addition of denominate numbers. It is worked to show you just what to do.

PROBLEM I. One summer Mary's older brother won first prize for raising corn. His reward was a trip to Washington, D. C. On the train to Washington, he kept a record of the time between stations.

Between home and the first station it took 1 hr. and 14 min.; between the first and second stations, 56 min.; between the second and third, 1 hr. and 29 min. How long had he been traveling when he reached the third station?

STEP I. Write the numbers which mean hours under ***Hr.*** Write the numbers which mean minutes under ***Min.***

	Hr.	*Min.*
	1	14
		56
	1	29
STEP II. *Add:*	2	99*
	1	39
STEP III. *Add:*	3 hr.	39 min., *Ans.*

After adding (Step II), the 99 min. must be changed to hr. and min.

99* min. = 1 hr. 39 min.

The 99 min. in reduced form (1 hr. 39 min.) is added to the 2 hr. Put a line around the 99 min. to be sure that the 1 hr. 39 min. is added only to the 2 hr.

277. Six Helps in Adding Denominate Numbers. (FOR SELF-HELP STUDY AND PRACTICE.) Study the following statements about forms of answers in denominate numbers.

1. When an answer comes out 67 min., it should be written as 1 hr. 7 min. Can you tell why this is true?

2. 15 in. in an answer should be written 1 ft. 3 in.

3. 21 oz. in an answer should be written 1 lb. 5 oz.

4. When there are more feet than 3 in an answer, change the feet to yards and feet.

5. When there are more ounces than 16 in an answer, change the ounces to pounds and ounces.

6. When there are more minutes than 60 in an answer, change the minutes to hours and minutes.

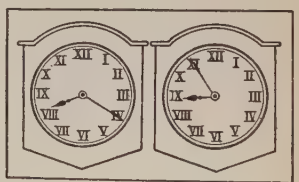
PRACTICE

1.	2.	3.	4.
4 ft. 8 in.	9 hr. 27 min.	4 da. 16 hr.	4 gal. 1 qt.
3 ft.	6 hr. 12 min.	3 da.	3 qt.
6 ft. 1 in.	7 hr. 36 min.	5 da. 6 hr.	4 gal. 1 qt.
2 ft. 6 in.	4 hr. 45 min.	4 da. 10 hr.	4 gal. 2 qt.

☛ The correct answers are given on page 468.

278. How to Subtract Denominate Numbers. (For Self-Help Study and Practice.) Study Problems I and II to see how to subtract denominate numbers.

Problem I. I started for school at 8:20 and got there at 8:55. How long did it take me to go to school?



8 hr. 55 min.
8 hr. 20 min.
35 min., *Ans.*

Problem II contains the one "hard spot" in subtracting denominate numbers. When we write the numbers on our paper, we find that we have 2 in. minus 9 in. Study the work and the explanation below to see what is done.

Problem II. Mr. Jones is 6 ft. 2 in. tall, and Mr. White is 5 ft. 9 in. tall. How much taller is Mr. Jones?

6 ft. 2 in. — 5 ft. 9 in.

Think → 5 ft. 14 in. — 5 ft. 9 in.

5 in., *Ans.*

9 in. cannot be subtracted from 2 in. But the number 6 ft. 2 in. can be changed in form. Since 1 ft. = 12 in., we

can think of the number 6 ft. 2 in. as 5 ft. 14 in. We can change the form of a denominate number to help us in subtraction.

PRACTICE

Subtract:

1.	2.	3.	4.
6 ft. 1 in.	9 gal. 1 qt.	1 hr. 10 min.	8 lb. 4 oz.
4 ft. 8 in.	7 gal. 3 qt.	55 min.	6 lb. 11 oz.

☛ Answers are on page 468. Use them **only** as a check.

279. Multiplying Denominate Numbers. (For Self-Help Study and Practice.) The problem below is worked and explained in order to show you how to multiply denominate numbers.

Problem. How much maple syrup could a Vermont farmer put in 10 cans, if each can holds 3 gal. 3 qt.?

3 gal. 3 qt.
10
7 2
30
37 gal. 2 qt., *Ans.*

(*a*) Think 10×3 qt.=30 qt. Then reduce the 30 qt. to 7 gal. 2 qt.
(*b*) Write 7 gal. 2 qt. as shown in the work at the left.
(*c*) 10×3 gal.=30 gal.
(*d*) 7 gal. 2 qt.+30 gal.=37 gal. 2 qt.

The one "hard spot" in multiplying denominate numbers is in changing the forms of the numbers.

If changing forms of denominate numbers bothers you, study articles 274 and 275 again, and **learn** the Tables of Measure on page 470.

PRACTICE

The correct answers to these multiplication examples are given on page 468.

1.	2.	3.	4.
4 min. 30 sec.	2 gal. 3 qt. 1 pt.	4 bu. 2 pk.	5 ft. 3 in.
7	12	9	8

280. Dividing Denominate Numbers. (For Self-Help Study and Practice). The next work shows how to divide denominate numbers. In dividing denominate numbers there is a "hard spot" in the carrying for which you must watch out. Thus:

$$4\overline{)9 \text{ ft. } 8 \text{ in.}} = 2 \text{ ft. } 5 \text{ in.}$$

9 ft. ÷ 4 = 2 ft. and **1 ft. left over.** This 1 ft. is changed to 12 in. so that it can be added to the 8 in. 8 in. + 12 in. = 20 in. Now divide the 20 in. by 4. This gives 5 in., and the answer becomes 2 ft. 5 in.

$$6\overline{)8 \text{ lb. } 4 \text{ oz.}} = 1 \text{ lb. } 6 \text{ oz.}$$

The 2 lb. left over when you divide 8 lb. by 6 becomes **32 oz.** 32 oz. + 4 oz. = 36 oz. 36 oz. ÷ 6 = 6 oz.

$$3\overline{)11 \text{ bu. } 2 \text{ pk.}} = 3 \text{ bu. } 3\tfrac{1}{3} \text{ pk.}$$

The 2 bu. left over when you divide 11 bu. by 3 becomes **8 pk.** 8 pk. + 2 pk. = 10 pk., to be divided by 3.

In "carrying" in division of denominate numbers, be sure to change numbers correctly.

PRACTICE

1. 7 yd. 2 ft. ÷ 4
2. 9 pk. 4 qt. ÷ 4
3. 5 gal. 3 qt. ÷ 4
4. 11 hr. 35 min. ÷ 5
5. 8 lb. ÷ 6
6. 13 yd. 2 ft. ÷ 4

☛ Correct answers are given on page 468.

MENSURATION

281. How to Use the Lessons on Mensuration. (Silent Reading.) The chart below shows you the articles to use in restudying anything about mensuration which you do not now understand very well.

> Art. 282 explains the meaning of important words used in mensuration.
>
> Art. 283 explains the finding of areas of parallelograms.
>
> Art. 284 explains finding areas of triangles.
>
> Art. 285 explains and gives practice in finding volumes of rectangular solids.

282. Words Used in Mensuration. (For Self-Help Study.) This article explains the meaning of many important words often used in mensuration.

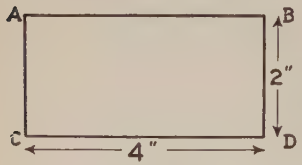


Fig. 1

1. Fig. 1 is called a **rectangle.** A rectangle is a surface or area with opposite sides that are equal and parallel and which has four **right** angles (square corners).

2. A **square** is a special kind of rectangle. Its four sides are equal, and its angles are all right angles.

3. Fig. 1 has a perimeter of 12″. **Perimeter** means the **distance around.** It is the **sum** of the **sides.** Is 12″ correct for the perimeter of Fig. 1?

4. The area of Fig. 1 is 8 sq. in. **Area** is the **amount of space** or **surface** covered.

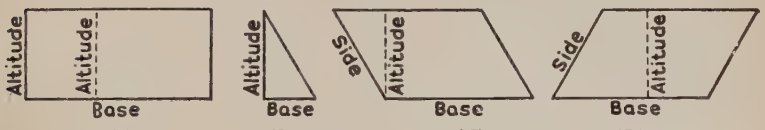
5. The **length, width,** and **depth** or **height** of a figure are called its **dimensions.**

6. Parallel lines are lines which are everywhere the same distance apart.

7. A **parallelogram** is a figure bounded by two pairs of parallel lines. A **rectangle** is a special kind of parallelogram; its four corners are right angles. Figs. A, C, and D, below are all parallelograms. Fig. A is also a rectangle.

8. A **triangle** is any closed figure that has just three straight sides. A **right triangle** is a special kind of triangle. One of its angles is a right angle (square corner). Fig. B below is a right triangle.

9. The **base** of a figure is the side on which it is thought of as standing. Any side may be taken as a base.



10. The **altitude** of a figure is the height of that figure measured along a line drawn at right angles to the base.

11. A **rectangular solid** has three dimensions: **length, width,** and **height** or **depth.**

283. How to Find Areas of Parallelograms. (For Self-Help Study and Practice.) Areas of parallelograms are easy to find. The important things to remember are that in finding areas **dimensions must be expressed in the *same* unit and the answer in *square* units.** Be sure you know the rule below.

Area of a parallelogram = base × altitude.

Remember that a side of the rectangle is also the altitude, because the side makes a right angle with the base

PRACTICE

Use the rule in finding the areas of these parallelograms:

1. A floor 10 ft. 6 in. by 12 ft.
2. A window-pane 28 in. by 52 in.
3. A table top 28 in. square.
4. Base, 8 yd.; altitude, $5\frac{1}{2}$ ft.
5. Base, 16 in.; altitude, 1 ft.
6. Base, 2 rd.; altitude, 4 yd.

☛Correct answers are given on page 468.

284. How to Find Areas of Triangles. (For Self-Help Study and Practice.) The arithmetic needed to find areas of triangles is easy. Here is the rule:

Area of any triangle $=\frac{1}{2}$ of the base $\times$ the altitude.

You must be careful that you know what the base and the altitude are. The altitude must always be measured along a line drawn from the **vertex** (uppermost point of the triangle), at right angles to the base. 3′ is the altitude in Fig. 1. If the triangle is a right triangle, the altitude is easy to find. Just let one of the two shorter sides be the base; then the other of the shorter sides is the altitude, because the two shorter sides form a right angle. 6 in. is the altitude in Fig. 2.

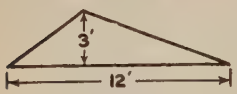


Fig. 1

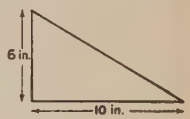


Fig. 2

PRACTICE

Find the areas of the triangles having these dimensions:

1. $a=4$ in., $b=6$ in.
2. $a=8$ in., $b=1$ ft.
3. $a=7$ ft., $b=2$ yd.
4. $a=8$ mi., $b=15$ mi.

☛Correct answers are given on page 469.

VOLUMES

285. How to Find Volumes of Rectangular Solids. (For Self-Help Study and Practice.) In this article you can review the finding of volumes.

Problem 1. How much water is there in a swimming pool 18 yd. long and 4 yd. wide? The water is 6 ft. deep on the average.

The amount of water in the tank is called the **volume** of water. The unit of measure of the volume of the water is a **cubic unit** of water—a cubic yard, a cubic foot, or any other cubic unit. The number of cubic yards of water in this tank is found like this:

$$18 \text{ yd.} \times 4 \text{ yd.} \times \mathbf{2 \text{ yd.}} = 144 \text{ cu. yd.}$$

Length × width × height = volume, or **length × width × depth = volume.**

Notice that before multiplying, all dimensions must be in the same unit. Here we changed the 6 ft. to 2 yd. Our answer is in **cubic** yards. We could have changed 18 yd. to 54 ft. and 4 yd. to 12 feet. Then, $54 \times 12 \times 6$ would have given us our answer in **cubic feet.**

Problem 2. How many cubic inches are there in a block of ice which is 6 in. wide, 8 in. high, and 10 in. long?

$$6'' \times 10'' \times 8'' = 480 \text{ cu. in.}$$

Length × width × height = volume.

A block of ice is in the form called a **rectangular solid.** The word "solid" has a special meaning in arithmetic. It really means shape. Thus air and water can be in the form of a rectangular solid.

Find the volumes of the solids shown below. The correct volumes are given on page 469.

FIG. 1

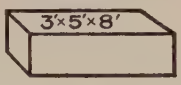


FIG. 2

FIG. 3

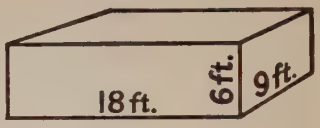


FIG. 4

PERCENTAGE

286. How to Use the Lessons on Percentage. (SILENT READING.) The chart below will help you find articles that you may use in restudying percentage.

Art. 287 contains a chart giving fraction-decimal-per cent relations most often used. These are important. You should learn these thoroughly and use the practice in this article.

Art. 288 explains how to find a per cent of a number.

Art. 289 gives practice in finding per cents of numbers.

Art. 290 explains and gives practice in the second use of percentage, namely, finding what per cent one number is of another.

Art. 291 reviews these two uses of percentage, giving you valuable suggestions to keep them clear in your mind.

Art. 292 gives practice in the two uses of percentage.

Art. 293 explains percentage as used in figuring interest and discount.

287. The Three Languages of Arithmetic. (FOR SELF-HELP STUDY AND PRACTICE.) In your study of history and geography you know that people in different countries often speak different languages. In arithmetic we may say that

there are three different languages. These are: fraction language, decimal language, and percentage language. The chart below shows the same facts written in the three arithmetic languages. You should know these facts.

Fraction	*Decimal*	*Per Cent*	*Fraction*	*Decimal*	*Per Cent*
$\frac{1}{2}$ ($\frac{2}{4}, \frac{3}{6}$, etc.)	.5 or .50	50%	$\frac{1}{8}$	.12$\frac{1}{2}$ or .125	12$\frac{1}{2}$%
$\frac{1}{3}$ ($\frac{2}{6}, \frac{4}{12}$)	.33$\frac{1}{3}$	33$\frac{1}{3}$%	$\frac{3}{8}$	.37$\frac{1}{2}$ or .375	37$\frac{1}{2}$%
$\frac{2}{3}$ ($\frac{4}{6}, \frac{8}{12}$)	.66$\frac{2}{3}$	66$\frac{2}{3}$%	$\frac{5}{8}$	.62$\frac{1}{2}$ or .625	62$\frac{1}{2}$%
$\frac{1}{4}$ ($\frac{2}{8}, \frac{3}{12}$)	.25	25%	$\frac{7}{8}$	.87$\frac{1}{2}$ or .875	87$\frac{1}{2}$%
$\frac{3}{4}$ ($\frac{6}{8}, \frac{9}{12}$)	.75	75%	$\frac{1}{10}$	.1 or .10	10%
$\frac{1}{5}$ ($\frac{2}{10}$)	.2 or .20	20%	1 ($\frac{1}{1}, \frac{2}{2}$)	1 or 1. or 1.0	100%
$\frac{2}{5}$ ($\frac{4}{10}$)	.4 or .40	40%	2	2. or 2.0	200%
$\frac{3}{5}$ ($\frac{6}{10}$)	.6 or .60	60%	$1\frac{1}{2}$	1.5 or 1.50	150%
$\frac{4}{5}$ ($\frac{8}{10}$)	.8 or .80	80%	$2\frac{1}{2}$	2.5 or 2.50	250%
$\frac{1}{6}$	.16$\frac{2}{3}$	16$\frac{2}{3}$%	$1\frac{1}{4}$	1.25	125%
$\frac{5}{6}$	.83$\frac{1}{3}$	83$\frac{1}{3}$%	$\frac{1}{100}$	.01	1%
			$\frac{4}{100}$	.04	4%
			$\frac{4\frac{1}{2}}{100}$	.04$\frac{1}{2}$ or .045	4$\frac{1}{2}$%
			$\frac{4\frac{1}{4}}{100}$	.04$\frac{1}{4}$ or .0425	4$\frac{1}{4}$%

➡The next samples are worth studying.

5 = 500%	$4\frac{1}{2}$ or 4.5 = 450%	$3\frac{3}{4}$ or 3.75 = 375%
.5 = 50%	.45 = 45%	.375 = 37$\frac{1}{2}$% or 37.5%
.05 = 5%	.045 = 4.5% or 4$\frac{1}{2}$%	.0375 = 3.75% or 3$\frac{3}{4}$%
.005 = .5 of 1% or $\frac{1}{2}$%		

➡You will notice from a study of the above charts that percentage language is based on fractions with denominators of 100. Thus 4% means $\frac{4}{100}$. In fact, you can think of per cents as fractions whose denominators are 100.

4% = $\frac{4}{100}$. The % sign is a short way of writing $\frac{\quad}{100}$.

PRACTICE IN CHANGING FORMS

From the several answers given, choose the one correct answer for each example. List your answers on a sheet of paper. When you have done this, compare your answers with those given in the Answer Key on page 469.

1. 4% means	400%	.4	4	.04
2. .125 means	125	12.5	$12\frac{1}{2}\%$	125%
3. $3\frac{1}{2}\%$ means	.35	3.5	.035	35%
4. $4\frac{1}{4}$ means	.425	425%	$4\frac{1}{4}\%$	42.5%
5. .06 means	6%	$\frac{6}{10}\%$	.6	$\frac{6}{1000}$
6. $2\frac{1}{2}\%$ means	2.5	$.2\frac{1}{2}$	25%	.025
7. $7\frac{1}{4}\%$ means	.0725	.725	7.25	725
8. $\frac{1}{2}$ of 1% means	.05	5	.005	.5

FINDING A PER CENT OF ANY NUMBER

288. How to Find a Given Per Cent of Any Number. (For Self-Help Study.) There are always two methods of finding a given per cent of a number, as shown below and on the next page. In some cases it saves time to use Method I, and in other cases it saves time to use Method II.

Method I. **Multiply by a decimal.** Find 25% of 324.

Think: 25% of means 25%×, and 25% = .25

Thus: 25% of 324 is the same as

$$\begin{array}{r} 324 \\ .25 \\ \hline 1620 \\ 648 \\ \hline 81.00 \end{array}$$ or 81, *Ans.*

METHOD II. **Multiply by a fraction.** Find 25% of 324.

Think: 25% **of** means $\frac{1}{4}$ of; and $\frac{1}{4}$ **of** means $\frac{1}{4}\times$.

Thus: 25% of 324 is the same as $\frac{1}{\cancel{4}_{1}}\times\cancel{324}^{81}=81$, *Ans.*

You see that the first method is thinking a per cent as a **decimal;** the second method is thinking a per cent as a **fraction.**

"Of" means times. But in finding a % of a number we **never** multiply by the % itself. We **always change the** % to its equivalent **decimal** or **fraction.**

289. Finding a Per Cent of a Number. (FOR SELF-HELP PRACTICE.) For each example below, you are shown both methods of finding a per cent of a number. Either method is correct. Do not work the examples **now,** but decide which method you think is better for each example.

Example	*Method I*	*Method II*
1. Find $16\frac{2}{3}$% of 126.	$126 \times .16\frac{2}{3}$	$\frac{1}{6}\times 126$
2. Find 36% of 294.	$294 \times .36$	$\frac{36}{100}\times$ or $\frac{9}{25}\times 294$
3. Find 7% of 531.	$531 \times .07$	$\frac{7}{100}\times 531$
4. Find 120% of 475.	475×1.20	$\frac{120}{100}\times$ or $\frac{6}{5}\times 475$

5. Find 60% of 84.	84 ×.60	$\frac{60}{100}\times$ or $\frac{3}{5}\times 84$
6. Find 5% of 750.	750 ×.05	$\frac{5}{100}\times$ or $\frac{1}{20}\times 750$
7. Find 265% of 629.	629 ×2.65	$\frac{265}{100}\times$ or $\frac{53}{20}\times 629$
8. Find $62\frac{1}{2}$% of 360.	360 ×.625	$\frac{5}{8}\times 360$
9. Find 125% of 484.	484 ×1.25	$\frac{5}{4}\times 484$

Work Examples 1-9. Use the method you think is the better one. Compare your answers with those on page 469.

FINDING WHAT PER CENT ONE NUMBER IS OF ANOTHER

290. How to Find What Per Cent One Number Is of Another. (For Self-Help Study and Practice.) In school and later on in life you must know how to find **what per cent one number is of another.**

Here is a good way to think of any question like this one: "56 is what per cent of 112?"

Think the question: "56 is what per cent of 112?", as **"56 is what fraction of 112?"** "56 is what per cent of 112" is simply another way of saying: "56 is what part or what fraction of 112?"

56 is what { per cent / or / fraction } of 112?

56 is $\frac{56}{112}$ of 112.

$\frac{56}{112}$ is changed to "decimal language" like this $112\overline{)56.}$

Any question like: "56 is what $\left\{\begin{matrix}\text{per cent}\\ \text{fraction}\end{matrix}\right\}$ of 112?" can be thought of as $\frac{56}{112}$ or $112\overline{)56}$.

We can see that 56 is $\frac{1}{2}$, or 50%, of 112. It is also .50 of 112. Let us see if 56 as a per cent of 112 would work out right as $\frac{56}{112}$ or $112\overline{)56}$.

```
        .50, or 50% Ans.
112)56.00
    56 0
    ----
       0
```

Do you see that this is an example in the **division of decimals?**

The .50 is changed to 50% because the question asks "what per cent?"

Many examples in finding what per cent one number is of another, can be solved by writing the fraction, reducing the fraction, and stating the per cent which is equal to the fraction. Thus: 9 is what per cent of 36?

Think: $\frac{9}{36}=\frac{1}{4}=25\%$. 9 is 25% of 36.

PRACTICE

1. 2 is what per cent of 8?
2. 6 is what per cent of 36?
3. 9 is what per cent of 18?
4. 17 is what per cent of 34?
5. 3 is what per cent of 24?
6. 21 is what per cent of 28?
7. 75 is what % of 100?
8. 22 is what % of 55?
9. 3 is what % of 2?
10. 200 is what % of 300?
11. 18 is what % of 16?
12. $\frac{1}{3}$ is what % of 1?

Oftentimes you will have a problem in finding what per cent one number is of another, when the fraction is too large to be handled mentally. In such cases the problem is worked as shown in the two samples on the next page.

Sample 1. 93 is what % of 124? Think: $\frac{93}{124}$ or $124\overline{)93}$.

.75, or 75% *Ans.*

$124\overline{)93.00}$

86 8

6 20

6 20

$.75=\frac{75}{100}$ or 75%

93=93.=93.00. Divide as shown at the left. Do not forget the decimal point in the quotient. **Change the quotient from a decimal to a per cent,** because you are asked what %, not what decimal.

Check: $.75\times124=93$.

Check Sample 1 to see if 93 really is $\frac{3}{4}$, or 75%, of 124.

Sample 2. 45 is what % of 218? Think: $\frac{45}{218}$ or $218\overline{)45}$.

.206+, or about 20.6%

$218\overline{)45.00}$

43 6

1 400

1 308

.206+ = a little more than 20.6%

The work is shown at the left. To change .206+ to per cent, move the decimal point two places to the right and attach the % sign.

Is 45 about $\frac{1}{5}$ of 218?

The rule given below always works. Remember it.

Rule: **To find what per cent one number is of another—Think what fraction is made by the numbers. Reduce the fraction if you can. Try to see the per cent which equals the fraction.**

When you cannot reduce to a small fraction, then divide the numerator by the denominator. Point off quotient carefully. Change quotient to per cent form by moving decimal point 2 places to the right and attaching per cent sign.

Thus: $\frac{5}{12}$ (fraction) = $12\overline{)5.00}$ .41$\frac{2}{3}$ (decimal) = 41$\frac{2}{3}$%.

PRACTICE

1. 19.92 is what % of 24? (19.92 is what fraction of 24?) $\frac{19.92}{24}$ or $24\overline{)19.92}$ Change quotient to a %.

2. 24.45 is what % of 67? Or $67\overline{)24.45}$. (Change quotient to %.)

3. Change $\frac{19}{95}$ to a decimal; then to a per cent. $(95\overline{)19.00}$

4. Change $\frac{21}{50}$ to a decimal; then to a per cent.

5. Change $\frac{11}{12}$ to a decimal; then to a per cent.

291. Summary of the First Two Uses of Percentage. (For Self-Help Study and Practice.) You have learned these two uses of percentage:

I. To find a given per cent of a number.

II. To find what per cent one number is of another.

Your work with these two uses will be made clearer if you keep the following suggestions in mind.

Problems and examples calling for either one of these two uses of percentage may be stated in different ways.

Thus, we would find 16% of 759 or $\begin{array}{r} 759 \\ \times .16 \\ \hline \end{array}$ for each of the following questions:

(*a*) What is 16% of 759?

(*b*) If 759 equals 100% of a number, what is 16% of it?

(*c*) What number equals 16% of 759?

No matter what way a problem is stated, if the real question is to find a certain per cent of a number, think: "Per cent of a number equals **decimal times the number.**"

The question, "50.12 is what per cent of 83?" can be thought of as: "50.12 is what fraction of 83?" To solve this example, we would write it as $\frac{50.12}{83}$, or $83\overline{)50.12}$.

We could ask the question in several ways: "What per cent of 83 is 50.12?" or, "If 83 is the whole, what per cent of the whole is 50.12?" or, "If 83 equals 100% of a number, what per cent of the number is 50.12?"

No matter what way the question is asked, if you are to find what per cent one number is of another, you may think: What fraction is the one number of the other? Then write the fraction as a division example.

It pays to study work which is done correctly. Be sure you understand the work in the two samples below.

Sample of Use I

Find 136% of \$362.

Think: "per cent of" means "decimal times."

$$\begin{array}{r} \$362 \\ \times 1.36 \\ \hline 2172 \\ 1086 \\ 362 \\ \hline \$492.32 \end{array}$$ *Ans.*

\$492.32 is 136% of \$362.

Sample of Use II

\$492.32 is what per cent of \$362?

Think: "what per cent of" means "what fraction of."

$$\frac{492.32}{362} \text{ or } 362\overline{)492.32}$$

$$\begin{array}{r} 1.36 = 136\% \\ 362 \\ \hline 1303 \\ 1086 \\ \hline 2172 \\ 2172 \end{array}$$

\$492.32 is 136% of \$362.

These two uses of percentage work just as well when the per cent is larger than 100% as when smaller per cents are called for. In the Sample of Use I above, why is the decimal 1.36 used? In the Sample of Use II above, why is the decimal 1.36 in the quotient changed to 136%?

PRACTICE IN SELECTION

For each example below, several solutions are shown. Only one is correct. Which one is it?

Compare your choice of solutions with those listed on page 469.

1. Find $4\frac{1}{2}\%$ of 936.

(*a*) $936 \times 4\frac{1}{2}$ (*b*) 936×45 (*c*) $\frac{45}{936}$ or $936\overline{)45.}$ (*d*) $936 \times .045$

2. 362 is what per cent of 1351?

(*a*) 1351×3.62 (*b*) $\frac{362}{1351}$ or $1351\overline{)362.}$ (*c*) $\frac{1351}{362}$ or $362\overline{)1351.}$

3. What per cent of 450 is 932?

(*a*) $\frac{450}{932}$ or $932\overline{)450.}$ (*b*) 932×4.50 (*c*) $\frac{932}{450}$ or $450\overline{)932.}$

4. What number equals 125% of 249?

(*a*) 249×125 (*b*) 249×1.25 (*c*) $\frac{125}{249}$ or $249\overline{)125.}$ (*d*) 249×1.25

292. The First Two Uses of Percentage. (For Self-Help Practice.) Remember: (*a*) Find 14% of 1926, means $.14 \times 1926$; (*b*) What per cent of 136 is 34? means 34 is what fraction of 136? Correct answers are on page 469.

1. Find 14% of 1926.
2. What per cent of 136 is 34?
3. What % of 216 is 72?
4. Find 88% of 2167.
5. What per cent of 264 is 33?
6. 24 is what % of 600?
7. 26.5 is what % of 300?
8. 24 is what % of 150?
9. Find 125% of 84.
10. 84 is what % of 42?
11. Does $4\frac{1}{2}\%$ of $451 = 202.95$ or 2.0295 or 20.295?

293. Some Everyday Uses of Percentage. (For Self-Help Study.) In this article are given some important facts you should know about **interest** and **discount.**

1. **Interest** is money earned by money that is loaned.

2. Interest is also money paid for the use of money.

3. Interest is reckoned by the year, or fractions of a year.

Thus, \$1000 at 4% interest for 2 years means 4% of \$1000 for each year, or \$80 (2×\$40). \$1000 for 6 mo. at 4% would be $\frac{1}{2}$ of 4% of \$1000, or \$20 (6 mo. = $\frac{1}{2}$ yr.).

4. The usual interest rates are 3%, $3\frac{1}{2}$%, 4%, $4\frac{1}{2}$%, 5%, $5\frac{1}{2}$%, 6%, $6\frac{1}{2}$%, 7%, and $7\frac{1}{2}$%.

5. To find interest for a number of months, divide the months in a way that will give fractions easy to work with.

Thus, 6 mo. = $\frac{1}{2}$ yr., 3 mo. = $\frac{1}{4}$ yr., 4 mo. = $\frac{1}{3}$ yr., 8 mo. = $\frac{8}{12} = \frac{2}{3}$ yr., 7 mo. = $\frac{7}{12}$ yr., etc.

6. The usual times for loans are 30 da. (1 mo.), 2 mo., 3 mo., 6 mo., 1 year, and over. (The interest rate is always stated in % per annum, or per year.)

Finding interest at a given rate (%) of interest is merely finding a per cent of a number. The only other thing you have to do is to multiply by the correct number of years.

Another application of percentage is found in figuring **discounts.** When the price of an article or the amount of money due for work done is reduced from its full amount, we call the amount taken off a **discount.** The discount is generally a certain **per cent** of the full amount of money due.

There are four main kinds of discount:

1. Discounts are sometimes given for **prompt payment.** These are often called **cash discounts.**

2. Quantity discounts are given for buying goods in large amounts.

3. Special-sale discounts are given when a store-keeper has some particular reason for wanting to sell certain articles or when he wants especially to attract people to his store. So he reduces his prices, as 10% off or 25% off.

4. A trade discount is one which a wholesaler (one who sells to store-keepers) gives to the retailer (the store-keeper).

In wholesalers' catalogues the prices quoted are often the **retail prices,** that is, the amounts which the retailer must charge the customer for the article. In order to pay his business expenses and make a profit, the store-keeper must buy the article **at a discount;** that is, he must buy of the wholesaler for **less** than the catalogue price.

For this reason catalogues usually give **retail prices, less a certain discount,** such as 20%, 25%, $33\frac{1}{3}$%, 40%, etc.

Sometimes there may be more than one discount of a bill. Look at the part of a bill shown below.

6 Window shades @ $4.20.....	$25.20
Less trade discount 40%	10.08
	$15.12
Less discount for cash 2%	.30
	$14.82

Notice that the larger discount is figured first. Always figure discounts **separately.** Do you see that figuring discount is merely finding a per cent of a number? But don't get caught if a problem asks you to find the amount to be paid on the bill; amount to be paid (or amount due) means full amount of bill **less** discount.

294. Bank Checking Accounts. (For Self-Help Study.) In sixth grade you studied certain things about checking accounts at banks.

1. People **deposit** money in checking accounts in banks.

2. The bank will pay money from a person's account only on a written order signed by the person. This written order is called a **check.**

DENVER, COLO., Jan. 3 1928 No. 91

CITIZENS STATE BANK

PAY TO THE ORDER OF James D. Worden $913.00

Nine hundred thirteen and no/100 DOLLARS

David A. Lawson

3. Study the picture of the check above and note these important facts that each check must show:

(*a*) The date on which the check was written (*Jan. 3, 1928*).

(*b*) The number of the check (*91*).

(*c*) The name of the bank and the town in which it is located (*Citizens State Bank, Denver, Colo.*).

(*d*) The name of the person to whom the money is to be paid (*J. D. Worden*).

(*e*) The amount to be paid, written in both words and figures (*$913*).

(*f*) The name of the person paying the money (*David A. Lawson*).

The person who receives the money must write his name on the back of the check. This is called **indorsing.**

ANSWER KEY FOR SELF-HELP LESSONS

These answers are placed here to help you learn how to work different sorts of examples correctly and accurately. They will not do this if you use the answers before you have solved the examples. Be honest with yourself and use the answers **after** you have worked the examples.

Answers for Article 217. (*Helping Yourself in Addition.*)

1. 302	**3.** 286	**5.** 23,749	**7.** 27,335
2. 1837	**4.** 281	**6.** $207.01	**8.** $34.75

Answers for Article 222. (*Final Practice in Subtraction.*)

1. 4634	**7.** 5819	**13.** 3806	**19.** 1416
2. 2539	**8.** 56,417	**14.** 10,244	**20.** 5220
3. 35,090	**9.** 7709	**15.** $91.31	**21.** 8158
4. 357	**10.** 7589	**16.** 6304	**22.** $23.87
5. 1852	**11.** 9686	**17.** 4099	**23.** 2578
6. 2752	**12.** $45.84	**18.** 869	**24.** 2335

Answers for Article 228. (*Final Practice in Multiplication.*)

1. 168,701	**6.** 136,354	**11.** 107,379	**16.** 82,688
2. 136,202	**7.** 167,331	**12.** $4801.50	**17.** 74,893
3. 48,216	**8.** 94,068	**13.** 343,332	**18.** $266 (26600¢)
4. 40,460	**9.** 87,725	**14.** 290,146	**19.** 107,973
5. 271,312	**10.** 82,440	**15.** 187,627	**20.** $1086.78

Answers for Article 233. (*Practice in Finding Quotients, Page 406.*)

1. $9\frac{7}{67}$	**4.** $8\frac{11}{12}$	**7.** $8\frac{55}{57}$	**10.** $8\frac{21}{23}$	**13.** $8\frac{45}{46}$
2. $8\frac{7}{8}$	**5.** $8\frac{6}{29}$	**8.** $8\frac{3}{7}$	**11.** $8\frac{6}{19}$	**14.** $8\frac{5}{39}$
3. $6\frac{29}{31}$	**6.** $8\frac{41}{42}$	**9.** $4\frac{1}{14}$	**12.** $8\frac{25}{27}$	**15.** $6\frac{57}{86}$

Answers for Article 234. (*Example B, Page 406.*) 804.

(*Practice with Zero in Quotient, Page 407.*)

1. 602 **2.** $206\frac{7}{73}$ **3.** 509 **4.** $603\frac{23}{89}$ **5.** 802

6. 507 **7.** $907\frac{16}{97}$ **8.** $206\frac{1}{14}$ **9.** $805

Answers for Article 234. (*Practice with Final Zero, Page 409.*)

1. 830 **2.** $290\frac{22}{93}$ **3.** 620 **4.** $720\frac{5}{66}$ **5.** 580

6. $960\frac{7}{83}$ **7.** $420\frac{7}{19}$ **8.** 130 **9.** $1.40

Answers for Article 235. (*Final Practice in Division.*)

1. $48\frac{3}{17}$ **5.** $23\frac{12}{59}$ **9.** 95 **13.** 89 **17.** 408 **21.** 71

2. 74 **6.** 73 **10.** $96\frac{16}{51}$ **14.** 72 **18.** $900\frac{13}{27}$ **22.** 83

3. 40 **7.** 62 **11.** 601 **15.** $50\frac{7}{31}$ **19.** 23 **23.** 85

4. 87 **8.** 61 **12.** 41 **16.** 26 **20.** 160 **24.** $59\frac{7}{96}$

Answers for Article 236. (*Checking Decimals.*)

1. *Check:* $3.7\times.72=2.664$

2. *Check:* $46\times.7=32.2$ $\quad 32.2+.5=32.7$

3. *Check:* $6.4\times.38=2.432$ $\quad 2.432+.050=2.482$

Answers for Article 237. (*Estimating, Page 413.*)

1. $60\times30=1800$

2. $800\div20=40$

3. $40\times900=36{,}000$

4. $45+30+80=155$

5. $20\times520=10{,}400$

6. $\$80+\$50=\$130$

7. $12\times\$20=\$240$

8. $\$470\div10=\$47$

9. $5700-3500=2200$

10. $2400\div40=60$

Answers for Article 241. (*Finding Common Denominators.*)

1. $\frac{}{4}$ **2.** $\frac{}{9}$ **3.** $\frac{}{12}$ **4.** $\frac{}{24}$ **5.** $\frac{}{24}$ **6.** $\frac{}{12}$ **7.** $\frac{}{6}$ **8.** $\frac{}{12}$

9. $\frac{}{24}$ **10.** $\frac{}{12}$ **11.** $\frac{}{10}$ **12.** $\frac{}{12}$ **13.** $\frac{}{12}$ **14.** $\frac{}{20}$ **15.** $\frac{}{12}$ **16.** $\frac{}{24}$

Answers for Article 243. (*Practice in Reducing Answers.*)

1. $\frac{2}{3}$ **2.** $2\frac{2}{3}$ **3.** $8\frac{1}{2}$ **4.** $6\frac{1}{2}$ **5.** 1 **6.** $7\frac{1}{2}$

7. $1\frac{1}{4}$ **8.** $3\frac{3}{4}$ **9.** $\frac{2}{3}$ **10.** $5\frac{5}{8}$ **11.** $4\frac{1}{3}$ **12.** $3\frac{3}{8}$

Answers for Article 245. (*Improper Fractions to Correct Form.*)

1. $7\frac{1}{3}$ **2.** $1\frac{7}{8}$ **3.** $5\frac{2}{5}$ **4.** 8 **5.** $4\frac{1}{2}$ **6.** $3\frac{1}{2}$ **7.** $2\frac{1}{2}$

Answers for Article 246. (*How to Add Mixed Numbers, Page 419.*)
All the examples are correct.

Answers for Article 247. (*Adding Fractions and Mixed Numbers.*)

1. 1
2. $6\frac{1}{12}$
3. $\frac{4}{5}$
4. $\frac{9}{10}$
5. $6\frac{1}{4}$
6. $1\frac{1}{9}$
7. $10\frac{2}{3}$
8. $12\frac{5}{18}$
9. $12\frac{1}{6}$
10. $1\frac{13}{44}$
11. $1\frac{1}{20}$
12. $1\frac{13}{20}$
13. $\frac{7}{8}$
14. $4\frac{3}{4}$
15. $\frac{6}{7}$
16. $1\frac{2}{3}$
17. $9\frac{12}{17}$
18. $5\frac{11}{16}$
19. $8\frac{1}{4}$
20. $\frac{2}{3}$
21. $13\frac{3}{5}$
22. $1\frac{12}{35}$
23. $9\frac{5}{12}$
24. $\frac{18}{77}$
25. $3\frac{11}{15}$
26. 14
27. $\frac{5}{9}$
28. $7\frac{4}{5}$
29. $5\frac{31}{40}$
30. $3\frac{29}{30}$
31. $5\frac{2}{3}$, 1, $2\frac{1}{2}$, 4
32. $\frac{}{12}$

Answers for Article 250. (*Denominators in Easy Subtraction.*)

1. $\frac{1}{15}$ **2.** $\frac{5}{12}$ **3.** $\frac{17}{24}$ **4.** $\frac{3}{10}$ **5.** $\frac{13}{20}$ **6.** $\frac{19}{30}$ **7.** $\frac{1}{12}$ **8.** $\frac{17}{60}$ **9.** $\frac{1}{24}$ **10.** $\frac{1}{30}$ **11.** $\frac{17}{40}$ **12.** $\frac{19}{60}$ **13.** $\frac{8}{15}$ **14.** $\frac{7}{40}$

Answers for Article 252. (*Subtracting Fractions and Mixed Nos.*)

1. $\frac{1}{8}$
2. $\frac{1}{36}$
3. $\frac{14}{15}$
4. 9
5. $4\frac{1}{10}$
6. $10\frac{1}{18}$
7. $\frac{2}{9}$
8. $2\frac{5}{11}$
9. $\frac{2}{17}$
10. $\frac{1}{12}$
11. $1\frac{19}{21}$
12. $1\frac{1}{6}$
13. $\frac{19}{20}$
14. $2\frac{3}{4}$
15. $2\frac{5}{12}$
16. $6\frac{2}{3}$
17. $\frac{1}{6}$
18. $8\frac{5}{8}$
19. $7\frac{7}{20}$
20. $4\frac{1}{2}$
21. 10
22. $2\frac{4}{7}$
23. $\frac{1}{10}$
24. $7\frac{3}{5}$
25. $5\frac{7}{11}$
26. $\frac{1}{5}$
27. $4\frac{1}{6}$
28. $6\frac{1}{2}$
29. $5\frac{7}{12}$
30. $6\frac{19}{22}$

Answers for Article 255. (*Easy Practice in Multiplication of Fractions.*)

1. $\frac{8}{15}$
2. $\frac{15}{28}$
3. $\frac{2}{25}$
4. $\frac{8}{15}$
5. $\frac{1}{8}$
6. $\frac{5}{12}$
7. $3\frac{1}{3}$
8. $4\frac{1}{2}$
9. $\frac{1}{24}$
10. $4\frac{1}{5}$
11. $\frac{16}{49}$
12. $\frac{1}{4}$
13. $4\frac{4}{5}$
14. 3
15. $2\frac{2}{5}$
16. $4\frac{1}{2}$
17. $6\frac{2}{3}$
18. $\frac{3}{16}$
19. 49
20. 0

Answers for Article 257. (*Practice in Changing Mixed Numbers to Improper Fractions.*)

1. $\frac{9}{2}$	4. $\frac{25}{7}$	7. $\frac{39}{8}$	10. $\frac{55}{6}$	13. $\frac{58}{3}$	16. $\frac{143}{12}$
2. $\frac{77}{12}$	5. $\frac{10}{9}$	8. $\frac{14}{5}$	11. $\frac{62}{8}$	14. $\frac{78}{9}$	17. $\frac{28}{5}$
3. $\frac{25}{4}$	6. $\frac{11}{3}$	9. $\frac{33}{10}$	12. $\frac{171}{16}$	15. $\frac{7}{3}$	18. $\frac{151}{12}$

Answers for Article 260. (*Practice in Cancellation.*)

1. $\frac{\overset{1}{2}}{\underset{1}{3}} \times \frac{\overset{1}{3}}{\underset{2}{4}} = \frac{1}{2}$

2. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

3. $\frac{\overset{1}{4}}{\underset{1}{5}} \times \frac{1}{\underset{1}{4}} = \frac{1}{5}$

4. $\frac{\overset{1}{2}}{\underset{6}{12}} \times \frac{1}{\underset{1}{5}} = \frac{1}{6}$

5. $\frac{1}{3} \times \frac{\overset{7}{21}}{1} = 7$

6. $\frac{\overset{1}{2}}{\underset{1}{3}} \times \frac{\overset{1}{3}}{\underset{2}{4}} = \frac{1}{2}$

7. $\frac{5}{\underset{3}{6}} \times \frac{\overset{1}{2}}{3} = \frac{5}{9}$

8. $\frac{7}{\underset{2}{8}} \times \frac{\overset{1}{4}}{5} = \frac{7}{10}$

9. $\frac{1}{\underset{1}{2}} \times \frac{\overset{2}{4}}{5} = \frac{2}{5}$

10. $\frac{\overset{3}{9}}{\underset{5}{10}} \times \frac{1}{\underset{1}{2}} = \frac{3}{5}$

11. $\frac{1}{5} \times \frac{1}{\underset{1}{5}} = 1$

12. $\frac{\overset{1}{3}}{8} \times \frac{1}{\underset{2}{6}} = \frac{1}{16}$

13. $\frac{\overset{1}{2}}{5} \times \frac{1}{\underset{4}{8}} = \frac{1}{20}$

14. $\frac{5}{\underset{3}{9}} \times \frac{\overset{1}{3}}{4} = \frac{5}{12}$

15. $\frac{25}{\underset{2}{26}} \times \frac{1}{18} = \frac{25}{2} = 12\frac{1}{2}$

16. $\frac{7}{\underset{2}{8}} \times \frac{3}{12} = \frac{21}{2} = 10\frac{1}{2}$

17. $\frac{4}{\underset{1}{25}} \times \frac{\overset{2}{50}}{1} = 8$

18. $\frac{1}{\underset{1}{2}} \times \frac{\overset{9}{18}}{1} = 9$

19. $\frac{\overset{1}{3}}{\underset{1}{5}} \times \frac{\overset{4}{20}}{\underset{7}{21}} = \frac{4}{7}$

20. $\frac{\overset{1}{2}}{\underset{1}{3}} \times \frac{\overset{3}{9}}{\underset{5}{10}} = \frac{3}{5}$

21. $\frac{3}{4} \times \frac{6}{24} = 18$

22. $\frac{\overset{1}{5}}{16} \times \frac{3}{\underset{2}{10}} = \frac{3}{32}$

23. $\frac{\overset{1}{4}}{\underset{1}{5}} \times \frac{\overset{1}{5}}{\underset{3}{12}} = \frac{1}{3}$

24. $\frac{1}{\underset{1}{6}} \times \frac{\overset{2}{12}}{25} = \frac{2}{25}$

Answers for Article 262. (*Multiplying Fractions and Mixed Numbers.*)

1. 10	**7.** \$15.75	**13.** $2\frac{8}{21}$	**19.** $\frac{7}{10}$	**25.** 1
2. $3\frac{1}{2}$	**8.** $\frac{1}{6}$	**14.** $\frac{10}{33}$	**20.** $\frac{1}{10}$	**26.** $\frac{1}{12}$
3. $\frac{2}{9}$	**9.** $1\frac{1}{10}$	**15.** $4\frac{1}{2}$	**21.** $1\frac{1}{4}$	**27.** $5\frac{3}{5}$
4. $\frac{3}{14}$	**10.** $1\frac{2}{3}$	**16.** $\frac{1}{20}$	**22.** $\frac{1}{4}$	**28.** $\frac{1}{6}$
5. $\frac{1}{2}$	**11.** 9	**17.** $\frac{1}{5}$	**23.** $6\frac{3}{4}$	
6. $1\frac{1}{3}$	**12.** $\frac{20}{21}$	**18.** \$2.00	**24.** $1\frac{2}{9}$	

Answers for Article 265. (*Inverting the Divisor.*)

1. $\times\frac{3}{1}$	**7.** $\times\frac{8}{9}$	**13.** $\times\frac{3}{16}$	**19.** $\times\frac{1}{3}$
2. $\times\frac{2}{5}$	**8.** $\times\frac{8}{3}$	**14.** $\times\frac{3}{7}$	**20.** $\times\frac{5}{12}$
3. $\times\frac{5}{9}$	**9.** $\times\frac{8}{73}$	**15.** $\times\frac{1}{7}$	**21.** $\times\frac{2}{13}$
4. $\times\frac{6}{1}$	**10.** $\times\frac{2}{9}$	**16.** $\times\frac{2}{3}$	**22.** $\times\frac{3}{2}$
5. $\times\frac{2}{3}$	**11.** $\times\frac{3}{5}$	**17.** $\times\frac{4}{15}$	**23.** $\times\frac{8}{41}$
6. $\times\frac{3}{1}$	**12.** $\times\frac{2}{17}$	**18.** $\times\frac{4}{25}$	**24.** $\times\frac{1}{2}$

Answers for Article 266. (*Dividing Fractions and Mixed Numbers.*)

1. $\frac{3}{4}$	**7.** $\frac{1}{6}$	**13.** $2\frac{5}{6}$	**19.** $\frac{2}{5}$
2. $1\frac{4}{5}$	**8.** $\frac{18}{25}$	**14.** $3\frac{1}{8}$	**20.** $\frac{7}{11}$
3. $1\frac{23}{27}$	**9.** $\frac{5}{51}$	**15.** $\frac{1}{2}$	**21.** $\frac{4}{9}$
4. $4\frac{1}{2}$	**10.** $\frac{1}{2}$	**16.** 30	**22.** $7\frac{1}{2}$
5. $\frac{2}{5}$	**11.** $\frac{26}{45}$	**17.** $10\frac{2}{3}$	**23.** $\frac{56}{123}$
6. $\frac{56}{73}$	**12.** 4	**18.** 30	**24.** 12

Answers for Article 267. (*Division of Fractions and Mixed Numbers.*)

1. 6	**4.** $1\frac{1}{3}$	**7.** $4\frac{4}{5}$	**10.** 32	**13.** 4
2. $\frac{1}{5}$	**5.** 3	**8.** $\frac{48}{55}$	**11.** $\frac{1}{4}$	**14.** 124
3. $\frac{2}{3}$	**6.** $\frac{5}{12}$	**9.** $2\frac{7}{36}$	**12.** 3	**15.** 0

Answers for Article 268. (*Mixed Fundamentals of Fractions.*)

1. $12\frac{7}{8}$ **3.** $16\frac{3}{4}$ **5.** $\frac{2}{15}$ **7.** $9\frac{4}{5}$ **9.** $2\frac{11}{24}$ **11.** $4\frac{1}{2}$

2. $3\frac{3}{4}$ **4.** $\frac{3}{10}$ **6.** $6\frac{1}{2}$ **8.** $22\frac{7}{8}$ **10.** $\frac{21}{80}$ **12.** $38\frac{1}{4}$

Answers for Article 270. (*The "Hard Spot" in Addition and Subtraction of Decimal Fractions.*)

1. 42.34 **2.** 9.295 **3.** 122.17 **4.** 50.75 **5.** 13.2 **6.** 6.004

Answers for Article 271. (*How to Multiply Decimals.*)

1. 110.72 **2.** 1026.944 **3.** 9.39162 **4.** 309520

5. 3412.2 **6.** .008 **7.** $21.25 **8.** $81.25

Answers for Article 272. (*Division of Decimals.*)

1. 19.82 **2.** .043 **3.** 6429.34 **4.** .065 **5.** 1497.52+ **6.** 525

Answers for Article 275. (*Form of Answers in Addition of Denominate Numbers.*)

1. 7 ft. 6 in. **4.** 10 lb. 5 oz. **7.** 7 sq. ft. 6 sq. in.

2. 6 yd. 1 ft. **5.** 5 pk. 3 qt. **8.** 5 sq. yd. 5 sq. ft.

3. 5 qt. 1 pt. **6.** 7 hr. 50 min. **9.** 3 cu. yd. 5 cu. ft.

Answers for Article 277. (*Adding Denominate Numbers, Page 443.*)

1. 16 ft. 3 in. **2.** 28 hr. **3.** 17 da. 8 hr. **4.** 13 gal. 3 qt.

Answers for Article 278. (*Subtracting Denominate Numbers.*)

1. 1 ft. 5 in. **2.** 1 gal. 2 qt. **3.** 15 min. **4.** 1 lb. 9 oz.

Answers for Article 279. (*Multiplying Denominate Numbers.*)

1. 31 min. 30 sec. **2.** 34 gal. 2 qt. **3.** 40 bu. 2 pk. **4.** 42 ft.

Answers for Article 280. (*Dividing Denominate Numbers.*)

1. 1 yd. $2\frac{3}{4}$ ft. **2.** 2 pk. 3 qt. **3.** 1 gal. $1\frac{3}{4}$ qt.

4. 2 hr. 19 min. **5.** $1\frac{1}{3}$ lb. **6.** 3 yd. $1\frac{1}{4}$ ft.

Answers for Article 283. (*Areas of Parallelograms.*)

1. 126 sq. ft. **2.** 1456 sq. in. **3.** 784 sq. in.

4. 132 sq. ft. **5.** 192 sq. in. or $1\frac{1}{3}$ sq. ft. **6.** 44 sq. yd.

Answers for Article 284. (*Areas of Triangles.*)

1. 12 sq. in.
2. 48 sq. in. or $\frac{1}{3}$ sq. ft.
3. 21 sq. ft. or $2\frac{1}{3}$ sq. yd.
4. 60 sq. mi.

Answers for Article 285. (*Volumes of Rectangular Solids.*)

1. 24 cu. in. **2.** 120 cu. ft. **3.** 48 cu. ft. **4.** 972 cu. ft.

Answers for Article 287. (*Changing Forms in Percentage.*)

1. .04 **3.** .035 **5.** 6% **7.** .0725
2. $12\frac{1}{2}$% **4.** 425% **6.** .025 **8.** .005

Answers for Article 289. (*Finding a Per Cent of a Number.*)

1. 21 **3.** 37.17 **5.** 50.4 **7.** 1666.85 **9.** 605
2. 105.84 **4.** 570 **6.** 37.5 **8.** 225

Answers for Article 290. (*How to Find What Per Cent One Number is of Another, Page 455.*)

1. 25% **3.** 50% **5.** $12\frac{1}{2}$% **7.** 75% **9.** 150% **11.** $112\frac{1}{2}$%
2. $16\frac{2}{3}$% **4.** 50% **6.** 75% **8.** 40% **10.** $66\frac{2}{3}$% **12.** $33\frac{1}{3}$%

Answers for Article 290. (*Page 457.*)

1. 83% **2.** 36.4+% **3.** 20% **4.** 42% **5.** $91\frac{2}{3}$%

Answers for Article 291. (*Practice in Selection, Page 459.*)

1. (*d*) **2.** (*b*) **3.** (*c*) **4.** (*b*)

Answers for Article 292. (*The First Two Uses of Percentage.*)

1. 269.64 **3.** $33\frac{1}{3}$% **5.** $12\frac{1}{2}$% **7.** $8\frac{5}{6}$% **9.** 105 **11.** 20.295
2. 25% **4.** 1906.96 **6.** 4% **8.** 16% **10.** 200%

TABLES OF MEASURE

LENGTH

12 inches (in.) = 1 foot (ft.)
3 feet = 36 inches = 1 yard (yd.)
$5\frac{1}{2}$ yards = $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods = 5280 feet = 1 mile (mi.)

SQUARE MEASURE (AREA)

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

CUBIC MEASURE (VOLUME)

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord

WEIGHT

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

TIME

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
30 days = 1 month (mo.)
12 months = 1 year (yr.)
365 days = 1 year
366 days = 1 leap year
10 years = 1 decade
100 years = 1 century

LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

DRY MEASURE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

PUPIL'S SELF-HELP INDEX

295. How to Direct Your Own Review Work. (SILENT READING.) In order to do good work in arithmetic you must make sure really to understand each new topic as you study it for the first time. You must also keep from forgetting how to do many things which you may not have studied for a long time.

An important question for all of us is: When I have forgotten how to work some kind of example or problem, how can I direct my own review work so that I can restudy and practice? The Pupil's Self-Help Index on pages 471-478 will direct you to articles for study and practice.

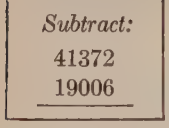
The article numbers in heavy type (Art. **214**) refer to articles **to study** in order to understand **how to do something.** The other article numbers refer to good material **for practice.**

ESSENTIALS OF WHOLE NUMBERS

Few business men are willing to excuse mistakes in adding, subtracting, multiplying, and dividing whole numbers. Every seventh-grade pupil should make sure that he can work with whole numbers accurately and easily.

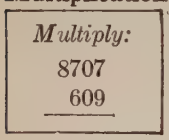
Tests in Whole Numbers		Arts. 3, 115
Addition		
783 49 856 375	Six Trouble Spots	Art. **214**
	Easy Practice	Arts. 215, 216
	Harder Practice	Art. 217
	How to Check	Arts. **29, 236**

Subtraction



Easy Practice Arts. 219, 221
Eight Questions on Subtraction Art. **220**
Harder Practice Art. 222
How to Check Arts. **29, 236**

Multiplication



Easy Practice Arts. 224, 226
"Hard Spots" in Multiplication Art. **225**
Harder Practice Arts. 227, 228
How to Check Arts. **29, 236**

Division

Easy Practice Art. 230
The Steps in Long Division Art. **231**
Finding Quotient Figures Arts. **232, 233**
Zero in the Quotient Art. **234**
Final Practice Art. 235
How to Check Art. **236**

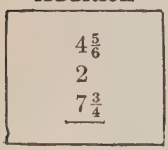
Words Often Used with Whole Numbers Art. **238**

FRACTIONS AND MIXED NUMBERS

There are many examples and problems in this book which use fractions and mixed numbers. If you need to review fractions, this Index will help you to find the right kind of material to use.

Tests in Fractions and Mixed Numbers Arts. 4, 116

Addition



Common Denominators Arts. **240,** 241
Reduction of Answers Arts. **242,** 243, **244,** 245
How to Add Mixed Numbers Art. **246**
Practice Art. 247

Subtraction

$6\frac{1}{8}-3\frac{1}{6}=$

Common Denominators Art. **249**
Easy Practice Art. 250
How to Subtract Mixed Numbers Art. **251**
Harder Practice Art. 252

Multiplication

$2\frac{3}{4}\times6\frac{3}{5}=$

How to Multiply Fractions Arts. **254,** 255
Changing Mixed Numbers to Improper Fractions Arts. **256,** 257
Cancellation Arts. **258, 259,** 260
How to Multiply Mixed Numbers Arts. **261,** 262

Division

$4\frac{2}{3}\div7=$

How to Divide Arts. **264, 267**
Practice in Inverting Divisor Art. 265
Practice with Fractions and Mixed Numbers Art. 266
Mixed Practice with Fractions Art. 268
Words Often Used with Fractions Art. **238**

DECIMAL NUMBERS

Decimal numbers appear frequently in the business uses of arithmetic. Seventh-grade pupils should clearly understand and work accurately with the kinds of decimal numbers in this book.

Tests in Decimal Numbers Arts. 5, 117

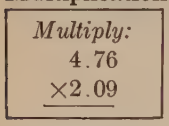
Addition and Subtraction

$$\begin{array}{r} 4.07 \\ .62 \\ 3.8 \\ 7.21 \\ \hline \end{array}$$

$3.81-7.06=$

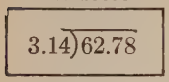
The "Hard Spot" in Addition and Subtraction Art. **270**

Multiplication



How to Multiply — Art. **271**

Division



How to Divide — **Art. 272**

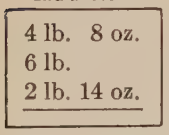
How to Check Decimals — Art. **236**

DENOMINATE NUMBERS

Denominate numbers are used to measure most of the things we buy and use in every day life.

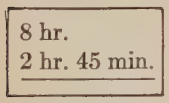
Tests on Denominate Numbers — Arts. 6, 7, 119, 120
Changing the Forms of Denominate Numbers — Arts. **274,** 275

Addition



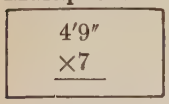
How to Add — Arts. **276, 277**

Subtraction



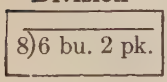
How to Subtract — Art. **278**

Multiplication



The "Hard Spot" in Multiplication — Art. **279**

Division



The "Hard Spot" in Division — Art. **280**

PERCENTAGE

Problems using per cents are one of the main topics of seventh-grade arithmetic. Almost everyone thinks that seventh-grade pupils should understand the uses of percentage very well indeed.

Informal Tests

Changing Forms of Numbers	Arts. 8, 118
Judgment Tests	Arts. 9, 118
Ability to Use Percentage	Arts. 10, 118
Changing Per Cent to Fractions	Art. 23

Fraction-Decimal-Per Cent Chart Art. **287**

Finding a Per Cent of a Number

Find 25% of 248

How to Find a Per Cent of a Number Arts. **288,** 289

Finding What Per Cent One Number Is of Another

62 is what per cent of 248?

How to Find What Per Cent One Number Is of Another Arts. **290, 291,** 292

Finding 100%, or the Whole, when a Per Cent is Known

62 is 25% of a number. What is the number?

Important Study and Practice References	Arts. **21, 22,** 24
Problems	Art. 27

Drills and Problems on the Three Uses of Percentage Arts. 28, 32, 69, 70, 177

MENSURATION

Whenever you are troubled about your work in mensuration, you will find the Index below useful in directing you to the right things to study and practice.

Tests on Finding Areas Arts. 11, 122

Words Used in Mensuration Art. **282**

Kinds of Measurements Art. **78**
Linear Measure Arts. **82,** 83

Changing Distances from One Unit to Another Arts. **79,** 80

Scale Drawing Arts. **84,** 85

Finding Areas
Rectangles and Parallelograms Arts. **93, 94, 283**
The Right Triangle Arts. **95, 284**
Any Triangle Arts. **96, 284**

Finding Volumes of Rectangular Solids
How to Find Volumes Arts. **129,** 130, **285**
Names of Important Types of Solids Art. **125**

The Circle
Important Facts about the Circle Arts. **99, 100,** 101
Finding the Area Arts. **102,** 103

The Angle as a Measure of Direction Arts. **188, 189, 192,** 193, 199
The Protractor Arts. **190,** 191
The Compass Art. **196**

Mixed Practice Arts. 134, 183

USES OF ARITHMETIC

The Index below contains references to important uses of arithmetic in everyday life.

Electric and Gas Meters	Arts. **38,** 39, **40,** 41
Checks and the Checking Account	Arts. **51, 52, 53,** 54, **56,** 57, 58, 59
Savings Bank Accounts	Arts. **63, 64,** 65
Accuracy in Making Change	Arts. **66,** 67
The Family Budget	Arts. **71,** 72
Commission	Arts. **109,** 110
Interest and the Interest Formula	Arts. **140,** 142, **143,** 144, 146
The Amount	Art. **145**
The Interest Table	Arts. **150,** 151, 152
Promissory Notes	Arts. **154,** 155
Bank Discounts	Arts. **156,** 157
Graphs	Arts. **165, 166,** 167, 168
Interest in Business	Art. **171**
Finding the Rate	Arts. **172,** 173, 174
Profit and Loss	Arts. **178, 179,** 180
Discounts (cash, quantity, trade, special-sale)	Art. **293**

USING ARITHMETIC TO SOLVE PROBLEMS

Practice in problem solving while in school will aid you better to use your knowledge of arithmetic to help solve your own problems when you are grown up.

How to Solve Problems. Arts. **18, 105, 203**

General Practice. The Problem Scales. Arts. 20, 33, 43, 60, 74, 92, 106, 128, 136, 149, 161, 176, 184, 204.

PRACTICE TO KEEP FROM FORGETTING

It is not enough for seventh-grade pupils to "have the right idea." They must be skillful and accurate in doing actual work. The articles listed in this section are good ones to use for practice.

Self-Testing Drills. Arts. 17, 26, 31, 37, 42, 50, 55, 62, 68, 76, 87, 98, 104, 108, 112, 126, 132, 138, 141, 148, 153, 160, 170, 175, 182, 186, 195, 202, 207, 210

Rapid Oral Drills. Arts. 25, 35, 49, 61, 75, 86, 97, 107, 131, 133, 137, 147, 159, 169, 181, 194, 205, 206, 208, 209

INVENTORY DRILL ON USEFUL FACTS AND IDEAS

Any seventh-grade pupil who can do the work in the articles listed below can feel confident that he understands the facts and ideas he is expected to have mastered by the end of the seventh grade.

Arts. 14, 44, 77, 113, 121, 139, 164, 187, 211

MISCELLANEOUS

Checking for Accuracy	Arts. 12, **29, 30, 34, 36,** 123, **236**
The Formula	Arts. **88,** 89, **90,** 91, **100, 143,** 144
Factors	Art. **46**
Estimating	Arts. **47,** 48, 73, 124, 158, 200, **237**

GENERAL INDEX

(References are to pages.)

Accounts, checking. *See* Checking accounts
Acute angles, 154, 354
Addition, checking of, 47; review of the uses of, 235-236; Self-Help Lessons on: 388-391 (whole number), 415-420 (fractions), 436 (decimals), 441-443 (denominate numbers)
Altitude, 152, 155, 447
Amount, meaning of, 254-255; practice in finding the, 256-258
Angles, acute, 154, 354; construction of, 352-354; measuring, 349-351; obtuse, 154, 354; right, 152, 154; straight, 354; vocabulary of, 357
Angular measurement, 345-356; review of, 362-365
Angular motion, 345-346, 355
Arc, defined, 347, 355
Area, measurement of, 130; of circles, 166-170; of parallelograms, 154-156; of rectangles, 152-153; of triangles, 156-159; Self-Help Lessons on, 446-448
Average, illustrated, 414

Bank discount. *See* Discount, bank
Banking and banks. *See* Checking accounts; Savings accounts
Bar graph, the, 292-294
Base, of a figure, defined, 447
Bonds, 309-318
Budgets, 119-123

Capacity, defined, 131
Capital, defined, 308-309
Change, making, 110-113
Checking accounts, 78-87, 90-95; balancing, 90-94; Self-Help Lessons on, 462
Checking for accuracy, new methods of, 47-49 (integers), 56-57 (fractions), 59 (percentage); practice in, 185-186, 337-338; Self-Help Lessons on, 410-411, tests on, 17, 211; value of, 49
Checks (bank), 83-84, 85-87, 94-95; indorsing, 85-86; Self-Help Lessons on, 462
Check stubs, 83-84, 91-93
Circle, area of the, 166-169; circumference of the, 163-165; facts about the, 161-163, 347, 354
Circle graph, the, 294-295
Circumference, defined, 161, 355; formula for finding, 164
Class records, comparing, 230
Commission, 179-184
Common denominator, defined, 414; rule for finding, 415
Common fraction, defined, 414
Compass, the magnetic, 359-362
Cube, defined, 216
Cubic measure, table of, 470. *See* also Volume

Decimals, defined, 414; location tests on, 7-8, 195-196; review of, 228, 375; Self-Help Lessons on, 436-439
Degree (angular measurement), 347-348
Demand notes. *See* Promissory notes

Denominate numbers, defined, 414; tests on, 10-11, 205-206; review of, 378-379; Self-Help Lessons on, 439-445
Denominator, defined, 414
Deposit book, 81
Deposit slip, 80
Diameter, defined, 161, 355
Dimensions, defined, 446
Discount, bank, 277-279; cash, quantity, trade, special sale, 460-461
Dividends, stock, 310-311
Division, checking of, 411; drill on, 281; review of the uses of, 239-241; Self-Help Lessons on, 401-409 (whole-number), 431-435 (with fractions), 438-439 (with decimals), 438-439 (with denominates)
Drawing to scale. *See* Scale drawing
Drills. *See* Oral Drills and Self-Testing Drills

Electric meters, 61-63; problems on, 64-65
Equations, 145-147
Estimating answers, practice in, 30-31, 76-77, 122-123, 218, 280; Self-Help Lessons on, 412-413; tests on, 212, 366

Face value, defined, 273-274
Factors, 73-74, 223-224
Formula, the, 144-149; for area of a circle, 166-167; for area of a parallelogram, 154; for area of a rectangle, 153; for area of triangles, 156, 158; for circumference of a circle, 164; for interest, 251-254; for rate of interest, 312-314; for volume of a rectangular solid, 221-222
Fraction-decimal-per cent chart, 451
Fractions and mixed numbers, checking, 56-57; location tests on, 5-6, 193-195; reviews of, 227, 374; Self-Help Lessons on: 415-420 (addition of), 420-423 (subtraction of), 424-431 (multiplication of), 431-435 (division of)

Gas meters, 65-66; problems on, 66-67
Graphs, 290-303. *See also* Circle graph, Bar graph, Line graph, Picture graph

Improper fraction, defined, 414
Indorsement, of a check, 85-86, of a promissory note, 274
Integers, checking of, 47-49; defined, 414; location tests on, 3-5, 192-193; review of, 373-374; Self-Help Lessons on, 388-409
Interest, 103-108, 242-247, 250, 251-258, 277-279, 307-318; formula for, 251-254; Self-Help Lessons on, 460
Interest slip, 105
Interest tables, 263-265; using, 262, 266-270
Interest term, defined, 101
Investments, 243-246, 307-318

Kilowatt-hour, 62

Leg (of an angle), defined, 356
Linear measures, 130, 132-141, 161-166; table of, 470
Line graph, the, 296-297
Location Tests, explained, 2-3, 191-192; in decimals, 7-8, 195-196; in fractions and mixed numbers, 5-6, 193-195; in integers, 3-5, 192-193; in percentage, 197-203
Lowest terms, explained, 416

Maker (of a note), defined, 273
Margin, defined, 325

Measurement, 130-141, 152-159, 161-170, 220-225, 229, 334-336, 345-356; tests on, 8-9, 14-16, 203-204, 208-210. *See also* Angles, Angular Measurement, Area, Volume
Mensuration. *See* Measurement
Meters. *See* Electric meters, Gas meters
Mixed number, defined, 414
Monthly statement, 81-82
Multiplication, checking of, 48, 410; review of the uses of, 237-239; Self-Help Lessons on: 396-400 (whole-number), 424-431 (with fractions), 437 (of decimals), 444 (of denominate numbers)

Numerator, defined, 414

Oblique triangles, 157
Obtuse angles, 154, 374
Oral drills, 41, 58, 77, 98, 125-126, 159, 177, 225, 233, 258, 282, 304-305, 332, 357, 376-377
Overdrawing (of an account), 86
Overhead, 324

Parallel lines, defined, 447
Parallelogram, area of a, 154-156, 447-448, defined, 447
Par value, 308
Payee (of a note), defined, 273
Per annum, explained, 460
Perimeter, defined, 446
Percentage, checking, 59; reviews of, 43-46, 115-119, 228-229, 249, 320-323, 370-380; Self-Help Lessons on, 450-461; tests on, 11-13, 197-203; third use of, 34-40
Perpendicular, meaning of, 152, 355
Pi, defined, 163
Pictogram. *See* Picture graph
Picture graph, the, 297-298
Principal, meaning of, 101, 310
Prisms, explained, 214-215
Problem Scales, explained, 32; list of: I, 32-34; II, 54-55; III, 68-70; IV, 96-97; V, 123-124; VI, 150-151; VII, 174-177; VIII, 218-220 IX, 231-232; X, 260-262; XI, 284-285; XII, 319-320; XIII, 335-336; XIV, 371-372
Problem solving, helps in, 29-32, 171-174, 217-218, 369-370
Proceeds, defined, 278
Profit and loss, 323-331
Progress Chart, explained, 25-27; blank charts, 483-488
Promissory notes, 271-276
Protractor, defined, 349; use of, 349-353

Radius, defined, 161, 354
Rate, interest, 243-246, 311-317; formula for, 312-314
Reciprocal, defined, 414
Rectangle, area of a, 152-154, 447-448; defined, 447
Rectangular prisms and solids. *See* Solids
Retail price, explained, 461
Reviews (other than Self-Help Lessons), of angular measurement, 362-365; of decimals, 228, 375; of denominate numbers, 378-379; of fractions, 227, 374; of integers, 373-374; of mensuration, 229, 334-335; of percentage, 34-40, 43-46, 115-119, 228-229, 249, 320-323, 379-380; general, 19-23, 71-72, 128-129, 187-189, 227-229, 235-241, 286-289, 339-343, 373-375, 381, 383-386
Review Tests, on checking, 17, 211; on denominate numbers, 10, 205-206; on estimating, 212, 366; on mensuration, 14-16, 208-210; on percentage, 11-13; on tables of measure, 8-9, 203-204
Right angles, 152

Savings accounts, 100-110; computing interest on, 103-108
Scale drawing, 138-141
Self-direction, 366-367
Self-Help Index, explained, 19, 471
Self-Help Study and Practice Lessons, explained, 3, 387
Self-Testing Drills, explained, 24-27; list of, *1*, 28; *2*, 42; *3*, 50; *4*, 60; *5*, 67; *6*, 78; *7*, 88; *8*, 99; *9*, 113; *10*, 126; *11*, 143; *12*, 160; *13*, 170; *14*, 178; *15*, 186; *16*, 216; *17*, 226; *18*, 234; *19*, 217; *20*, 259; *21*, 270; *22*, 282; *23*, 305; *24*, 318; *25*, 333; *26*, 338; *27*, 358; *28*, 368; *29*, 377; *30*, 382
Selling price, 325
Semicircle, defined, 356
Solids, 214-216, 449; measurement of, 220-224, 449-450
Square, defined, 446
Square measure, 153-154; table of, 470
Standards, for Location Tests, explained, 3-4; for Problem Scales, explained, 32; for Self-Testing Drills, explained, 25-26
Statement, monthly, bank, 81-82
Stocks, 308-309
Stubs, check, 83-84, 91-93
Subtraction, checking of, 48, 410; review of the uses of, 236-237; Self-Help Lessons on: 392-395 (whole-number), 420-423 (with fractions), 436 (with decimals), 443 (with denominate numbers)

Tests. *See* Location Tests, Review Tests, Self-Testing Drills, True-False Tests
Time notes. *See* Promissory notes
Triangle, area of a right, 156-157; area of any, 157-159; the oblique, 157; Self-Help Lessons on, 448
Triangular prisms. *See* Solids
True-False Tests, 141-142, 286-289

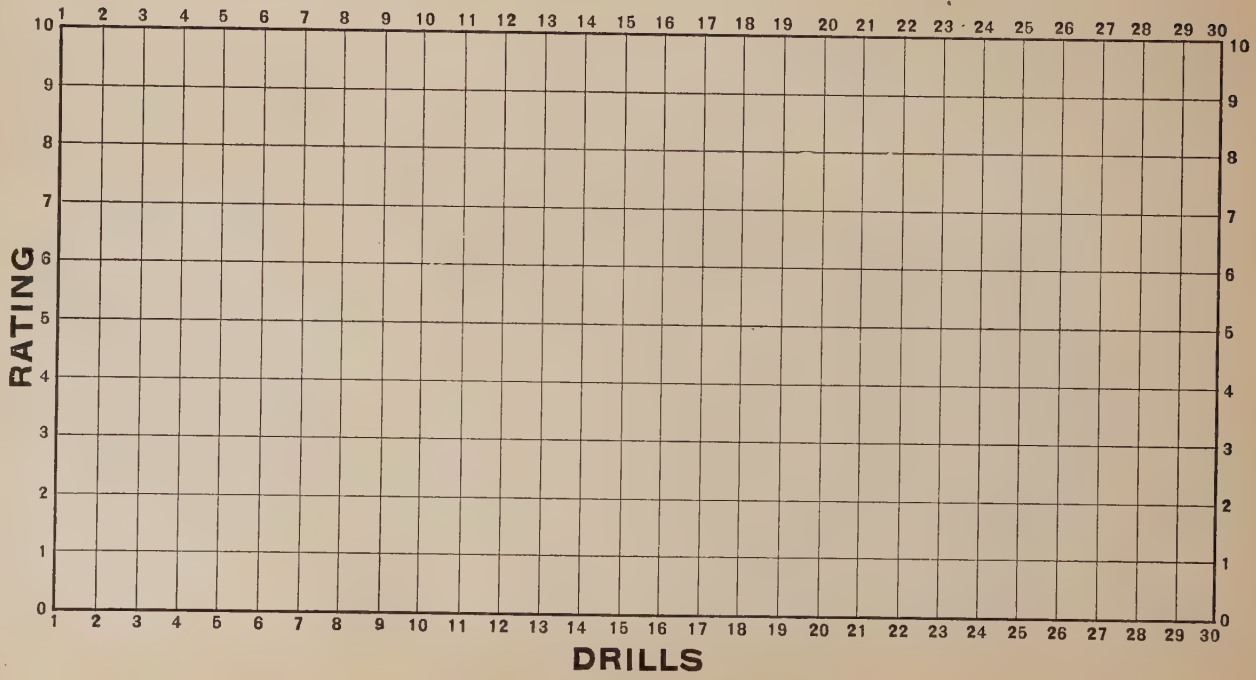
Vertex, defined, 354
Vocabulary, of arithmetic, Self-Help Lessons on, 414, 446-447
Volume, measurement of, 131; of rectangular solids, 220-224; Self-Help Lessons on, 449-450

Wholesaler, explained, 461

Yield (on bonds), 310-311

PROGRESS CHART

Name_____ Age_____ Grade_____ School_____



PROGRESS CHART

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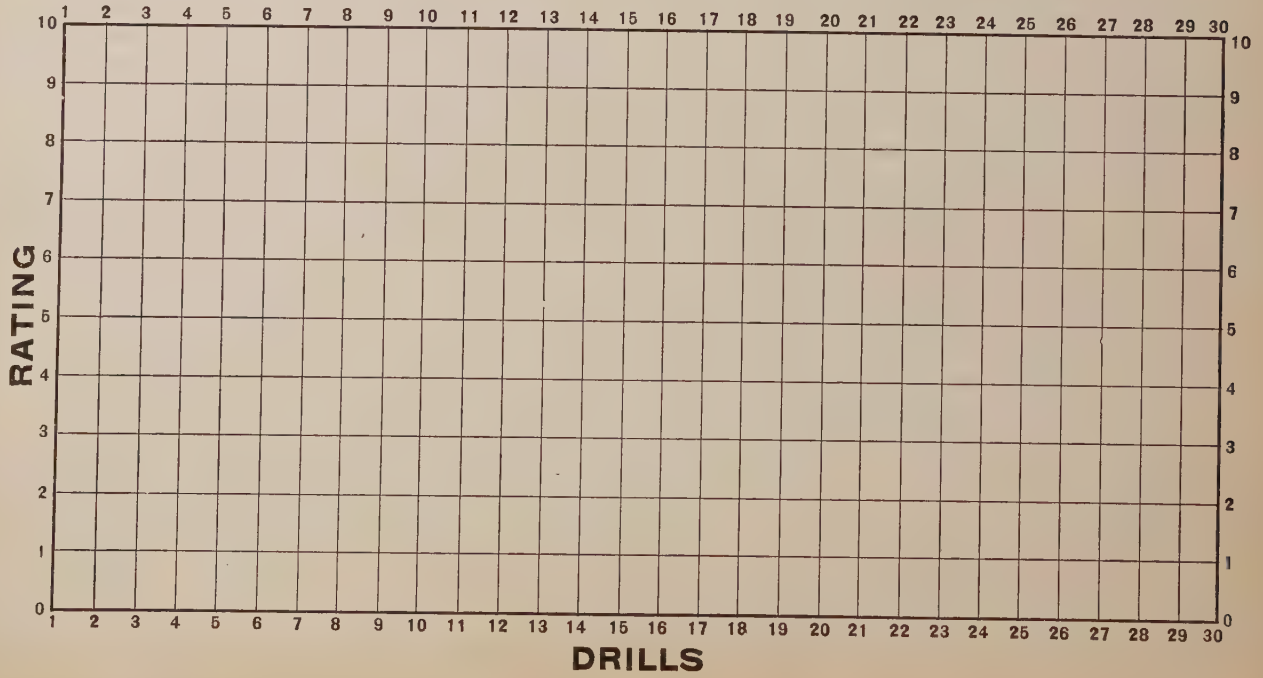
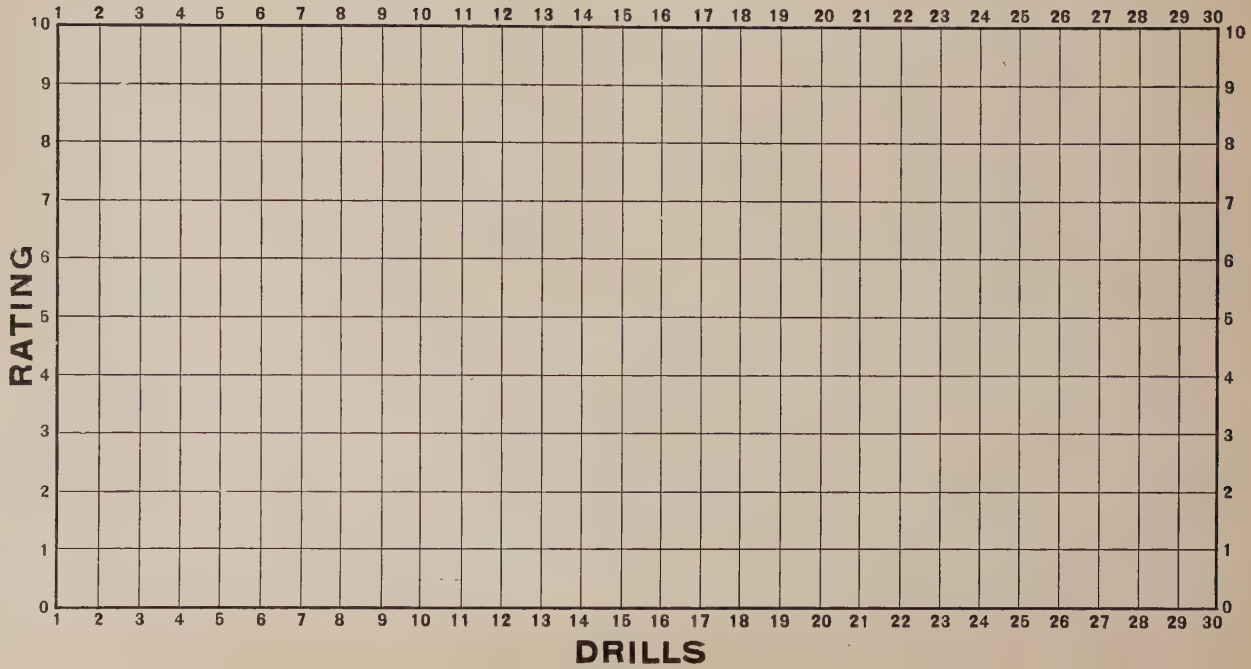


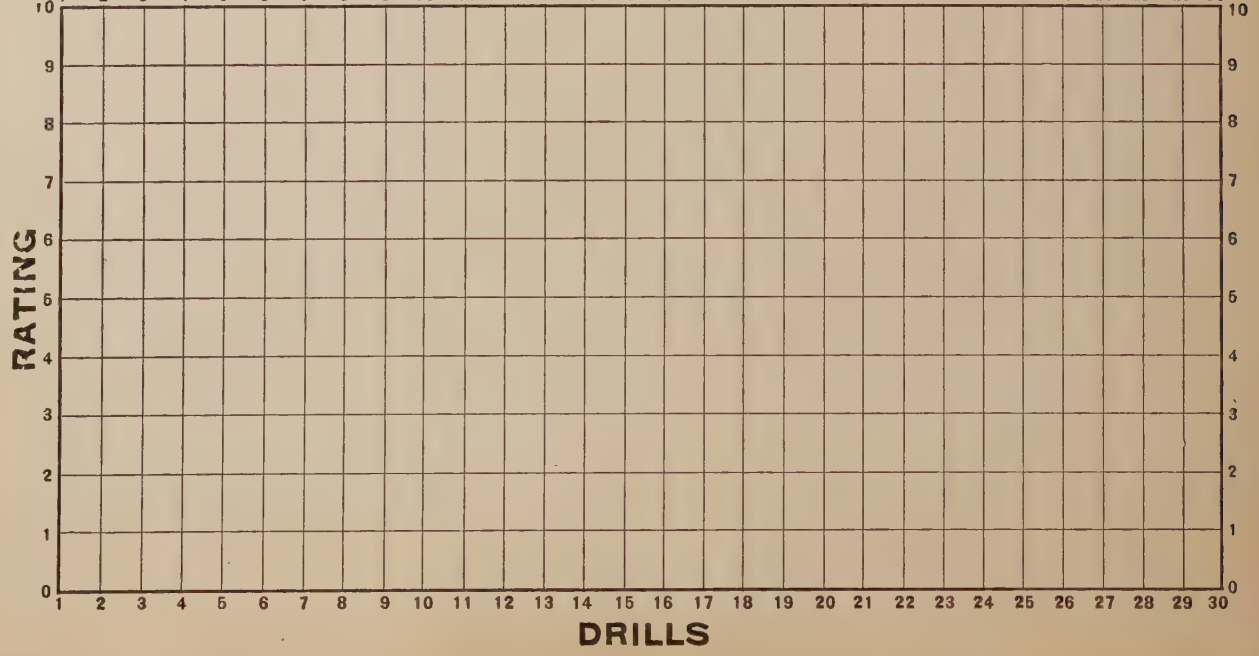
PROGRESS CHART

Name_____ Age_____ Grade_____ School_____



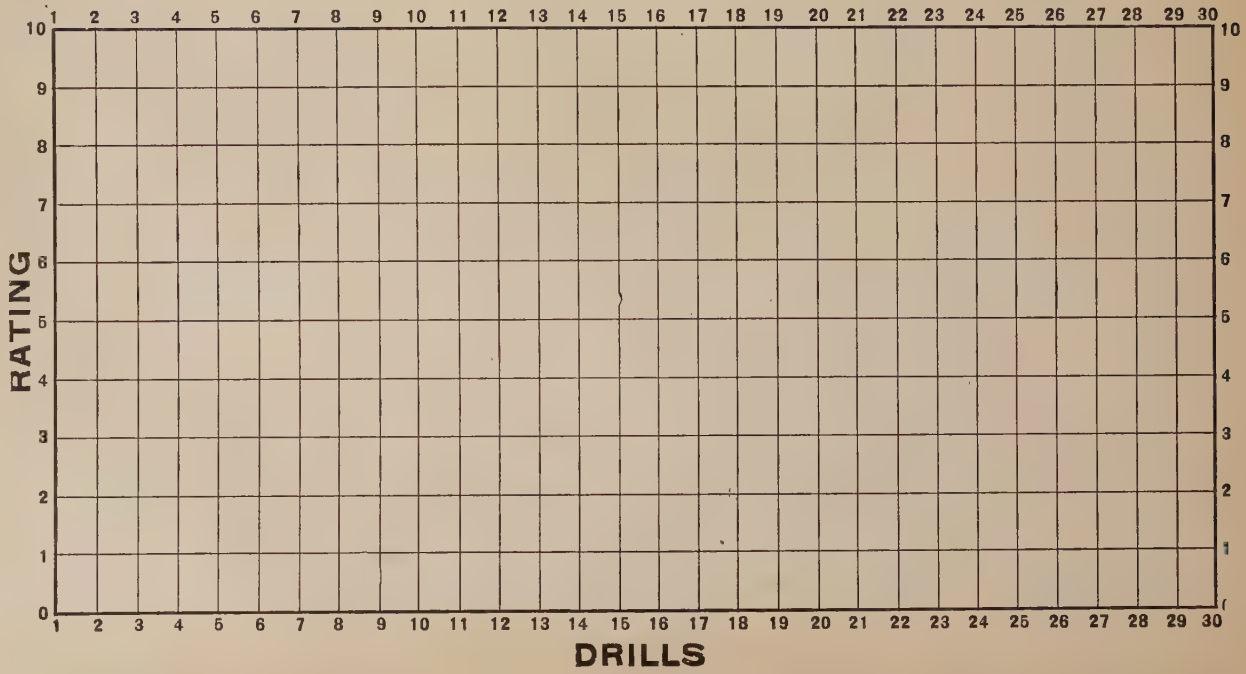
PROGRESS CHART

Name_____ Age_____ Grade_____ School_____



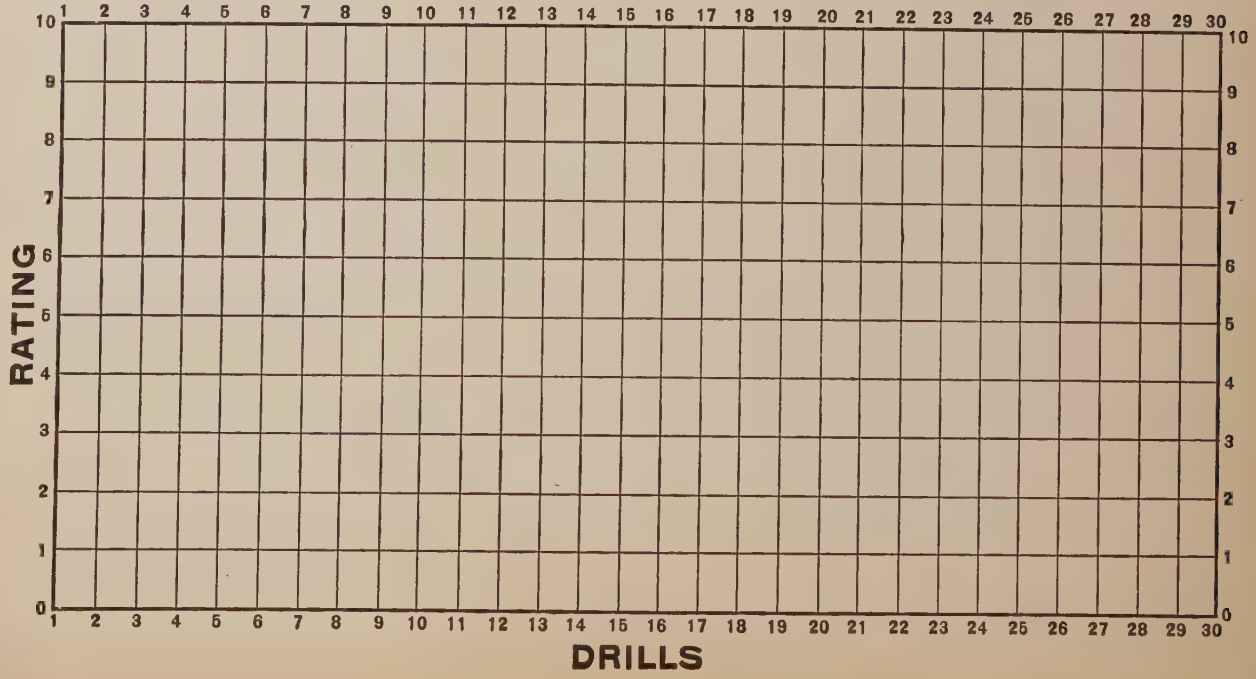
PROGRESS CHART

Name_____ Age_____ Grade_____ School_____



PROGRESS CHART

Name_____ Age_____ Grade_____ School_____



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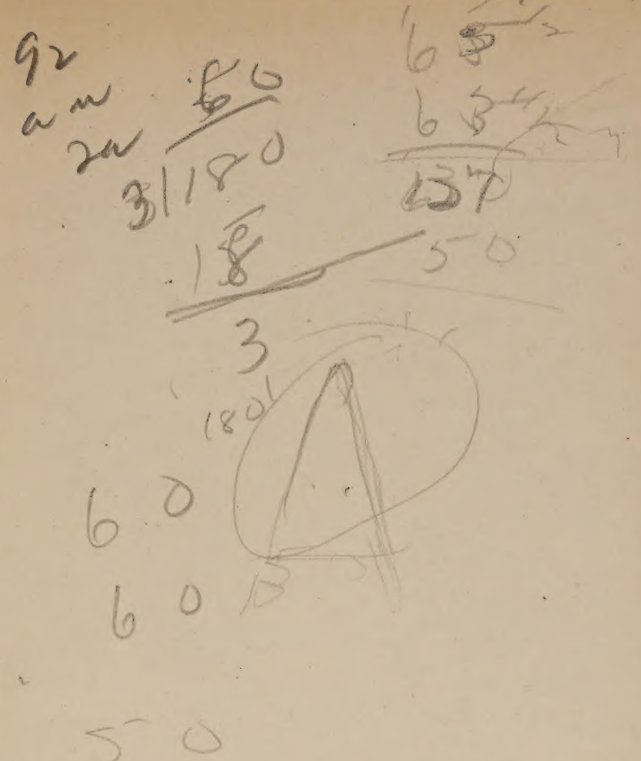
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